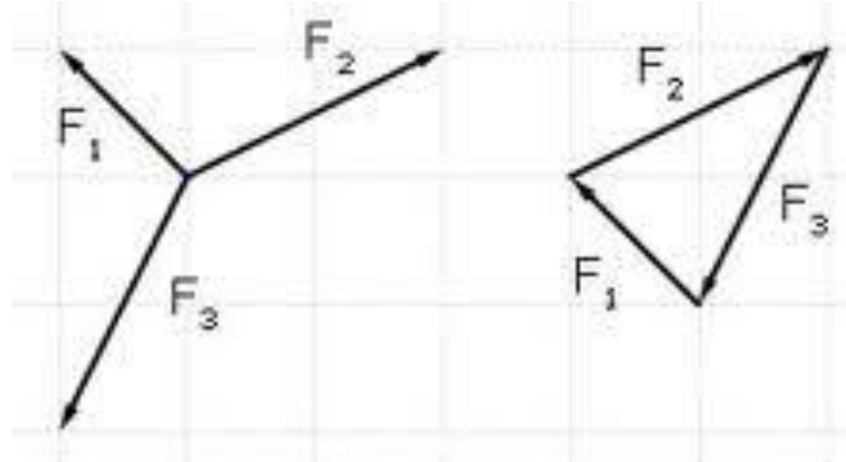
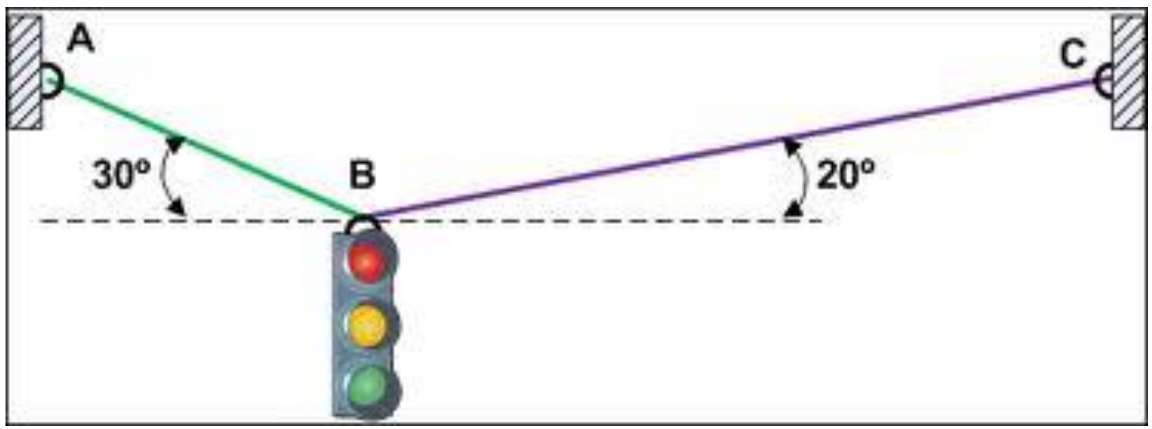
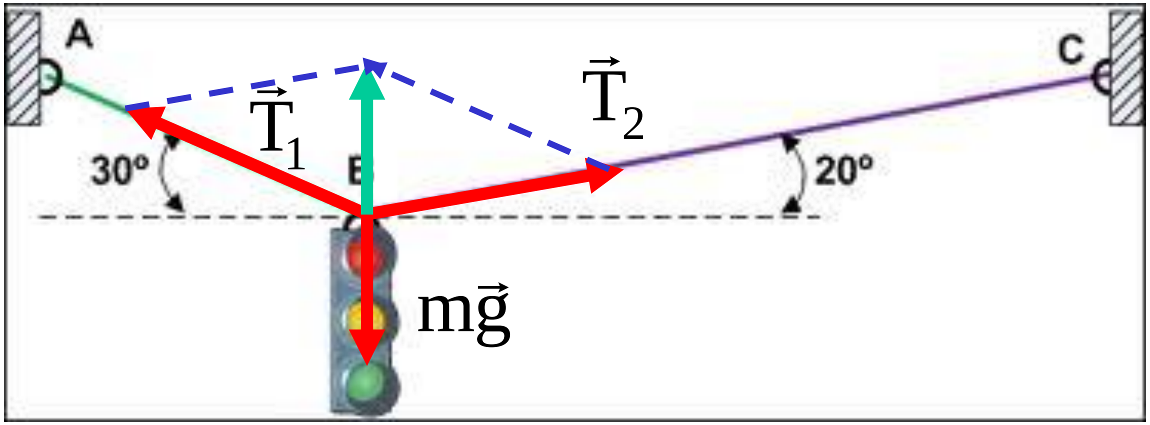
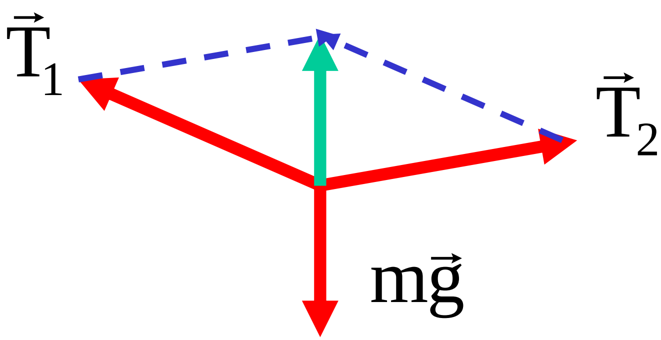




