Chapter 4. Global Positioning System

- *How does the Global Positioning System [GPS] work?*
- *How accurately can I locate myself on Earth with the GPS?*
- *Why does the GPS not work when I "turn off general relativity"?*
- *What are practical uses of the GPS?*

CHAPTER

4 Global Positioning System

Edmund Bertschinger & Edwin F. Taylor *

There is no better illustration of the unpredictable payback of fundamental science than the story of Albert Einstein and the Global Positioning System [GPS] . . . the next time your plane approaches an airport in bad weather, and you just happen to be wondering "what good is basic science," think about Einstein and the GPS tracker in the cockpit, guiding you to a safe landing.

—Clifford Will

4.1 ■ OPERATION OF THE GLOBAL POSITIONING SYSTEM

Relativistic effects of altitude and speed on clock rates

General relativity: Crucial to the operation of the GPS

Do you think that general relativity concerns only events far from common experience? Think again. Your hand-held Global Positioning System (GPS) receiver "listens" to overhead satellites and tells you where you are—anywhere on Earth! In this chapter you show that the operation of the GPS system depends fundamentally on general relativity.

GPS satellite system

The Global Positioning System includes a network of 24 operating satellites in circular orbits around Earth with orbital period of 12 hours, distributed in six orbital planes equally spaced in angle (Figure 1). Each satellite carries an operating atomic clock (along with several backup clocks) and emits a timed signal that also codes the satellite's location. By analyzing signals from at least four of these satellites (Box 1), your hand-held receiver on Earth displays your own location (latitude, longitude, and altitude). Consumer receivers provide a horizontal position accurate to approximately 5 meters. Among its almost endless applications, the GPS guides your driving, flying, hiking, exploring, rescuing, mapmaking, and locating your dog.

General relativity: position and motion effects

The timing accuracy required for the performance of the GPS is so great that general relativistic effects are central to its operation: First relativistic

*Draft of Second Edition of *Exploring Black Holes: Introduction to General Relativity*

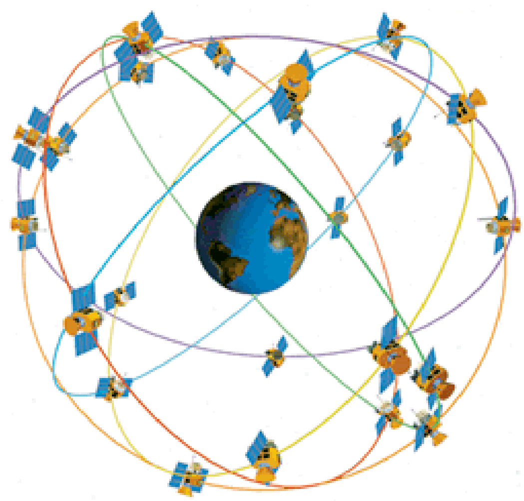

FIGURE 1 Schematic plot of GPS satellites in 12-hour orbits around Earth. Not to scale.

effect: different observed clock rates at different altitudes. Second relativistic effect, different observed rates of clocks in relative motion. (In this first analysis of the GPS, we assume all signal propagation occurs in a vacuum.)

4.2 ■ STATIONARY CLOCKS

Warping of t-coordinate at different altitudes.

Simplest case: clock on tower and no Earth rotation.

The Global Positioning System depends on the reception by a receiver on Earth's surface of microwave signals from multiple overhead satellites. Begin with the simplest possible case: Earth does not rotate and the higher clock is not in a satellite but rather sits on top of a tower at Higher r-coordinate, r_{H}. The tower clock communicates with us on Earth, at Lower r-coordinate, r_{L}. Calculate the radially-downward dr/dt of microwaves that move from the top to the bottom of the tower. Light and microwaves move at this same rate. For these conditions, $d\phi = 0$ and for light, $d\tau = 0$. Then the Schwarzschild metric, equation (3.5), yields the following radial motion in global coordinates:

$$\frac{dr}{dt} = -\left(1 - \frac{2M}{r}\right) \qquad \text{(light moving radially inward)} \qquad (1)$$

Map speed of light $\neq 1$.

