Contribution of the Hanbury Brown – Twiss experiment to the development of quantum optics

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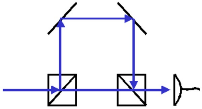
Light interference on a screen



- Electric field strength (complex analytic signal):

$$E(\boldsymbol{r},t) = |E(\boldsymbol{r},t)|e^{i\phi(\boldsymbol{r},t)}$$

- In general $E(\boldsymbol{r},t)$ is a wave packet:

$$E(\boldsymbol{r},t) = \int \mathrm{d}\omega \widetilde{E}(\omega) e^{i(\boldsymbol{k}\,\boldsymbol{r}-\omega t)}$$

The intensity distribution on the screen:

$$I = \frac{1}{2}\left(|E(\boldsymbol{r},t)|^2 + |E(\boldsymbol{r},t+\omega_0 L/c)|^2 + \mathrm{Re}\{E(\boldsymbol{r},t)^* E(\boldsymbol{r},t+\omega_0 L/c)\}\right)$$

For plane waves ($E = E_0 e^{i(\boldsymbol{k}\boldsymbol{r}-\omega t)}$)

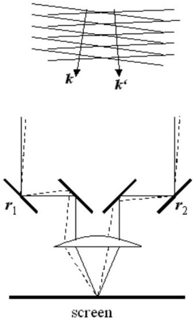
$$I = \frac{I_0}{2}(1+\cos(\omega L/c))$$

First order coherence

$$g^{(1)}(\tau) = \frac{\langle E(\boldsymbol{r},t)^* E(\boldsymbol{r},t+\tau)\rangle}{\langle E(\boldsymbol{r},t)^* E(\boldsymbol{r},t)\rangle}\,,$$

where $\langle ... \rangle$ means manifold or time average (ergodicity).

Measuring the angular diamater of a star



Spatial coherence of a wave front (astronomical Michelson interferometer):

$$
\begin{aligned}
I &= \frac{1}{2}\langle [E(\mathbf{r_1}) + E(\mathbf{r_2})]^* [E(\mathbf{r_1}) + E(\mathbf{r_2})] \rangle \\
&= \frac{1}{2}\left(\left\langle |E(\mathbf{r_1})|^2 \right\rangle + \left\langle |E(\mathbf{r_2})|^2 \right\rangle + 2\mathrm{Re}\{E(\mathbf{r_1})^* E(\mathbf{r_2})\} \right) \\
&= 2I_0 \left(1 + g^{(1)}(\mathbf{r}_1, \mathbf{r}_2) \right)
\end{aligned}
$$

For two wave fronts:

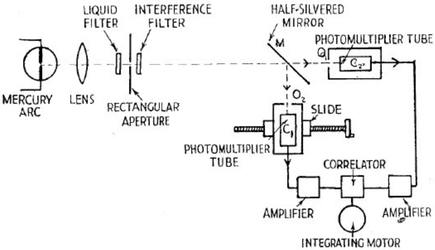
$$
I = \frac{1}{2}\left\langle |E_{\mathbf{k}}(\mathbf{r_1}) + E_{\mathbf{k}'}(\mathbf{r_1}) + E_{\mathbf{k}}(\mathbf{r_2}) + E_{\mathbf{k}'}(\mathbf{r_2})|^2 \right\rangle = 4I_0(1 + \cos([\mathbf{k} + \mathbf{k}']\mathbf{d}/2)\cos(kd\phi/2))
$$

where $\mathbf{d} = \mathbf{r}_1 - \mathbf{r}_2$. The term $\cos(kd\phi/2)$ depends on the angular diameter ϕ of the star.

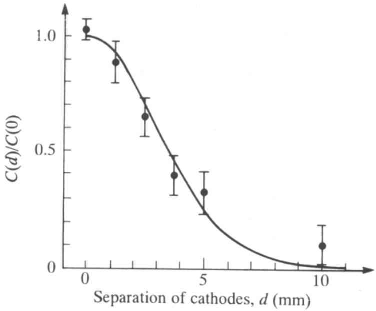
Drawback: the method has large uncertainty due to $\cos([\mathbf{k} + \mathbf{k}']\mathbf{d}/2)$

The Hanbury Brown – Twiss effect (1956)

What is the intensity correlation between different points of the wavefront?



$$C(d) = \frac{\langle \Delta J_1(t) \Delta J_2(t) \rangle}{\langle (\Delta J_1(t))^2 \rangle^{1/2} \langle (\Delta J_2(t))^2 \rangle^{1/2}}$$



What do we observe in the light intensity correlation experiment?

Answer: correlation of photocurrents!