Statics,
elastic properties of materials,

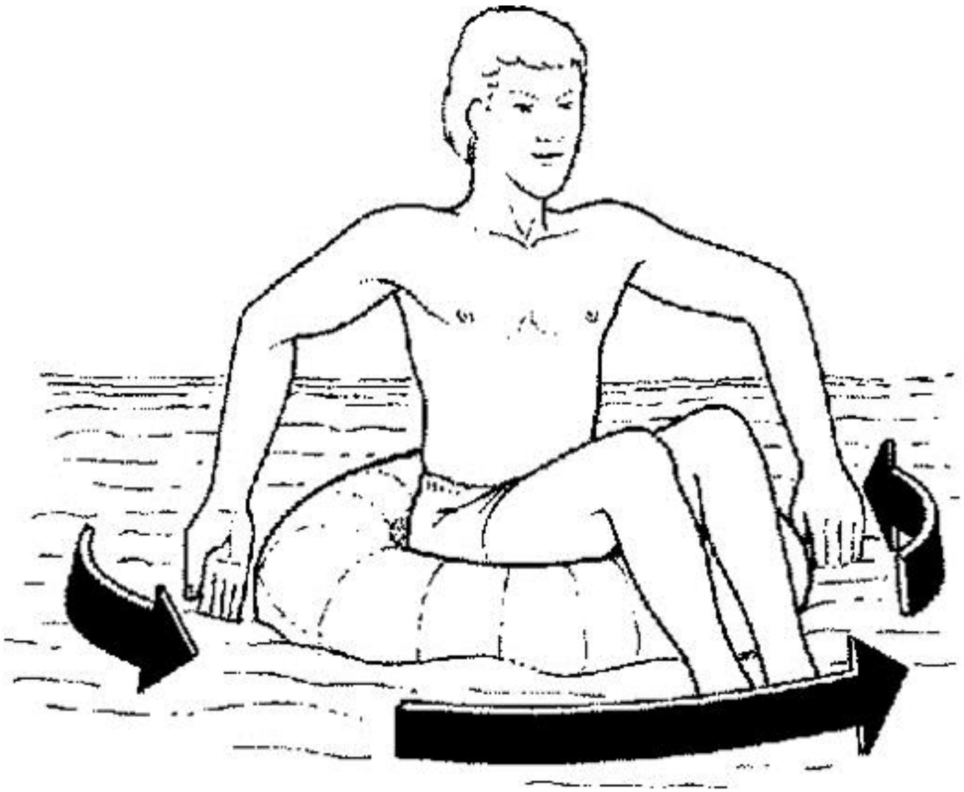


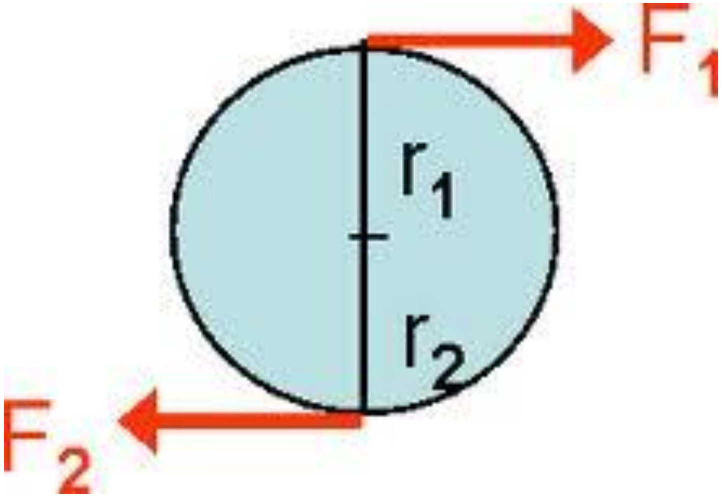


Condition of equilibrium:

$$m\vec{g} + \vec{T}_1 + \vec{T}_2 = 0$$

$$\vec{F}_{net} = 0$$





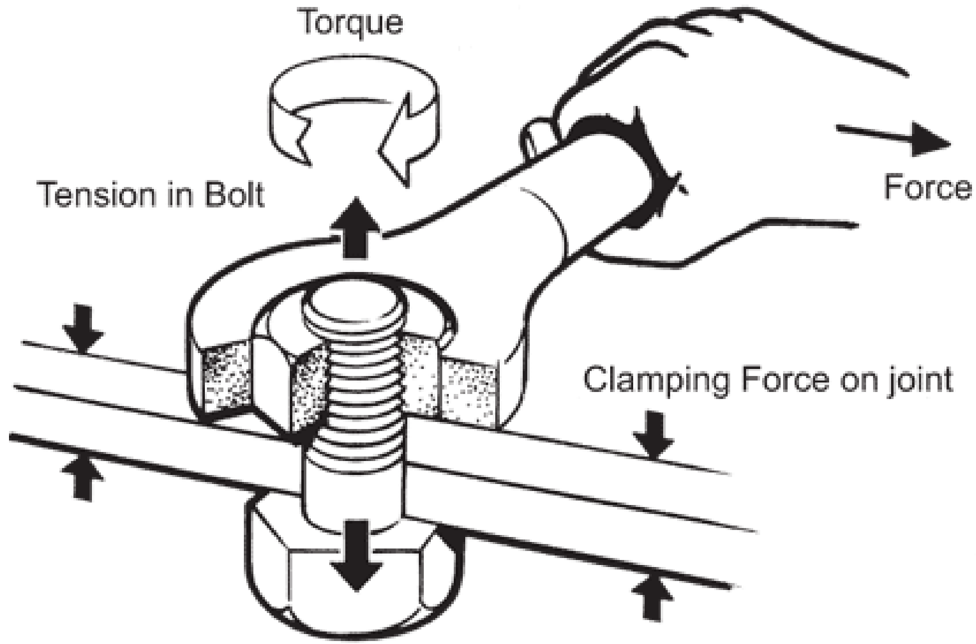
Condition of equilibrium:

$$\vec{F}_{net} = 0$$

$$\vec{F}_1 + \vec{F}_2 = 0$$

? ? ?