Is equation (1) a surprise? For the first time in our study of relativity, calculated light speed differs from one meter of distance per meter of time. Ah, but the expression dr/dt in *global* coordinates is a unicorn, not measured by anyone. We need to go back and determine the observable wristwatch time lapse between two flashes emitted from the clock at the top of the tower. As usual, the metric converts from global coordinate separations (on its right side) to measured wristwatch time lapse (on its left side).

Box 1. Practical Operation of the Global Positioning System

The goal of the Global Positioning System (GPS) is to determine your position on Earth in three dimensions: east-west, north-south, and vertical—longitude, latitude, and altitude. Signals from three overhead satellites provide this information. Each satellite emits a signal that encodes its local time of emission and the satellite's position in global coordinates at that emission event, this position continually revised using data uploaded from control stations on the ground. The local clock in your hand-held GPS receiver records the local time of reception of each signal, then subtracts the emission t (encoded with the incoming signal) to determine the lapse in t-coordinate and hence how far the signal has traveled at the speed of light in global coordinates. This is the map distance the satellite was from your position when it emitted the signal. In effect, the receiver constructs three spheres from these distances, one sphere centered on the emission point of each satellite. Simple triangulation locates the point where the three spheres intersect. That point is your location in global coordinates.

Of course there is a wrinkle: The local clock in your hand-held receiver is not nearly so accurate as the atomic clocks carried in a satellite. For this reason, the signal from a fourth overhead satellite is used to correct the local clock in your receiver. This fourth signal enables your hand receiver to process GPS signals as though it contained an atomic clock.

Signals exchanged between atomic clocks at different altitudes and moving at different speeds are subject to general relativistic effects. Neglect these effects and the GPS is useless (Box 3).

Tower clock emits two downward flashes.

The clock at the top of the tower emits two flashes radially downward (emission events A and B) differentially close together in global t-coordinate: $dt_{\rm AB}$. For this top tower clock, $dr = 0$ and $d\phi = 0$, the metric tells us the corresponding wristwatch time lapse $d\tau_{\rm H}$ recorded on the tower clock:

$$d\tau_{\rm H} = \left(1 - \frac{2M}{r_{\rm H}}\right)^{1/2} dt_{\rm AB} \qquad (d\phi = 0,\ dr = 0) \tag{2}$$

Map t-lapse between flashes is constant during descent.

Figure 2 traces on an $[r,t]$ slice the radially-downward global worldlines of the two flashes emitted by the tower clock at events A and B. The Earth clock receives these flashes at events C and D with t-coordinate separation $dt_{\rm CD}$. Equation (1) tells us that these worldlines have identical slopes (the radial global coordinate speed of light has the same value) at every intermediate r-coordinate. As a result, the two worldlines are parallel at every r-coordinate on the $[r,t]$ slice, so the global t-coordinate separation between them maintains its initial value $dt_{\rm AB}$. The two flashes arrive at the ground with the initial difference in global t-coordinate.

$$dt_{\rm CD} = dt_{\rm AB} \tag{3}$$

The clock on Earth's surface is also at fixed $r_{\rm L}$. Therefore its wristwatch time lapse on the ground between the reception of events is similarly given by (2):

$$d\tau_{\rm L} = \left(1 - \frac{2M}{r_{\rm L}}\right)^{1/2} dt_{\rm CD} = \left(1 - \frac{2M}{r_{\rm L}}\right)^{1/2} dt_{\rm AB} \tag{4}$$

The final step in equation (4) comes from (3). Equations (2) and (4) give us the relation between wristwatch time lapses of stationary clocks at higher and lower global r-coordinates:

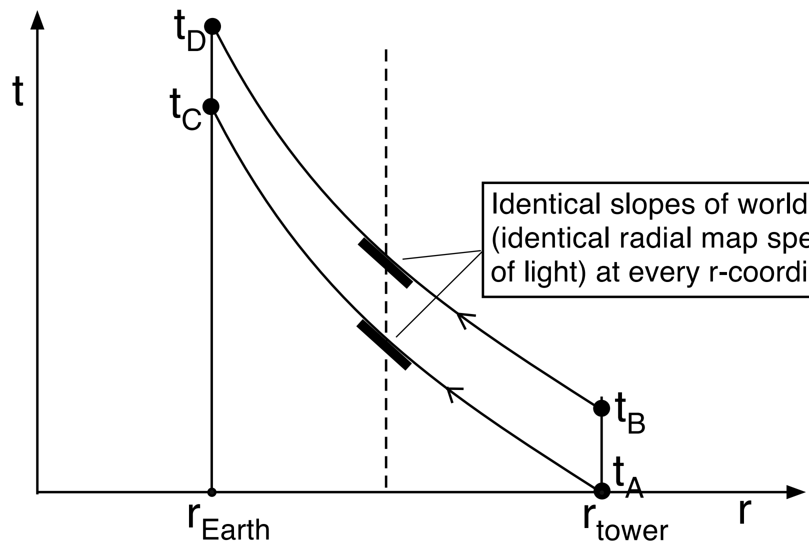

FIGURE 2 Schematic plot in Schwarzschild global coordinates (t, r) of worldlines of two sequential flashes moving downward from the top to the bottom of a tower. According to equation (1) the r-coordinate (map) speed depends only on the r-coordinate. As a result, the map t-coordinate difference between receptions of the flashes is identical to the map t-coordinate difference between emissions. However, the *wristwatch* time between the reception events C and D, measured by the bottom observer, is different from the *wristwatch* time between emission events A and B, measured by the top observer, equation (5). The figure greatly exaggerates the variation of radial map light speed with r-coordinate.

$$\frac{d\tau_{\rm H}}{d\tau_{\rm L}} = \left(\frac{1 - \dfrac{2M}{r_{\rm H}}}{1 - \dfrac{2M}{r_{\rm L}}}\right)^{1/2} \qquad \text{(stationary clocks)} \qquad (5)$$

Wristwatch time between flashes is different at different r.

The lapse dt in Schwarzschild global map t-coordinate between flashes is the same at the locations of upper and lower clocks, but the *wristwatch* time is different as recorded on these different clocks. Indeed, $r_{\rm H} > r_{\rm L}$, so equation (5) tells us that $d\tau_{\rm H} > d\tau_{\rm L}$; the lapse of wristwatch time on the higher clock is greater than the lapse of wristwatch time on the lower clock.

Gravitational red and blue shifts

The wristwatch time lapse $d\tau_{\rm H}$ on the higher clock can be extended to the measured period $T_{\rm H}$ of a sinusoidal signal emitted from the top of the tower. The measured period $T_{\rm L}$ of the signal as it reaches Earth's surface is therefore observed to be smaller. Frequency is inversely proportional to period, so the observed frequency of the signal increases as it descends. This is called the **gravitational blue shift**, and gives the lower observer the impression that clocks above him "run fast" compared with his. In contrast, for a signal rising from Earth's surface to be observed at the top of the tower, the period increases and the measured frequency decreases, an effect labeled **gravitational red shift**, which gives the higher observer the impression that clocks below him "run slow." (Impression or reality? See Box 2)

4.3 ■ APPROXIMATIONS

How small is small?

GR effects: small but crucial to GPS.

The general relativistic effects we study are small. How small? Small compared to what? When *can* we use approximations to general relativistic expressions? And when we do, which approximations are good enough? These questions are so central to the analysis of the GPS that it is useful to begin with a rough estimate of the expected effects, not worrying initially about the crudeness of this approximation.