Emission of photoelectrons

The Hamiltonian of the detector reads

$$H = \varepsilon_g |g\rangle\langle g| + \sum_q \varepsilon_{e,q} |e,q\rangle\langle e,q| - \sum_q [\mu_{(eq),g} |e,q\rangle\langle g| + \mathrm{H.c.}](E(t) + E^*(t))$$

The state vector is defined as: $|\psi\rangle = c_g|g\rangle + \sum_q c_{e,q}|e,q\rangle$

The Schrödinger equation for the state vector in the interaction picture

$$\begin{aligned} i\frac{d}{dt}c_g &= -\sum_q \int_0^\infty \mathrm{d}\omega \frac{\mu^*_{(eq),g}\tilde{E}^*(\omega)}{\hbar} e^{-i(\omega_q-\omega)t} c_{e,q} \\ i\frac{d}{dt}c_{e,q} &= -\int_0^\infty \mathrm{d}\omega \frac{\mu_{(eq),g}\tilde{E}(\omega)}{\hbar} e^{i(\omega_q-\omega)t} c_g \end{aligned}$$

where $\hbar\omega_q = \varepsilon_{e,q} - \varepsilon_g$, $\{c_g(t_0) = 1,\ c_{e,q}(t_0) = 0\}$

Apply the time-dependent perturbation theory, and compute $c_{e,q}(t)$, $(t = t_0 + \Delta t)$

$$c_{e,q}(t) = \frac{i\Delta t}{\hbar} \int \mathrm{d}\omega \mu_{(eq),g} \tilde{E}(\omega) e^{i(\omega_q-\omega)(t_0+\Delta t/2)} \frac{\sin\left(\frac{\omega_q-\omega}{2}\Delta t\right)}{\frac{\omega_q-\omega}{2}\Delta t}$$

The total transition probability per unit time to the excited state $((\omega-\omega')\Delta t \ll 1)$

$$\Pi(t) = \frac{1}{\Delta t}\sum_q |c_{e,q}(t)|^2 \approx \frac{1}{\Delta t}\int_0^\infty \mathrm{d}\omega_q |c_{e,q}(t)|^2 \varrho(\omega_q)$$

Emission of photoelectrons (cont.)

The probability of emitting a single photoelectron in the Δt interval ($\propto$ Fermi g.r.)

$$P(1,t,t+\Delta t) = 2\pi \frac{\varrho(\omega_0)}{\hbar^2} |\mu_{(ek_0),g}|^2 \cdot I(t) \cdot \Delta t \equiv \eta I(t) \Delta t$$

where $I(t) = \overline{|E(t)|^2}$, $E(t) = \int \mathrm{d}\omega \tilde{E}(\omega) \exp(-i\omega t)$, $k_0 = \omega_0 / c_0$

If there are N effective emitters

$$\begin{aligned} P(?,t,t+\Delta t) &= \sum_1^N \binom{N}{n} [\eta I(t)\Delta t]^n [1-\eta I(t)\Delta t]^{N-n} = 1 - [1-\eta I(t)\Delta t]^N \\ &= N\eta I(t)\Delta t - \frac{N(N-1)}{2!}[\eta I(t)\Delta t]^2 + \dots \end{aligned}$$

For $\eta I(t)\Delta t \ll 1$

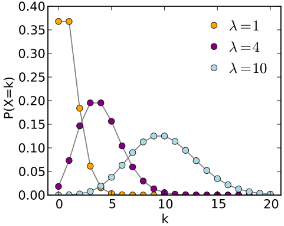
$$P(1,t,t+\Delta t) = N\eta I(t)\Delta t \equiv \eta I(t)\Delta t$$

The probability of emitting n photoelectrons in $[t, t+T]$, assuming independent emissions, follows the Poisson distribution

$$P(n,t,t+T) = \frac{[\eta U(t,T)]^n}{n!} \exp(-\eta U(t,T))$$

with $U(t,T) = \int_t^{t+T} \mathrm{d}\tau \, I(\tau)$ the fluence

Properties of the $P(n, t, t+T)$ distribution



Mean number of emitted photoelectrons:

$$\langle n \rangle = \eta \int_t^{t+T} \mathrm{d}\tau \, I(\tau)$$

Mean photoelectron number from $P(n, t, t+T)$

$$\langle n \rangle = \sum_{n=0}^{\infty} n \, P(n, t, t+T) = \eta U(t, T) = \eta \int_t^{t+T} \mathrm{d}\tau \, I(\tau)$$

Calculating the variance of the photoelectron number

$$\begin{aligned}
\langle n^2 \rangle &= \sum_{n=0}^{\infty} [n(n-1) + n] \, P(n, t, t+T) = [\eta U(t, T)]^2 + \eta U(t, T) \\
\langle (n - \langle n \rangle)^2 \rangle &= \langle n^2 \rangle - \langle n \rangle^2 = [\eta U(t, T)]^2 + \eta U(t, T) - [\eta U(t, T)]^2 = \eta U(t, T) \\
\Delta n &= \sqrt{\eta U(t, T)}
\end{aligned}$$

