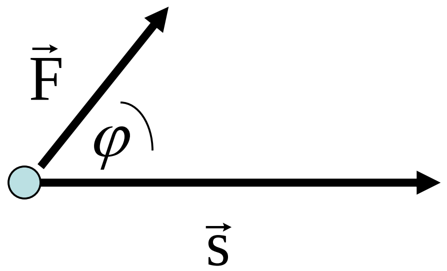
Work and energy

Def.: Work $W = \vec{F} \cdot \vec{s} = |\vec{F}| \cdot |\vec{s}| \cdot \cos\varphi$ $[\mathrm{Nm} = \mathrm{J}]$



If: $\vec{F} = \text{const.}$

$$W = F_x s_x + F_y s_y$$

Scalar product:

$$\vec{a} = a_x \vec{i} + a_y \vec{j}$$

$$\vec{b} = b_x \vec{i} + b_y \vec{j}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

$$\cos\varphi = \frac{a_x b_x + a_y b_y}{\sqrt{a_x^2 + a_y^2}\sqrt{b_x^2 + b_y^2}}$$

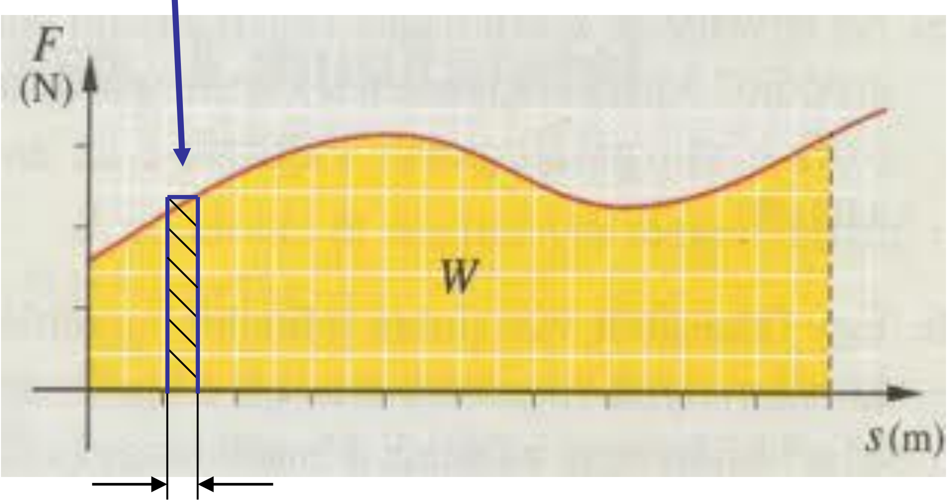
1. D. motion

$$\vec{F} \neq \text{const.}$$

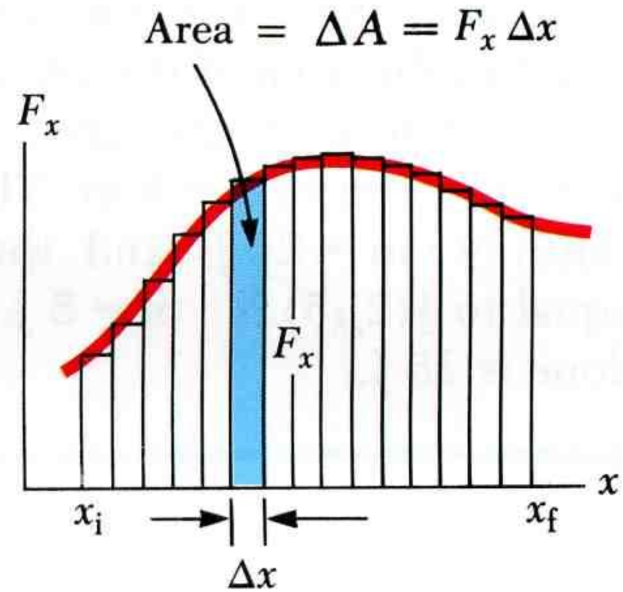
$$\Downarrow$$

$$F = F(x)$$

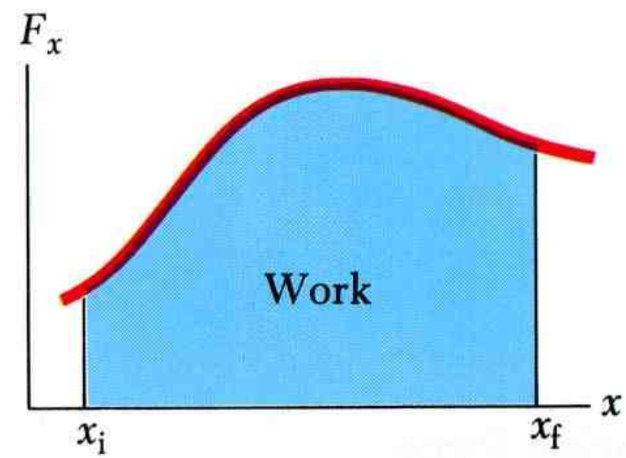
$$\Delta W = F(x)\Delta x \quad \Rightarrow \quad W = \sum_{i=1}^{N} F_i \Delta x_i$$



Δx

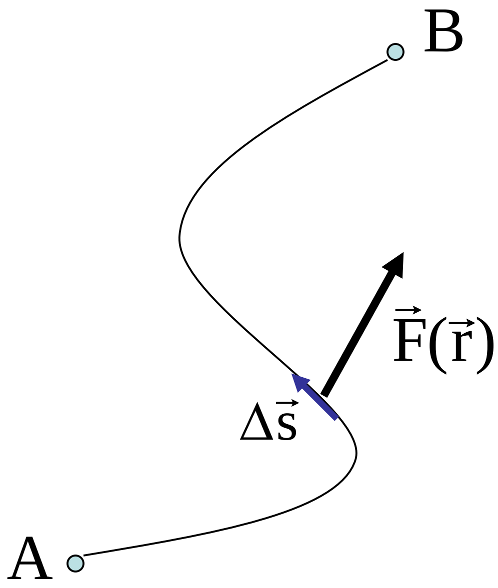


(a)



(b)