Assume that our tower—standing on a non-rotating Earth—extends to the height of the GPS satellite and that the satellite rests without moving on the top of the tower ($v = 0$). First write (5) in the form

$$\frac{d\tau_{\mathrm{H}}}{d\tau_{\mathrm{L}}} = \left(1 - \frac{2M}{r_{\mathrm{H}}}\right)^{1/2} \left(1 - \frac{2M}{r_{\mathrm{L}}}\right)^{-1/2} \qquad \text{(stationary clocks)} \qquad (6)$$

In Query 7 you show Newton's result that, for a 12-hour circular orbit, the orbital radius (from Earth's center) is about 26.6×10^6 meters. Inside the front cover are values for the radius and mass of Earth. We now make use of an approximation also written inside the front cover:

$$(1+\epsilon)^n \approx 1 + n\epsilon + O(\epsilon^2) \quad \text{provided} \quad |\epsilon| \ll 1 \quad \text{and} \quad |n\epsilon| \ll 1 \qquad (7)$$

Our approximations are "to first order," that is, we neglect the correction term $O(\epsilon^2)$, which means "terms of second (and higher) order in ϵ."

QUERY 1. Clock rate difference due to difference in altitude.

Apply approximation (7) to the two parenthetical expressions on the right side of equation (6). Multiply out the result to show that, to first order:

$$\frac{d\tau_{\mathrm{H}}}{d\tau_{\mathrm{L}}} \approx 1 - \frac{M}{r_{\mathrm{H}}} + \frac{M}{r_{\mathrm{L}}} \qquad \text{(to first order for } v = 0 \text{ and nonrotating Earth)} \qquad (8)$$

Verify that values of both M/r_{Earth} and $M/r_{\mathrm{satellite}}$ satisfy the criteria for approximation (7) that leads to the result (8).

QUERY 2. Numerical approximation, stationary clocks.

In the following equation, b stands for the sum of two terms added to the number one on the right side of equation (8). Substitute numbers into equation (8) and find the numerical value of b:

$$\frac{d\tau_{\rm H}}{d\tau_{\rm L}} \approx 1 + b \qquad (v = 0 \text{ and nonrotating Earth}) \tag{9}$$

Small fractional differences in clock rates affect GPS operation.

The numerical value of b in equation (9) gives us an estimate of the fractional difference in rates of signals between stationary clocks at the position of the satellite and at Earth's surface. Is this fractional difference negligible or important to the operation of the GPS? Suppose the timing of a satellite clock is off by one nanosecond (10^{-9} second). In one nanosecond a light signal (or microwave pulse) propagates approximately 30 centimeters, approximately one English foot. So a difference of, say, hundreds of nanoseconds will render GPS results inaccurate if we need a location precision of ten meters or so.

QUERY 3. Synchronization discrepancy after one day.

As long as Earth and satellite clocks do not move and the Earth does not rotate, the wristwatch time increments in equation (9) can be as long as we want, leading to the equation

$$\tau_{\rm H} \approx (1 + b)\, \tau_{\rm L} \qquad (v = 0 \text{ and nonrotating Earth}) \tag{10}$$

There are approximately 86 400 seconds in one day. (The fractional difference in rates is so small that it does not matter which local clock records this time.) To an accuracy of one significant digit, the satellite clock and Earth clock go out of synchronism by about 50 000 nanoseconds per day due to their difference in altitude alone. Find the correct value to three-digit accuracy.

The Earth observer thinks that the satellite clock above him "runs fast" by something like 50 000 nanoseconds per day compared with his local clock, due to position effects alone. Clearly we must use general relativity to analyze the operation of the Global Positioning System, even though the *fractional* difference between clock rates at the two locations (at least the part due to difference in r-coordinate) is small.

Speed effects opposite to altitude effects.

In addition to the effect of altitude, we must include the effect due to relative motion between satellite and Earth observer. Which way will this second effect influence the discrepancy in clock rates due to altitude introduced by general relativity: to increase it or decrease it? The satellite clock now moves with respect to a string of Earth clocks. Special relativity tells us (in an imprecise summary): "Speeding clocks run slow." Therefore we expect the effect of motion to *reduce* the amount by which the satellite clock runs fast compared to the Earth clock. In brief, when speed effects are taken into account, we expect the satellite clock to run faster than the Earth clock by *less* than the estimated 50 000 nanoseconds per day. In Query 8, you check your final result against this prediction.