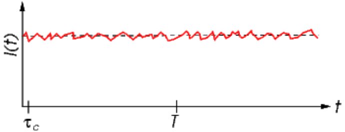
Fluctuating classical fields

The quantity $\eta U(t,T)$ is a random variable $W \longrightarrow$ ensemble averaging is necessary

$$\begin{aligned}
f(W) &: \quad \text{probability distribution} \\
\int_0^\infty f(W)\mathrm{d}W &= 1 \\
P(n,t,t+T) &= \left\langle \frac{W^n}{n!}\exp(-W)\right\rangle = \int \mathrm{d}W\, f(W)\,\frac{W^n}{n!}\exp(-W) \\
\langle n\rangle &= \langle W\rangle \\
\langle (n-\langle n\rangle)^2\rangle &= \langle W\rangle + \langle(\Delta W)^2\rangle
\end{aligned}$$

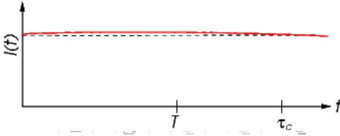
For $T \gg \tau_c$, τ_c is the correlation time of $I(t)$

$$\begin{aligned}
W &= \eta\langle I\rangle T\,, \quad \text{ergodic process} \\
P(n,t,t+T) &= \frac{[\eta\langle I\rangle T]^n}{n!}\exp(-\eta\langle I\rangle T)
\end{aligned}$$



For $T \ll \tau_c$, $I(t)$ is nearly constant in $[t, t+T]$

$$\begin{aligned}
W &= \eta I(t) T \\
P(n,t,t+T) &= \int \mathrm{d}I\, f(I)\,\frac{[\eta I T]^n}{n!}\exp(-\eta I T)
\end{aligned}$$



Fluctuating classical fields (cont.)

Example: classical thermal field

$$
\begin{aligned}
p(E)d^2E &= \frac{1}{2\pi\langle I\rangle}\exp(-|E|^2/\langle I\rangle)\mathrm{d}I\cdot\mathrm{d}\varphi \\
P(I) &= \frac{1}{\langle I\rangle}\exp(-I/\langle I\rangle) \\
P(n,t,t+T) &= \frac{(1+\eta\langle I\rangle T)^n}{(1+\eta\langle I\rangle T)^{n+1}}
\end{aligned}
$$

Boson statistics for the photoelectrons ...

Joint detection by two independent photodetectors:

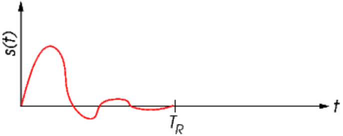
$$
\begin{aligned}
\Pi_1(\boldsymbol{r}_1,t_1)\Delta t_1 &= \eta_1 I(\boldsymbol{r}_1,t_1)\Delta t_1 \\
\Pi_1(\boldsymbol{r}_2,t_2)\Delta t_2 &= \eta_2 I(\boldsymbol{r}_2,t_2)\Delta t_2 \\
P_{\{1\}}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)\Delta t_1\Delta t_2 &= \eta_1\eta_2 I(\boldsymbol{r}_1,t_1)I(\boldsymbol{r}_2,t_2)\Delta t_1\Delta t_2
\end{aligned}
$$

For fluctuating fields

$$
P_{\{1\}}(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2)\Delta t_1\Delta t_2 = \eta_1\eta_2\langle I(\boldsymbol{r}_1,t_1)I(\boldsymbol{r}_2,t_2)\rangle\Delta t_1\Delta t_2
$$

Photoelectric current fluctuation

Usually the current of the electrons of the detector is amplified $\longrightarrow$ current pulses
The current pulses are not always resolved $\longrightarrow$ a continuous current $J(t)$ is studied



$$J(t) = \sum_r s(t - t_r)$$

Let's assume that n photoelectrons are emitted in time T. In a stationary field the average current is (emission prob. = $\mathrm{d}t_r/T$)

$$\langle J_{n,T}(t)\rangle_n = \sum_{r=1}^{n} \int_0^T s(t - t_r)\frac{\mathrm{d}t_r}{T} = \frac{1}{T}\sum_{r=1}^{n} \int s(t - t_r)\mathrm{d}t \approx \frac{1}{T}\sum_{r=1}^{n} Q = \frac{nQ}{T}$$

The average current

$$\langle J(t)\rangle = \sum_{n=0}^{\infty} P(n, t, t+T)\frac{nQ}{T} = \langle n\rangle \frac{Q}{T} = \eta\langle n\rangle Q$$

Photoelectric current correlation

Autocorrelation ($T_R \ll T$)

$$\begin{aligned}\langle \Delta J(t)\Delta J(t+\tau)\rangle &= \eta\langle I\rangle \int_{-\infty}^{\infty} s(t')s(t'+\tau)\mathrm{d}t' \\ &+ \eta^2 \iint_{-\infty}^{\infty} s(t')s(t'')\langle \Delta I(t)\Delta I(t+t'-t''+\tau)\rangle \mathrm{d}t'\mathrm{d}t''\end{aligned}$$