$\vec{F} \neq \text{const.}$

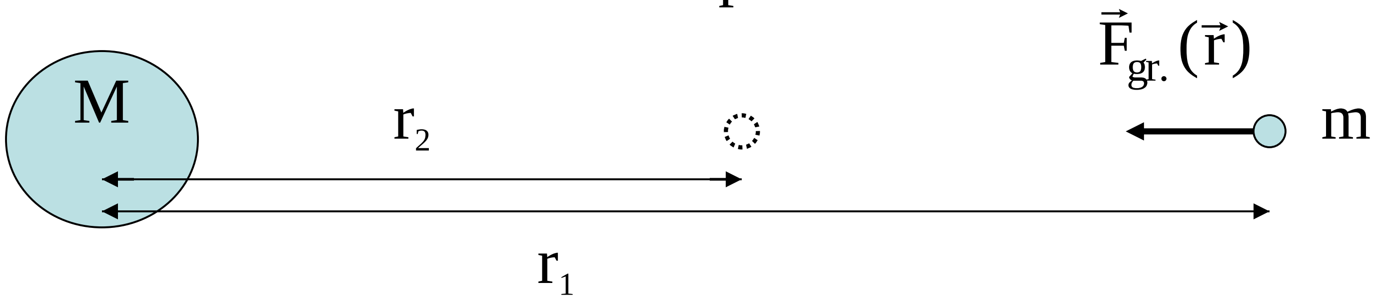


$$W = \sum_{i=1}^{N} \vec{F}_i(\vec{r}) \bullet \Delta\vec{s}_i$$

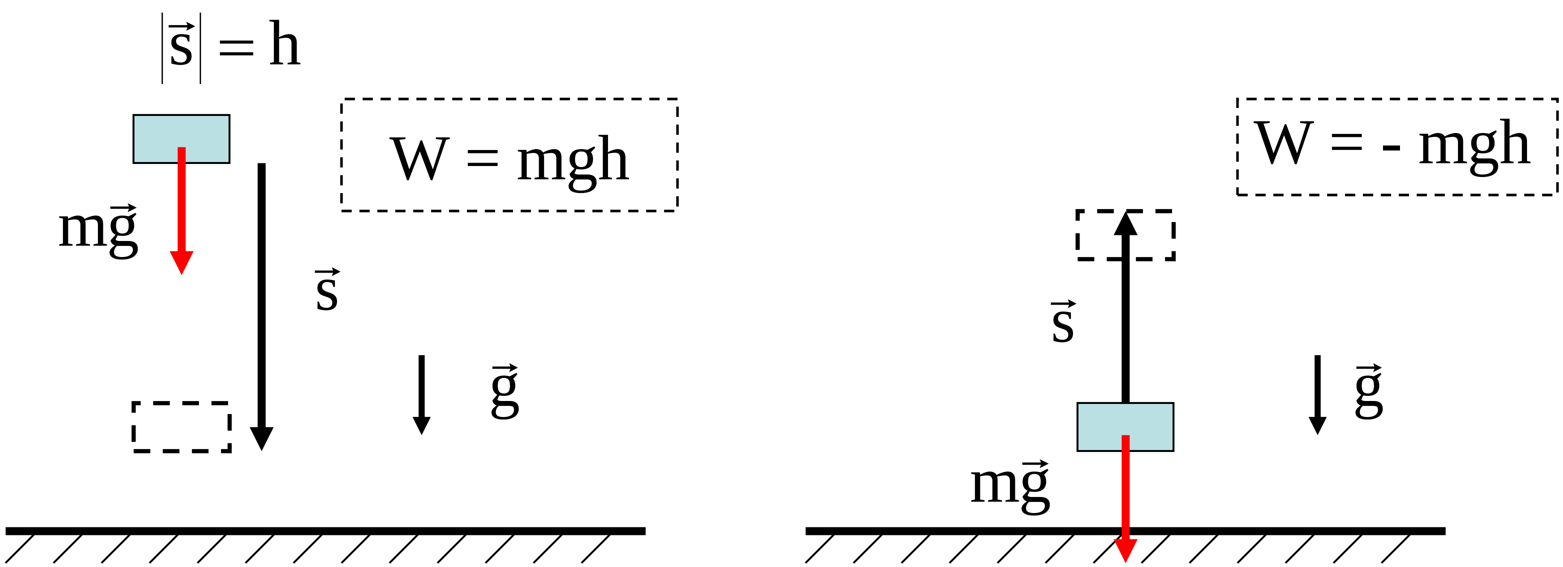
$$W = \sum_{i=1}^{N} \vec{F}_{xi}(\vec{r})\Delta x_i + \sum_{i=1}^{N} \vec{F}_{yi}(\vec{r})\Delta y_i + \sum_{i=1}^{N} \vec{F}_{zi}(\vec{r})\Delta z_i$$