Torque

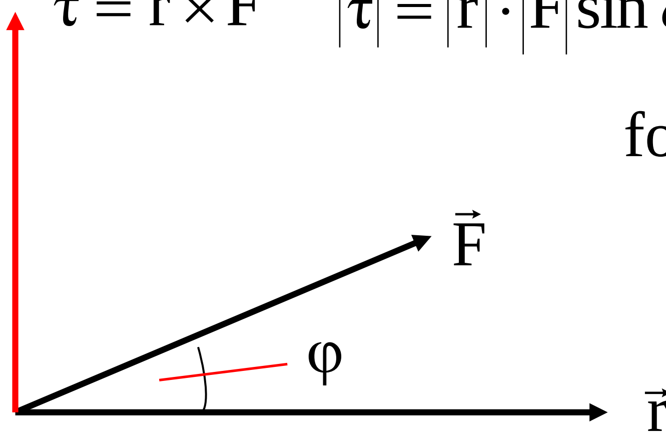


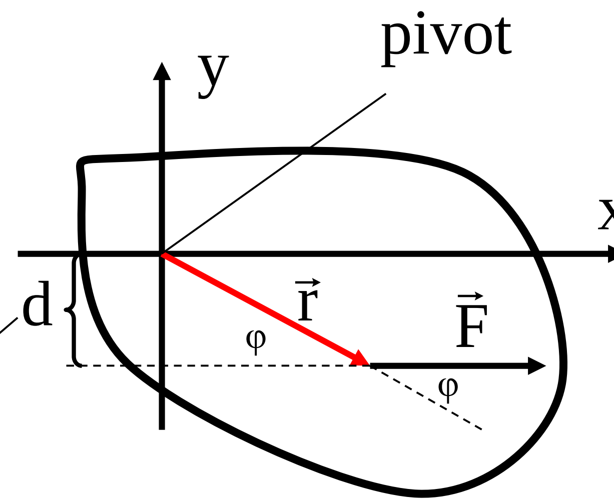
Def.: torque [Nm] $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} = \vec{r} \times \vec{F}$ $\quad |\vec{\tau}| = |\vec{r}| \cdot |\vec{F}| \sin\varphi = F \cdot d$

force

lever arm





rigid object

1. STATICS - Condition(s) of equilibrium

I. $\vec{F}_{net} = m\vec{a} \Rightarrow$ if $\vec{F}_{net} = 0 \Rightarrow \vec{a} = 0$

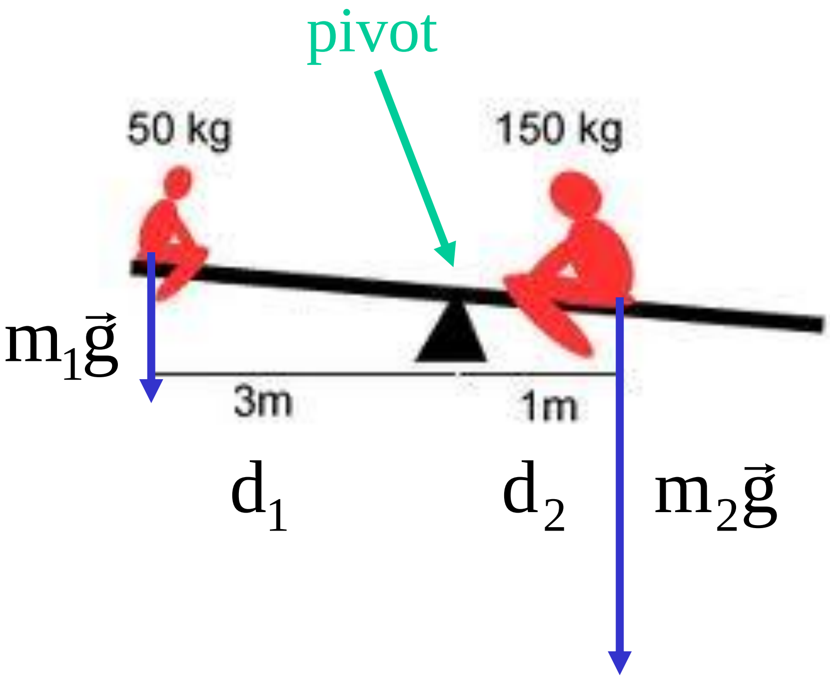
II. $\vec{\tau}_{net} = I\vec{\alpha} \Rightarrow$ if $\vec{\tau}_{net} = 0 \Rightarrow \vec{\alpha} = 0$

Condition(s) of equilibrium:

I. $\vec{F}_{net} = 0$

II. $\vec{\tau}_{net} = 0$

Example:

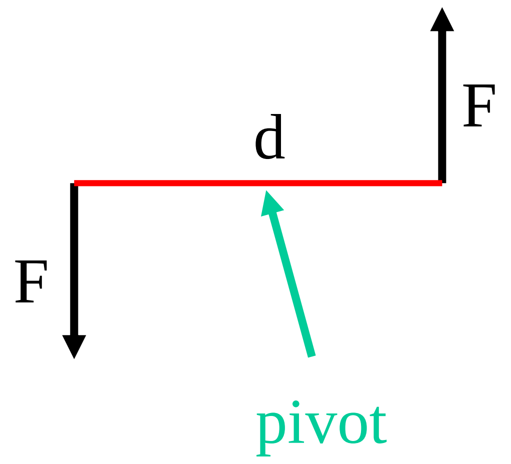


$$\vec{\tau}_{net} = 0$$

$$\vec{\tau}_1 + \vec{\tau}_2 = 0$$

$$m_1 g d_1 - m_2 g d_2 = 0$$

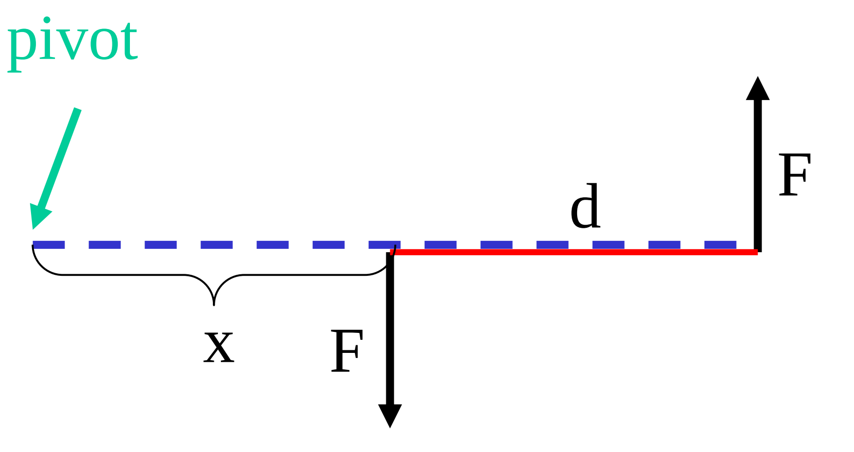
Couple of forces:



I. $\vec{F}_{net} = 0$

II. $\vec{\tau}_{net} \neq 0 \rightarrow \tau_{net} = Fd$

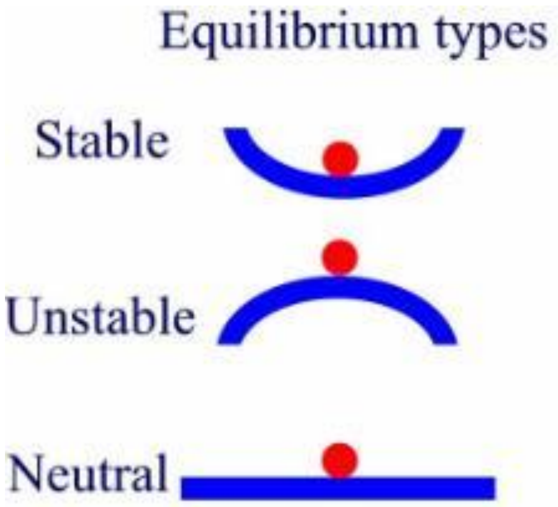
$$\tau = |\vec{\tau}_1| + |\vec{\tau}_2| = F\frac{d}{2} + F\frac{d}{2} = Fd$$