For slow response photodetector ($\tau_c \ll T_R$)

$$\langle \Delta J(t)\Delta J(t+\tau)\rangle = \left[\eta\langle I\rangle + \eta^2\langle(\Delta I)^2\rangle\tau_c\right]\int_{-\infty}^{\infty} s(t')s(t'+\tau)\mathrm{d}t'$$

Cross-correlation

$$\langle \Delta J_1(t)\Delta J_2(t+\tau)\rangle = \eta_1\eta_2 \iint_{-\infty}^{\infty} s(t')s(t'')\langle \Delta I_1(t)\Delta I_2(t+t'-t''+\tau)\rangle \mathrm{d}t'\mathrm{d}t''$$

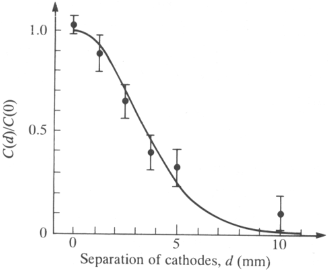
For slow response photodetector

$$\langle \Delta J_1(t)\Delta J_2(t+\tau)\rangle = \eta_1\eta_2\langle \Delta I_1\Delta I_2\rangle\tau_c \int_{-\infty}^{\infty} s(t')s(t'+\tau)\mathrm{d}t'$$

Explanation of the original HBT effect

Reacall : the normalized intensity correlation between two points of the wavefront

$$C(d) = \frac{\langle \Delta J_1(t)\Delta J_2(t)\rangle}{\langle(\Delta J_1(t))^2\rangle^{1/2}\langle(\Delta J_2(t))^2\rangle^{1/2}}$$



The autocorrelation is given by

$$\langle(\Delta J_i(t))^2\rangle = \eta_i \left[\langle I_i\rangle + \eta_i\langle(\Delta I_i)^2\rangle\tau_c\right] \int_{-\infty}^{\infty} s^2(t')\mathrm{d}t'$$

For unpolarized thermal field $\langle(\Delta I)^2\rangle = \frac{1}{2}\langle I\rangle^2$

$$\langle(\Delta J_i(t))^2\rangle = \eta_i\langle I_i\rangle \left[1 + \frac{1}{2}\eta_i\langle I_i\rangle\tau_c\right] \int_{-\infty}^{\infty} s^2(t')\mathrm{d}t'$$

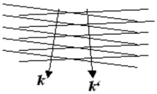
The cross cross correlation

$$\langle \Delta J_1(t)\Delta J_2(t)\rangle = \eta_1\eta_2\langle\Delta I_1\Delta I_2\rangle\tau_c \int_{-\infty}^{\infty} s^2(t')\mathrm{d}t'$$

Using the relation $\langle z_1^* z_1 z_2^* z_2\rangle = \langle z_1^* z_1\rangle\langle z_2^* z_2\rangle + \langle z_2^* z_1\rangle\langle z_1^* z_2\rangle$ (z_i Gaussian)

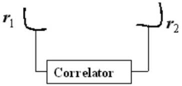
$$\langle \Delta J_1(t)\Delta J_2(t)\rangle = \frac{1}{2}\eta_1\eta_2\langle I_1\rangle\langle I_2\rangle\tau_c|\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, 0)|^2 \int_{-\infty}^{\infty} s^2(t')\mathrm{d}t'$$

Measuring the diameter of a star



Let's measure the normalized second order current correlation:

$$C(d) = \frac{\langle \Delta J_1(t) \Delta J_2(t) \rangle}{\langle (\Delta J_1(t))^2 \rangle^{1/2} \langle (\Delta J_2(t))^2 \rangle^{1/2}} = \eta \langle I \rangle \tau_c [1 + \cos(k\, d\, \phi)]$$



$$\langle (\Delta J_i(t))^2 \rangle = \eta_i \langle I_i \rangle \left[1 + \frac{1}{2} \eta_i \langle I_i \rangle \tau_c \right] \int_{-\infty}^{\infty} s^2(t') \mathrm{d}t'$$

$$\langle \Delta J_1(t) \Delta J_2(t) \rangle = \frac{1}{2} \eta_1 \eta_2 \langle \Delta I_1 \Delta I_2 \rangle \tau_c \int_{-\infty}^{\infty} s^2(t') \mathrm{d}t'$$

In case of two wave fronts:

$$\begin{aligned}
\langle \Delta I_1 \Delta I_2 \rangle &= \langle E(\mathbf{r_1})^* E(\mathbf{r_2})^* E(\mathbf{r_1}) E(\mathbf{r_2}) \rangle - \langle E(\mathbf{r_1})^* E(\mathbf{r_1}) \rangle \langle E(\mathbf{r_2})^* E(\mathbf{r_2}) \rangle \\
&= \langle [E_{\mathbf{k}} + E_{\mathbf{k'}}]^* (\mathbf{r_1}) [E_{\mathbf{k}} + E_{\mathbf{k'}}]^* (\mathbf{r_2}) [E_{\mathbf{k}} + E_{\mathbf{k'}}] (\mathbf{r_2}) [E_{\mathbf{k}} + E_{\mathbf{k'}}] (\mathbf{r_1}) \rangle \\
&- \langle [E_{\mathbf{k}} + E_{\mathbf{k'}}]^* (\mathbf{r_1}) [E_{\mathbf{k}} + E_{\mathbf{k'}}] (\mathbf{r_1}) \rangle \langle [E_{\mathbf{k}} + E_{\mathbf{k'}}]^* (\mathbf{r_2}) [E_{\mathbf{k}} + E_{\mathbf{k'}}] (\mathbf{r_2}) \rangle \\
&= 2 \left\langle |\tilde{E}_{\mathbf{k}}|^2 |\tilde{E}_{\mathbf{k'}}|^2 \right\rangle + \left\{ \left\langle \tilde{E}^*_{\mathbf{k}} \tilde{E}_{\mathbf{k}} \tilde{E}^*_{\mathbf{k'}} \tilde{E}_{\mathbf{k'}} \right\rangle e^{i(\mathbf{k} - \mathbf{k'})(\mathbf{r}_2 - \mathbf{r}_1)} + c.c. \right\} \\
&= 2 \langle I_{\mathbf{k}} \rangle \langle I_{\mathbf{k'}} \rangle [1 + \cos(k\, d\, \phi)]
\end{aligned}$$

Second order correlation function of quantized fields

Electric field operator for 1D, polarized e.m. field

$$\widehat{E}(z) = \widehat{E}^{(+)} + \widehat{E}^{(-)} = i\sum_k \left[\sqrt{\frac{\hbar}{2\omega\varepsilon_0 L}}\,\widehat{a}e^{ikz} + H.c.\right]$$

where $\widehat{E}^{(+)} \sim E(t)$ classical

Intensity : $I = \langle \widehat{E}^{(-)}(\boldsymbol{r})\widehat{E}^{(+)}(\boldsymbol{r})\rangle$

Second order correlation function:

$$G^{(2)} = \langle : \widehat{E}^{(-)}(\boldsymbol{r}_1)E^{(+)}(\boldsymbol{r}_1)\widehat{E}^{(-)}(\boldsymbol{r}_2)\widehat{E}^{(+)}(\boldsymbol{r}_2) : \rangle \equiv \langle \widehat{E}^{(-)}(\boldsymbol{r}_1)E^{(-)}(\boldsymbol{r}_2)\widehat{E}^{(+)}(\boldsymbol{r}_2)\widehat{E}^{(+)}(\boldsymbol{r}_1)\rangle$$

where $[: \widehat{a}^\dagger\widehat{a}\,\widehat{a}^\dagger\widehat{a} :] = \widehat{a}^\dagger\widehat{a}^\dagger\,\widehat{a}\,\widehat{a}$

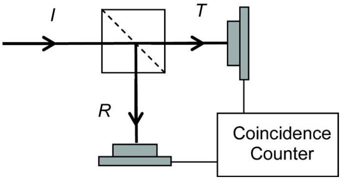
Stellar interferometer: assume $\langle \widehat{n}_k\rangle = \langle \widehat{n}_{k'}\rangle \equiv \langle \widehat{n}\rangle$, furthermore $\langle \widehat{n}_k^2\rangle = \langle \widehat{n}_{k'}^2\rangle \equiv \langle \widehat{n}^2\rangle$

$$G^{(2)} = 2\mathcal{E}^4\left(\langle \widehat{n}^2\rangle - \langle \widehat{n}\rangle + \langle \widehat{n}\rangle^2\left\{1 + \cos[(\boldsymbol{k} - \boldsymbol{k}')(\boldsymbol{r}_1 - \boldsymbol{r}_2)]\right\}\right)$$

light source	$G^{(2)}$	min	max
thermal	$2n^2 + n^2(1+\chi)$	$2n^2$	$4n^2$
laser	$n^2 + n^2(1+\chi)$	n^2	$3n^2$
single photon	$1+\chi$	0	2

where $n = \langle \widehat{n}\rangle$, $\chi = \cos[(\boldsymbol{k} - \boldsymbol{k}')(\boldsymbol{r}_1 - \boldsymbol{r}_2)]$

Observation of correlations through photon counting



Counting rates

$$P_T(0,T) = R_T(0,T)\Delta t = \left(\frac{N_T(0,T)}{\Delta T}\right)\Delta t$$

$$P_R(\tau,T) = R_R(\tau,T)\Delta t = \left(\frac{N_R(\tau,T)}{\Delta T}\right)\Delta t$$

$$P_{TR}(\tau,T) = R_{TR}(\tau,T)\Delta t = \left(\frac{N_{TR}(\tau,T)}{\Delta T}\right)\Delta t$$

Correlation between photo-counts (mode V is in vacuum)