Work of gravity

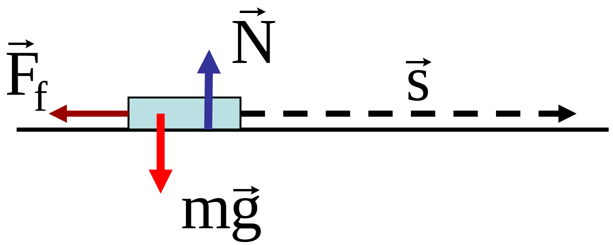
$$F_{gr.} = G\frac{mM}{r^2}$$



$$W_{gr.} = GmM\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$



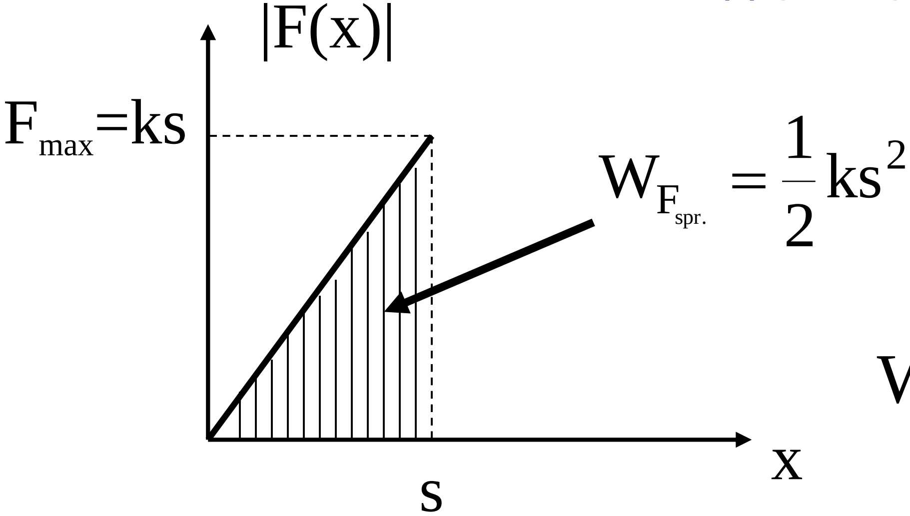
Work of the frictional force



$$W_{F_f} = \vec{F}_f \cdot \vec{s} = -F_f \cdot s < 0$$

$$W_N = 0$$

Work of the spring force

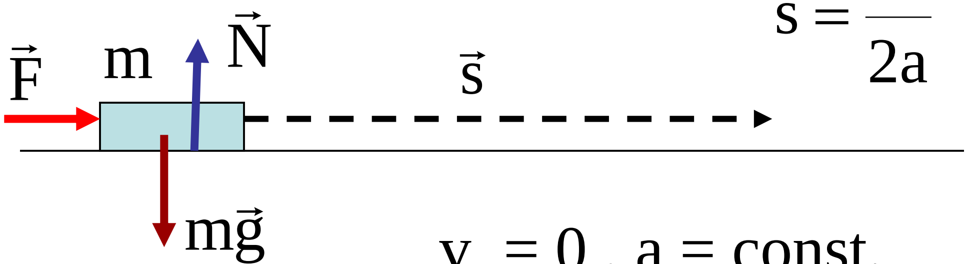


$$F(x) = -k \cdot x$$

$$W_{F_{spr.}} = \text{area} = \frac{1}{2} s \cdot F_{max} = \frac{1}{2} ks^2$$

Sign!!

Kinetic energy



$$s = \frac{v^2}{2a}$$

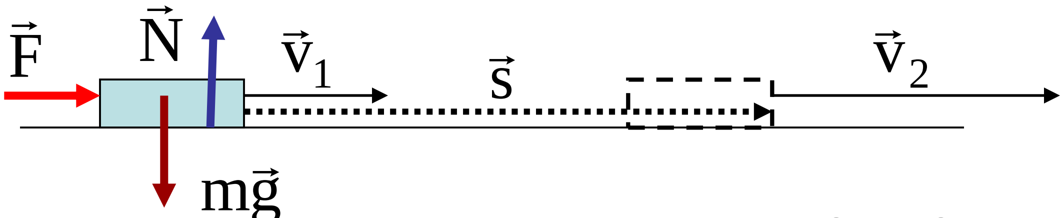
$$\vec{F} = m\vec{a}$$

$$v_o = 0\ ,\ a = \text{const.}$$

$$W = Fs = ma \cdot \frac{v^2}{2a} = \frac{1}{2} mv^2$$

$$E_k = \frac{1}{2} mv^2$$

Work-energy theorem