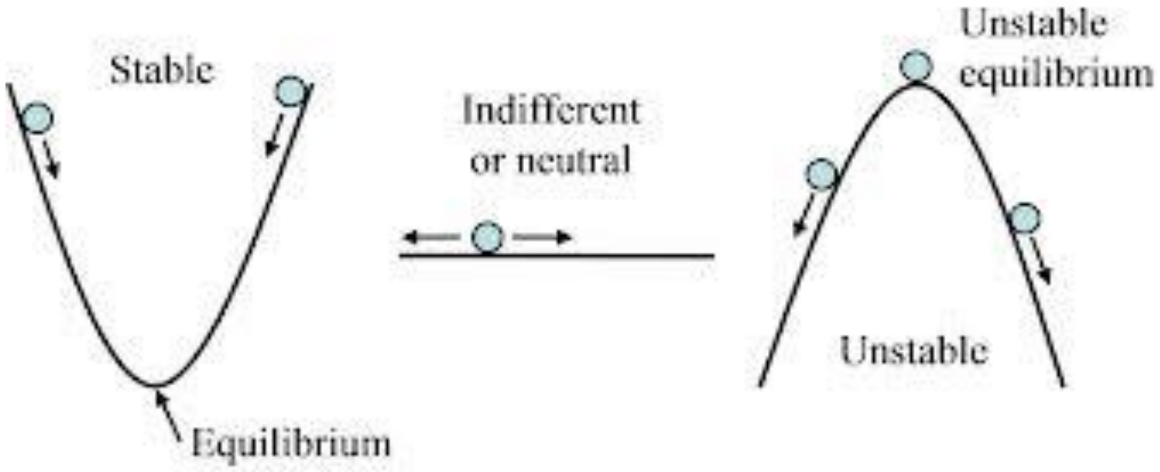


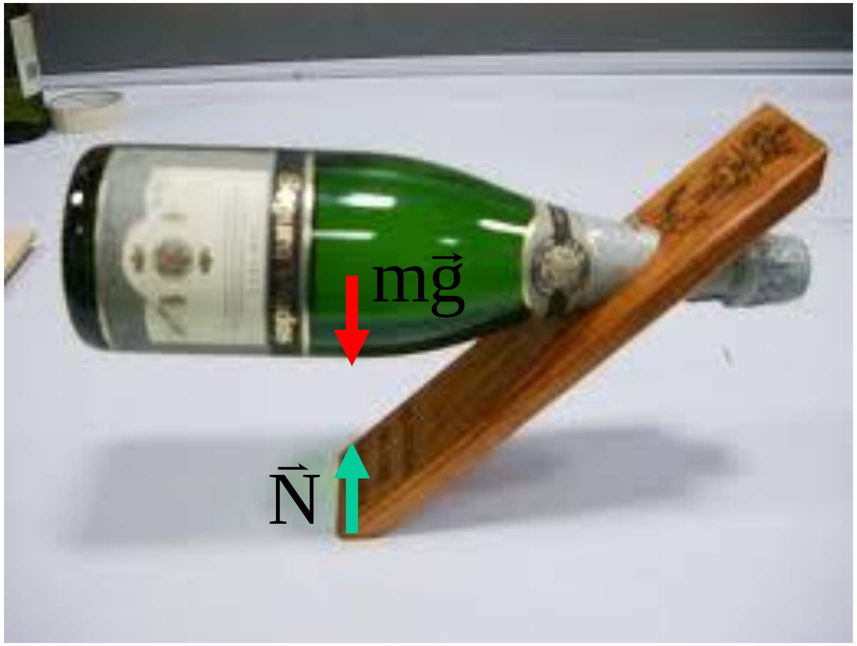
$\vec{\tau} = ?$

Def.: density

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V} \quad \left[\frac{kg}{m^3}\right]$$