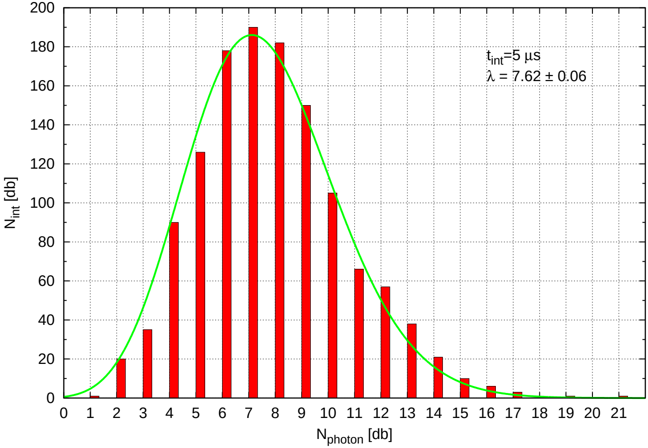
$$g^{(2)}_{T,R}(\tau) = \frac{\langle : \widehat{I}_T(0)\widehat{I}_R(\tau) : \rangle}{\langle \widehat{I}_T(0)\rangle\langle \widehat{I}_R(\tau)\rangle}\,, \qquad g^{(2)}_{T,R}(0) = \frac{\langle \widehat{a}^\dagger_T \widehat{a}^\dagger_R \widehat{a}_T \widehat{a}_R\rangle}{\langle \widehat{a}^\dagger_T \widehat{a}_T\rangle\langle \widehat{a}^\dagger_R \widehat{a}_R\rangle} = \frac{\langle \widehat{n}_I(\widehat{n}_I - 1)\rangle}{\langle \widehat{n}_I\rangle^2}$$

where

$$\widehat{a}_R = \frac{\widehat{a}_I + \widehat{a}_V}{\sqrt{2}}\,, \qquad \widehat{a}_T = \frac{\widehat{a}_I - \widehat{a}_V}{\sqrt{2}}$$

Some measurements (by Krisztián Lengyel)

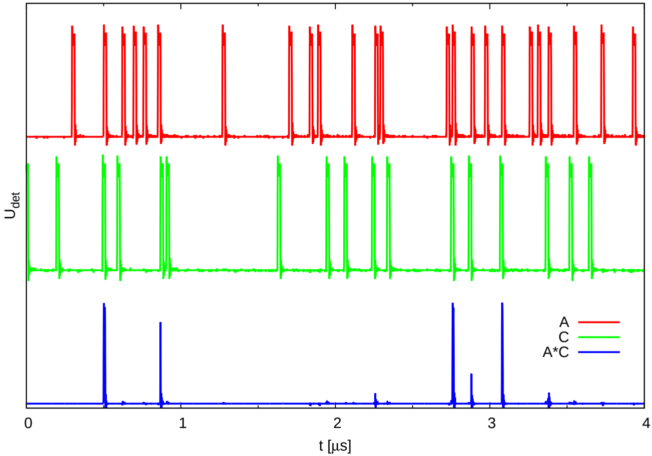
The probabilty distribution $P(n, t, t + T)$ for laser light



here $T = 5\mu\mathrm{s}$, detector pulse length $\approx 18\mathrm{ns}$, detector dead time $\approx 40\mathrm{ns}$, sampling time bin = $400\mathrm{ps}$

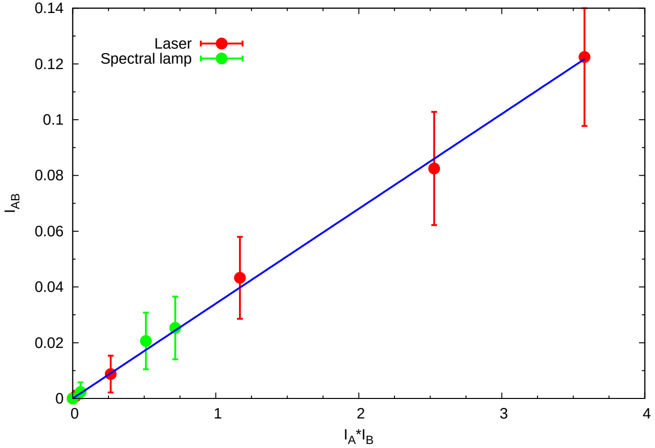
Some measurements (cont.)

Hanbury Brown–Twiss effect from photon counting



Some measurements (cont.)

There are no quantum effects here



Roy J. Glauber: the quantum theory of optical coherence

Coherent states

$$|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{n!}|n\rangle\,, \qquad \langle\alpha|\beta\rangle = e^{-|\alpha|^2/2-|\beta|^2/2-\alpha^*\beta}\,, \quad \text{non-orthogonal}$$

Completeness: $\widehat{I} = \frac{1}{\pi}\iint \mathrm{d}^2\alpha|\alpha\rangle\langle\alpha| \longrightarrow$ good to expand the quantum state of light

Glauber-Sudarshan $P(\alpha)$ representation: $\widehat{\varrho} = \iint \mathrm{d}^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$