4.4 ■ MOVING CLOCKS

Relative speed changes relative clock rates

Earth clock and satellite clock both move in circles.

Now we take account of the effects of relative motion on the relative rates of Earth and satellite clocks. Think of the Earth clock as fixed at the equator, so it moves in a circle as the Earth rotates. The satellite clock also circles the Earth, but in its own independent circular orbit. In each case $d\tau$ is the wristwatch time between ticks, the time recorded by a given clock. Set $dr = 0$ in the Schwarzschild metric and divide through by dt^2 to obtain, for either clock in its orbit,

$$\left(\frac{d\tau}{dt}\right)^2 = \left(1 - \frac{2M}{r}\right) - r^2\left(\frac{d\phi}{dt}\right)^2 = \left(1 - \frac{2M}{r}\right) - v^2 \qquad (dr = 0) \qquad (11)$$

Here $d\tau$ is the wristwatch time between ticks of either clock and $v = rd\phi/dt$ is the instantaneous tangential speed of that clock in global coordinates.

QUERY 4. Clock rate correction formula.
First apply equation (11) to the satellite clock, then apply (11) to the Earth clock. Divide the two sides of the satellite equation by the corresponding sides of the Earth equation. Take the square root of both sides of the result. For both numerator and denominator in the resulting equation, use the approximation (7). In the numerator, set

$$\epsilon_{\rm H} = -\frac{2M}{r_{\rm H}} - v_{\rm H}^2 \qquad \text{(subscript H means satellite)} \qquad (12)$$

Now do the same for the denominator. In the denominator the formula for $\epsilon_{\rm L}$ is the same as that for $\epsilon_{\rm H}$, but with L for "lower" as subscripts. Carry out an analysis similar to that in Query 1 to retain only the dominant terms. Show that the result is

$$\frac{d\tau_{\rm H}}{d\tau_{\rm L}} \approx 1 - \frac{M}{r_{\rm H}} - \frac{v_{\rm H}^2}{2} + \frac{M}{r_{\rm L}} + \frac{v_{\rm L}^2}{2} \qquad \text{(satellite directly overhead)} \qquad (13)$$

Newton orbits good enough for GPS analysis.

Now we need numerical values for the quantities on the right side of (13). Chapter 8 derives the map speed of a satellite in circular orbit according to general relativity. After completing that chapter you can verify that the following Newtonian derivation of orbit radius and satellite speed in that orbit are sufficiently accurate for our analysis of the Global Positioning System.

QUERY 5. Speed of a clock on the equator
Earth's center is in free fall as Earth orbits the Sun. The Earth also rotates on its axis, completing one full rotation with respect to the distant stars in what is called a **sidereal day**, which is 86 164.1 seconds long. (Even when we require 6-digit accuracy, which of our local clocks measures this time does not matter.) With respect to Earth's center, what is the speed v of a clock at rest on Earth's surface at the equator? Use Newtonian "universal time" t. Express your answer as a fraction of the speed of light.

Box 2. OPINION: Slogans and Observations

"Moving clocks run slow." Special relativity gives us this useful slogan, a slogan that follows us into general relativity, which adds a second useful slogan, "Clocks higher in a gravitational field run faster." What do these slogans mean?

First of all, we need to specify "faster or slower" with respect to what? More precisely, special relativity says, "An observer measures a clock moving past him to run slower than a set of synchronized clocks in his frame."' General relativity adds, "An observer at lower altitude in a gravitational field may interpret signals he receives from a clock above him to mean that the higher clock runs faster than his own clock." The GPS verifies these slogans and demonstrates their usefulness.

But there is a deeper issue: When you ride in a spaceship speeding past your friend, your wristwatch does not run slow *for you*; it ticks at its accustomed pace compared, for example, with your pulse—or your aging! Similarly, when you mount a ladder to climb vertically away from your friend in a gravitational field, you notice no change in your wristwatch rate; your wristwatch does not speed up *for you* as you gain altitude.