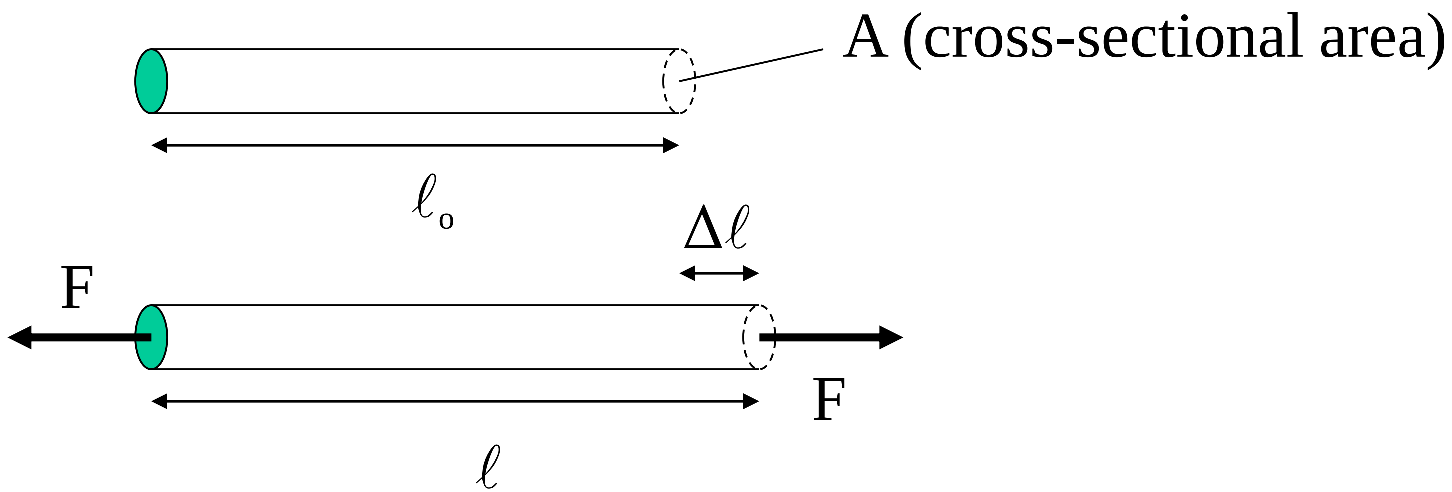


Position of center of mass: $$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

Net torque on the center of mass: $$\vec{\tau}_{net} = m_1\vec{g}\times\vec{r}_1 + m_2\vec{g}\times\vec{r}_2 + \ldots$$

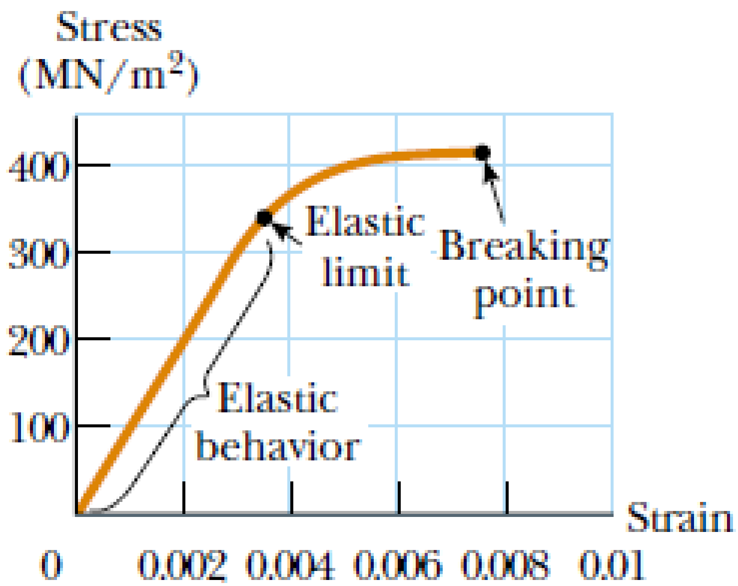
$$\vec{\tau}_{net} = \vec{g}\times\left(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\right) = \vec{g}\times\left(\vec{r}_{cm}M\right)$$

Elastic properties:



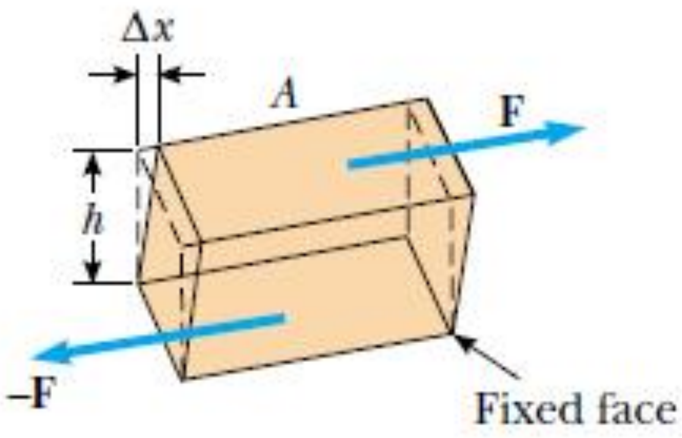
$$\left.\begin{aligned}\text{Tensile stress} &= \frac{F}{A} \\ \text{Tensile strain} &= \frac{\ell - \ell_o}{\ell_o}\end{aligned}\right\} \quad \text{Young's modulus} = \frac{\text{tensile stress}}{\text{tensile strain}} = Y$$