Examples:

source type	$P(\alpha)$
laser light β	$\delta(\alpha-\beta)$
thermal light	$\prod_{\boldsymbol{k},s}\frac{1}{\pi\langle\widehat{n}_{\boldsymbol{k},s}\rangle}e^{-\|\alpha\|^2/\langle\widehat{n}_{\boldsymbol{k},s}\rangle}$
number state $\|n\rangle$	$\frac{n!}{2\pi r(2n)!}e^{r^2}\left(-\frac{\partial}{\partial r}\right)^{2n}\delta(r)$

Higher order correlation function for electromagnetic fields

$$G^{(n)}_{\mu_1\dots\mu_{2n}}(x_1\dots x_n, x_{n+1}\dots x_{2n}) = \mathrm{Tr}\{\widehat{\varrho}\widehat{E}^{(-)}_{\mu_1}(x_1)\dots\widehat{E}^{(-)}_{\mu_n}(x_n)\widehat{E}^{(+)}_{\mu_{n+1}}(x_{n+1})\dots\widehat{E}^{(+)}_{\mu_{2n}}(x_{2n})\}$$

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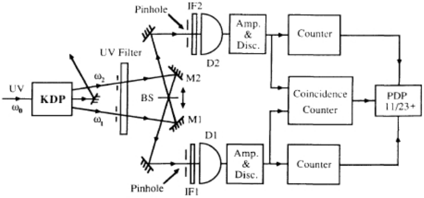
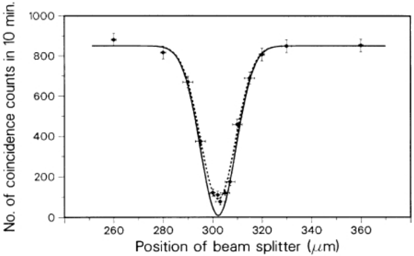
The Hong-Ou-Mandel interferometer

Interference of single-photon wave packets



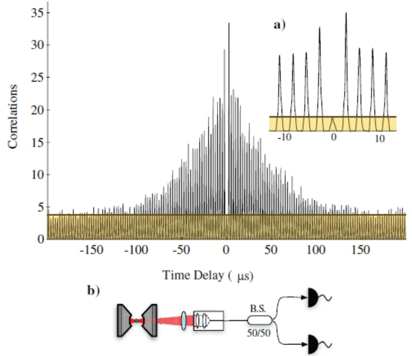


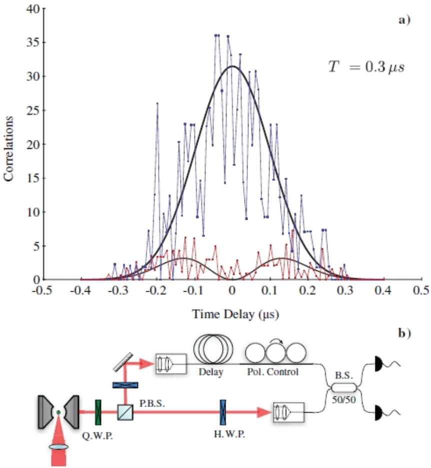
- Two photons impinge at the two imputs of the beam splitter: $|1_a, 1_b\rangle$
- Passing the beam splitter ($R = T = 50\%$) the photons „stick together”:

$$|\psi_{\text{out}}\rangle = \frac{i}{\sqrt{2}}(|2_A, 0_B\rangle + |0_A, 2_B\rangle)$$

The coincidence tends to zero if the two photons arrive at the same time to the beam splitter.

Design of single photon wave packets





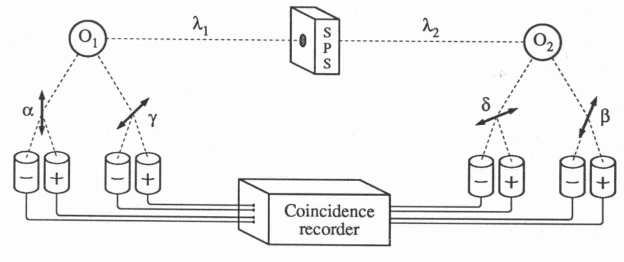
- Single photons source with an atom trapped inside a cavity: the atom is periodically excited (T = 3 μs), to get a stream of photons
- $G^{(2)} = \langle P_{\mathrm{D1}}(t) P_{\mathrm{D2}}(t-\tau) \rangle$ is measured in a HBT setup to prove the single photon state
- A Hong-Ou-Mandel interferometer is used to test the overlap between the photon wave packets.

Experimental test of the Bell's inequality

Einstein's locality: the outcome of a measurement cannot depend on parameters controlled by faraway agents.

Two observers test the polarization of photons emitted in an SPS cascade.

Observer A measures α, γ polarizations, the outcome ± 1.
Observer B measures β, δ polarizations, the outcome ± 1.



