MATHEMATICS

PLACEMENT TEST

1.) (10 points) Perform the indicated operations and express the final result in simplest form:

a) $\dfrac{a^2-9}{a^4-5a^3+6a^2} \div \dfrac{a+3}{a^3-a^2-2a}$ b) $\dfrac{2x}{x+2}+\dfrac{2x}{6-3x}+\dfrac{8x}{x^2-4}.$

2.) (10 points) Solve the following system of equations:

$$x-y=1\ ,\ \log_2(5x-1)=2\log_2(y-2).$$

3.) (22 points) Solve the following equations:

a) $\dfrac{x+1}{x^2+3x}+\dfrac{x-2}{x^2-3x}=\dfrac{1}{x}$ b) $\sqrt{3-x}-\sqrt{3+x}=\sqrt{x}$

c) $\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}=4\sin x\cdot\cos x.$

4.) (12 points) Give the domain of the following functions:

a) $f(x)=\sqrt{2x+6}+\dfrac{1}{3^x-9}$ b) $f(x)=\log_5\dfrac{x}{x^2-7x+10}.$

5.) (10 points) Given the points A(2,2), B(6,4) and C(5,6).

a) Show that the points A, B and C are vertices of a right triangle.

b) Give the equation of the circle through the points A, B and C.

6.) (10 points) Determine the equation of the circle whose center is the point (1,2) and touches the line $y-x+1=0$.

7.) (14 points) Find the solution set of the following inequalities:

a) $x+|2x-3|\le 4$ b) $2^{2x}-2\cdot 2^x-8\ge 0.$

8.) (12 points) Find the values of the real parameter *k* for which the equation

$$(1-k)x^2-4kx+4(1-k)=0$$

has at least one real root.