$$\frac{F}{A} = Y \frac{\Delta\ell}{\ell_o}$$



Typical Values for Elastic Moduli ? ?

Substance	Young's Modulus (N/m²)	Shear Modulus (N/m²)	Bulk Modulus (N/m²)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

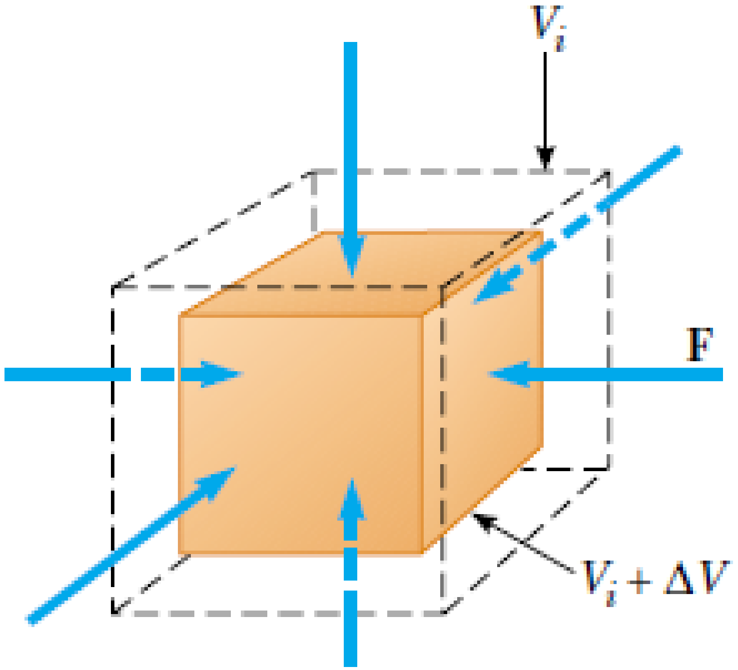


(a)

(b)

Shear modulus:

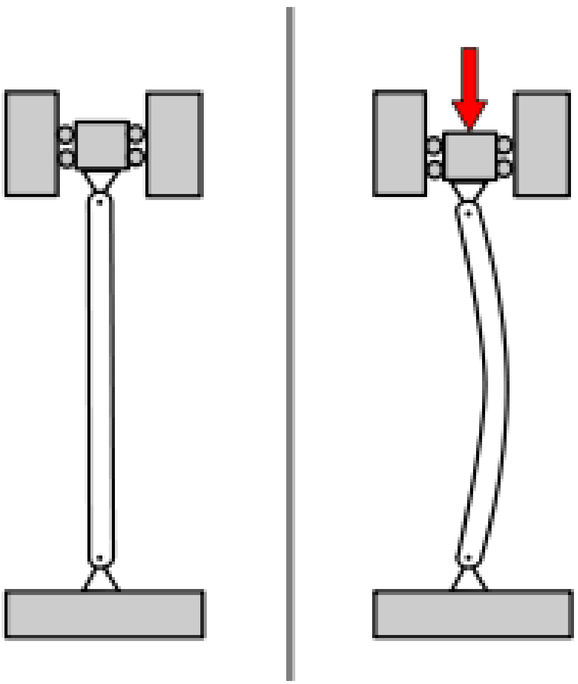
$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$



Bulk modulus:

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i}$$

Buckling:



Self buckling:

$$h_{crit} = \left(\frac{9B^2}{4} \frac{EI}{\rho g \pi r^2} \right)^{1/3}$$