So does a clock *really* slow down as it moves faster? Does a clock *really* speed up as it rises in a gravitational field? Welcome to relativity: *Observations depend on the observer!*

Physics does not tell us about *reality*—whatever that means. Physics formulates theories to describe our observations and to predict new ones.

What is the value of the speed v of the satellite? Newton tells us that in a circular orbit the center-directed acceleration has the magnitude v^2/r, where v is measured in conventional units, such as meters per second. Newton also tells us that the satellite mass m multiplied by this acceleration must equal Newton's gravitational force exerted by Earth:

$$F = ma = \frac{mv^2}{r} = m\frac{GM}{r^2} \qquad \text{(Newton, conventional units)} \qquad (14)$$

Newtonian orbit analysis

Equation (14) provides one relation between the speed of the satellite and the radius of its circular orbit. (This is Newtonian mechanics, where "radius" can, in principle, be directly measured.) A second relation connects satellite speed and orbit radius to the period of revolution. This period T is 12 hours for GPS satellites:

$$v = \frac{2\pi r}{T} \qquad \text{(Newton, conventional units)} \qquad (15)$$

QUERY 6. Units in meters

Convert equations (14) and (15) to units of meters, Earth mass M and satellite orbital period T to meters, and satellite speed v to the unitless fraction of light speed. Then eliminate r between these two equations to find an expression for v in terms of M and T and numerical constants.

4.5 ■ THE FINAL RECKONING

Effects of altitude AND relative speed on clock rates

Comment 1. Relative Motion Leads to Doppler Shift
Is the GPS satellite approaching the ground receiver or receding from it? If the satellite approaches, the receiver detects a Doppler increase in frequency of the clock-tick signals from the satellite. In contrast, when the satellite recedes from the Earth receiver it detects a Doppler decrease in frequency of the clock-tick signals. In this chapter we carry out calculations for satellite emissions when it is positioned directly above the Earth receiver. In this case the *change* in the detected clock-tick signal frequency as it passes overhead is due to the relative tangential motion between the satellite and ground clocks. For other relative motions of satellite and receiver, the computer in the receiver calculates the anticipated Doppler shift and adjusts the local receiver time lapses between incoming tick signals accordingly.

QUERY 7. Satellite orbital radius and speed, according to Newton.
Find the numerical value of the speed v (as a fraction of the speed of light) for a satellite in a 12-hour circular orbit. Find the numerical value of the radius r for this orbit—according to Newton and Euclid.

Now we have numerical values for all the terms in equation (13) and can estimate the difference in rate between satellite and Earth clocks.

QUERY 8. Clock rate correction, numerical
Substitute values for the various quantities in (13). Show that the satellite clock appears to run faster than the Earth clock by approximately 38 700 nanoseconds per day.

Section 4.3 described the difference in clock rates due only to difference in altitude. We predicted at the end of Section 4.3 that the full general relativistic treatment would lead to a *smaller* difference in clock rates than reckoned for the altitude effect alone. Your result in Query 8 verifies this prediction.

Before-launch settings of satellite and Earth clocks

Before launch, GPS satellite clocks are set to run at a rate of 38 700 nanoseconds per day *slower* than identical Earth clocks next to them, clocks that will remain on Earth's surface. *Result:* When the satellite clock passes overhead, the increased frequency (gravitational blue shift) of its signal received on Earth synchronizes with the Earth clock (Box 3).