If local hidden variables exist (Clauser, Horne, Shimony, Holt)

$$a_j b_j + b_j c_j + c_j d_j - d_j a_j \equiv \pm 2$$

After averaging

$$|\cos 2(\alpha - \beta) + \cos 2(\beta - \gamma) + \cos 2(\gamma - \delta) - \cos 2(\delta - \alpha)| \leq 2$$

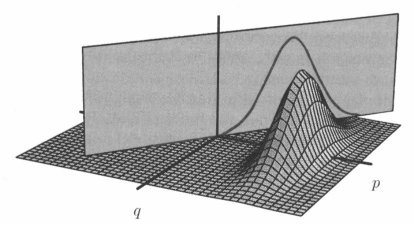
QM says: lhs = $2\sqrt{2}$, agrees with the measurement

Quantum state reconstruction of light

Quantum state reconstruction: find the density operator $\widehat{\varrho}$ ... or something equivalent

Quasiprobability distributions:

- Glauber-Sudarshan $P(\alpha)$ representation : not good, it's an ugly distribution
- Wigner's function, $W(q,p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(ipx)\langle q - x/2|\widehat{\varrho}|q + x/2\rangle \mathrm{d}x$
- The $Q(\alpha)$ function : $Q(\alpha) = \langle\alpha|\widehat{\varrho}|\alpha\rangle$



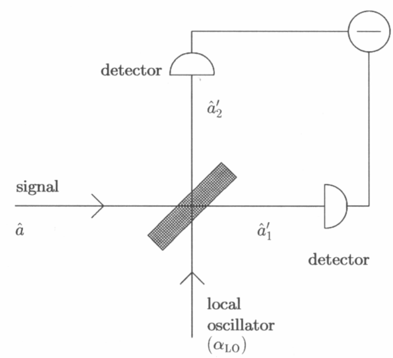
How to reconstruct the phase space distributions ?

Measure the quadrature distributions:

$$\widehat{q}_\theta = \widehat{q}\cos\theta + \widehat{p}\sin\theta$$

From the marginal distributions $\mathrm{pr}(q,\theta)$ a quasiprobability distribution between W and Q can be reconstructed.

Homodyne detection for measuring $\mathrm{pr}(q,\theta)$



There are two fields: 1. signal; 2. the local oscillator

Very important: they are phase locked

Beam splitter mixing:

$$\widehat{a}_1' = \frac{\widehat{a} + \alpha_{\mathrm{LO}}}{\sqrt{2}}$$
$$\widehat{a}_2' = \frac{\widehat{a} - \alpha_{\mathrm{LO}}}{\sqrt{2}}$$

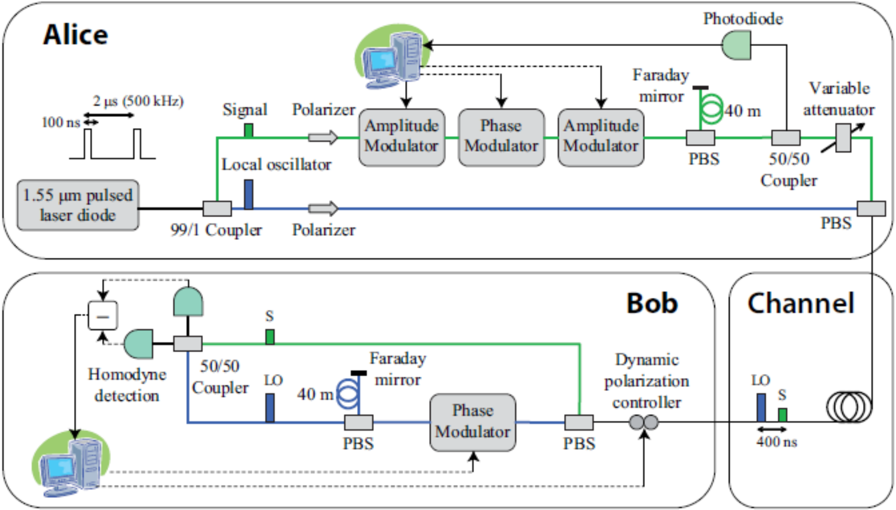
Measure the intensity at the output ports:

$$\widehat{I}_1 = \frac{1}{2}\{\langle\widehat{a}^\dagger\widehat{a}\rangle + |\alpha_{\mathrm{LO}}|^2 + |\alpha_{\mathrm{LO}}|(\widehat{a}e^{-i\theta} + \widehat{a}^\dagger e^{i\theta})\}\,,\ \widehat{I}_2 = \frac{1}{2}\{\langle\widehat{a}^\dagger\widehat{a}\rangle + |\alpha_{\mathrm{LO}}|^2 - |\alpha_{\mathrm{LO}}|(\widehat{a}e^{-i\theta} + \widehat{a}^\dagger e^{i\theta})\}$$

Record the difference

$$\widehat{I}_1 - \widehat{I}_2 = |\alpha_{\mathrm{LO}}|(\widehat{a}e^{-i\theta} + \widehat{a}^\dagger e^{i\theta}) \approx \widehat{q}_\theta$$

Continuous variable quantum key distribution



Thank you for your attention