$$a = \text{const.}$$

$$W = Fs = ma \cdot \frac{v_2^2 - v_1^2}{2a} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 = E_{k2} - E_{k1}$$

$$s = \frac{v_2^2 - v_1^2}{2a}$$

$$W = \Delta E_k$$

Power

Def.: **average power**

$$P_{ave} = \frac{\Delta W}{\Delta t} \qquad \left[\frac{J}{s} = W\right]$$

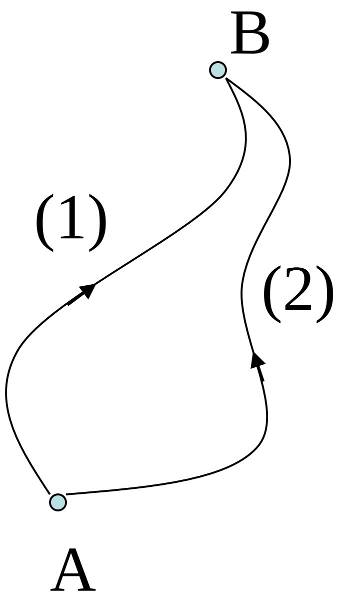
Def.: **instantaneous power**

$$P_{inst} = P(t) = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta(\vec{F} \cdot \vec{s})}{\Delta t} = \vec{F} \cdot \left(\lim_{\Delta t \to 0} \frac{\Delta \vec{s}}{\Delta t} \right) = \vec{F} \cdot \vec{v}$$

$$P(t) = \vec{F} \cdot \vec{v}$$

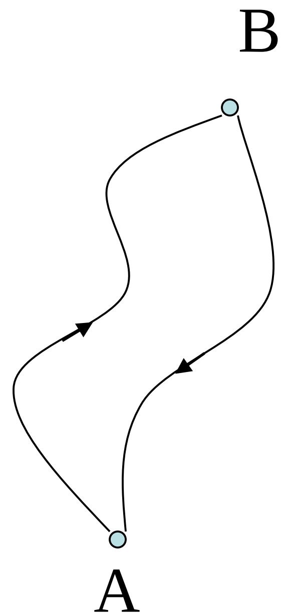
Potential energy and conservation of mechanical energy