Gravitational shifts: no single identifiable cause

An historical aside: Carroll O. Alley, a consultant to the original GPS project, had a hard time convincing the designers *not* to apply *twice* the correction given in (13): first to account for the different rate of time advance on wristwatches located at different altitudes and second to allow for the gravitational blue shift in frequency for the signal sent downward from satellite to Earth. There is only one correction; moreover there is no way to identify uniquely the "cause" of this correction. Listen to what Clifford Will says about

Box 3. General Relativity On/Off Switch

Launching the Global Positioning System was an immense military and civilian effort. Most participants were not skilled in general relativity and, indeed, wondered if the academic advisors were right about this strange theory. As one later publication put it:

> *There was considerable uncertainty among Air Force and contractor personnel designing and building the system whether these effects were being correctly handled, and even, on the part of some, whether the effects were real.*

The GPS prototype satellite called Navigation Technological Satellite 2 (NTS-2) was launched into a near-12-hour circular orbit on June 23, 1977, with its single atomic clock initially set (on Earth) to run at the same rate as Earth clocks. However, it had a general relativity on/off switch, leading to two possible modes of operation. In the first mode, with the switch set to ”off”, the satellite clock was simply left to run at the rate at which it had been set on Earth. It ran in this condition for 20 days. The satellite clock drifted, relative to Earth clocks, at the rate predicted by general relativity, “well within the accuracy capabilities of the orbiting clock.”

The NTS-2 satellite validated the general relativity results, so the general relativity on/off switch was flipped to “on.” This changed the satellite clock rate to a pre-arranged 38 700 nanoseconds per day slower than that of the Earth clock, also set before launch when the two clocks were side by side on Earth. Then the gravitational blue shift of the signal from an orbiting overhead satellite raised the frequency of the signal received on Earth to that of the Earth clocks. Since then, every GPS satellite goes into orbit with general relativity built into its design and construction. No more general relativity on/off switch!

the difference in rates between one clock emitting a signal from the top of a tower and a second identical clock receiving the signal on the ground:

> *A question that is often asked is, Do the intrinsic rates of the emitter and receiver or of the clock change, or is it the light signal that changes frequency during its flight? The answer is that it doesn't matter. Both descriptions are physically equivalent. Put differently, there is no operational way to distinguish between the two descriptions. Suppose that we tried to check whether the emitter and the receiver agreed in their rates by bringing the emitter down from the tower and setting it beside the receiver. We would find that indeed they agree. Similarly, if we were to transport the receiver to the top of the tower and set it beside the emitter, we would find that they also agree. But to get a gravitational red shift, we must separate the clocks in height; therefore, we must connect them by a signal that traverses the distance between them. But this makes it impossible to determine unambiguously whether the shift is due to the clocks or to the signal. The observable phenomenon is unambiguous: the received signal is blue shifted. To ask for more is to ask questions without observational meaning. This is a key aspect of relativity, indeed of much of modern physics: we focus only on observable, operationally defined quantities, and avoid unanswerable questions.*
>
> —Clifford Will

The ambiguity described by Clifford Will appears in our treatment of clock rates at different r-coordinates. Box 6 in Section 3.4 started with equal wristwatch times: $d\tau_{\mathrm{H}} = d\tau_{\mathrm{L}}$ and derived different global coordinate differentials: $dt_{\rm H} \neq dt_{\rm L}$. In contrast, Section 4.2 notes that the map dt between two radially-directed signals does not change as the signals travel between locations: $dt_{\rm H} = dt_{\rm L}$ and from this derives a difference in clock time lapses: $d\tau_{\rm H} \neq d\tau_{\rm L}$. Clifford Will tells us that both methods lead to the same conclusion about clock rates.

TWO COMMENTS

Comment 2. Newtonian orbit radius OK.
We assume in this chapter that the radius $r_{\rm H}$ of the circular orbit of the satellite and the speed v of the satellite in that orbit are both computed accurately enough using Newtonian mechanics. Exercise 2 in Section 8.7 validates this assumption.

Comment 3. Little latitude effect.
Our analysis considers an Earth clock fixed to the ground at the equator. One might expect that the speed-dependent correction would take on different values for an Earth clock fixed to the ground at different latitudes north or south of the equator, going to zero at the poles where there is no motion of the Earth clock due to rotation of Earth. In practice there is negligible latitude effect because Earth is not perfectly spherical; it bulges a bit at the equator due to its rotation, like a squashed balloon. The smaller r at the poles increases the $M/r_{\rm L}$ term in (13) by roughly the same amount that the speed term decreases. The outcome is that our calculation for the equator applies quite well to all latitudes.

QUERY 9. Orbit radius for zero time correction.
At a cocktail party one hears, "A speeding clock runs slow." and "A higher clock runs faster." This implies that there should be a radius at which the two effects cancel, so that two flashes received from a clock passing overhead would have the same time lapse between them as measured by an Earth observer directly below.

A. *Guess:* Do you expect the radius at which the two effects cancel to be smaller or larger than the actual orbital radius of GPS satellite orbits?
B. Use Newton to calculate the radius of the "cancellation orbit"?
C. A permanent circular orbit around Earth must be above the atmosphere. Is this true of the "cancellation orbit" you calculated in Item B?

4.6 ■ APPLICATIONS OF THE GLOBAL POSITIONING SYSTEM

GPS applications everywhere!

Uses of GPS? Look around!

Applications of the Global Positioning System have exploded. To ask how the GPS is used today is like asking about applications of the automobile or the telephone. Geologists measure the millimeters-per-year motion of the continents (motion with respect to what?); biologists track wildlife (Box 4). How is the GPS used? Look around and read the news!

Box 4. Tracking the Pack

Wolf 832F ventured out of her territory in Yellowstone's Lamar Valley. As soon as she left the park, she lost its protections, and the wolf, a 6-year-old alpha female, was shot and killed by a hunter. She had been wearing an expensive GPS tracking collar, which allowed scientists to follow her every move and gain crucial insight into the lives of gray wolves. Is this particular predator a pack leader or a lone wolf? A dedicated hunter or a mooch? How much time does it spend with its pups? Who are its associates, rivals and mates?

By using satellite and cellular tags to track free-ranging animals, biologists are providing us with intimate access to the daily lives of other species, drawing us closer to the world's wild things and making us more invested in their welfare.

Today's tags are capable of collecting months' or years' worth of data on an animal's location at a given moment, and can be used to track everything from tiny tropical orchid bees to blubbery, deep-diving elephant seals.

The Tagging of Pacific Predators project created a Web site broadcasting the movements of their subjects in real time (or close to it). While the project lasted, anyone with an Internet connection could follow the wanderings of Monty, the mako shark, Genevieve, the leatherback turtle, or Jon Sealwart and Stelephant Colbert, both northern elephant seals.

Bird lovers can follow the migrations of bald eagles through EagleTrak, run by the Center for Conservation Biology. The group provides detailed updates on the journeys of two eagles, Camellia and Azalea. Each bird has around a hundred "adoptive parents," proving how attached we can get to a wild creature when we have a name and a life story to assign to it.

We've only just scratched the surface of what's possible.

—Emily Anthes

QUERY 10. Aging on the International Space Station.
The International Space Station (ISS) circles the Earth at an altitude of approximately 350 kilometers, or at a radius of 6.73×10^6 meters from Earth's center, at an orbital speed of 7 707 meters per second. An astronaut lives on the ISS for one year. When she returns to Earth's surface, how much younger (or older?) is she than her twin sister who stayed on Earth?

4.7 ■ REFERENCES

Initial quote from "A Tale of Einstein, Congress and Safe Landings" by Clifford Will, an online posting by the Editorial Services, Washington University, St. Louis.

Box 3 quote about general relativity on/off switch: Carroll O. Alley, "Proper Time Experiments in Gravitational Fields With Atomic Clocks, Aircraft, and Laser Light Pulses," in *Quantum Optics, Experimental Gravity, and Measurement Theory,* edited by Pierre Meystre and Marlan O. Scully, Plenum Publishing, New York, 1983, pages 421-424.

Box 4 Tracking the Pack by Emily Anthes, New York Times, February 4, 2013. Excerpted and rearranged.

Quote about clocks on the ground and on a tower: Clifford M. Will, *Was Einstein Right? Putting General Relativity to the Test,* Second Edition, Basic Books/Perseus Group, New York, 1993, pages 49-50.