Conservative force



$$W_{(1)} = W_{(2)}$$

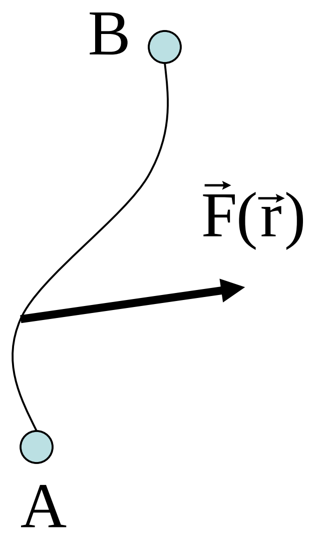
or



$$W_{A\to B} = -W_{B\to A}$$

$$W = \sum_{i=1}^{N} \vec{F}_i(\vec{r}) \bullet \Delta\vec{s}_i = 0$$

Potential energy (conservative forces)



$$W_{field} = \int_A^B \vec{F} \cdot d\vec{s}$$

$$\Delta U = U_B - U_A = -\int_A^B \vec{F} \cdot d\vec{s}$$

$$\boxed{\Delta U = -W_{field}}$$

Conservation of energy

$$W = \Delta E_k \Longrightarrow -\Delta U = \Delta E_k \Longrightarrow -\left(U_B - U_A\right) = E_{k_B} - E_{k_A}$$

$$\Downarrow$$

$$E_A = E_B \Longleftarrow \boxed{E_{k_A} + U_A = E_{k_B} + U_B}$$

$$F_{nonc.} \neq 0$$

$$W = \Delta E_k \quad \Longrightarrow \quad W = W_{cons.} + W_{nonc.}$$

$$\Downarrow$$

$$-\Delta U + W_{nonc.} = E_k$$

$$-(U_B - U_A) + W_{nonc.} = E_{k_B} - E_{k_A}$$

$$\Downarrow$$

$$\underbrace{E_{k_A} + U_A}_{E_A} + W_{nonc.} = \underbrace{E_{k_B} + U_B}_{E_B}$$

Generally: frictional force, air resistance, etc. : $W_{nonc.} \langle 0$

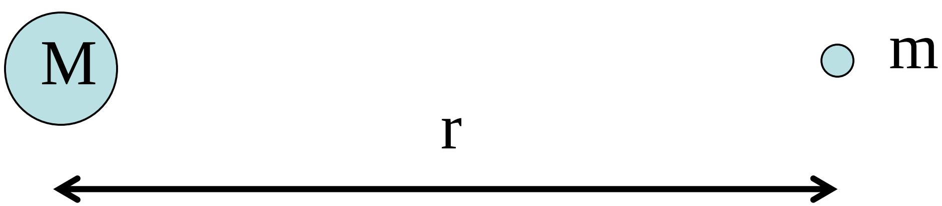
Potential energy of a spring

Work to stretch a spring: $W_{person} = \frac{1}{2} k s^2$

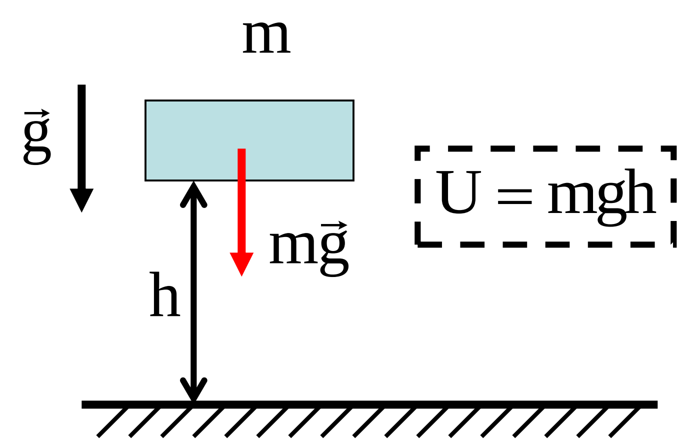
$$W_{F_{spr.}} = -\frac{1}{2} k s^2 \qquad \Delta U = -W_{F_{spr.}} = \frac{1}{2} k s^2$$

$$U(s=0) = 0 \longrightarrow \boxed{U = \frac{1}{2} k s^2}$$

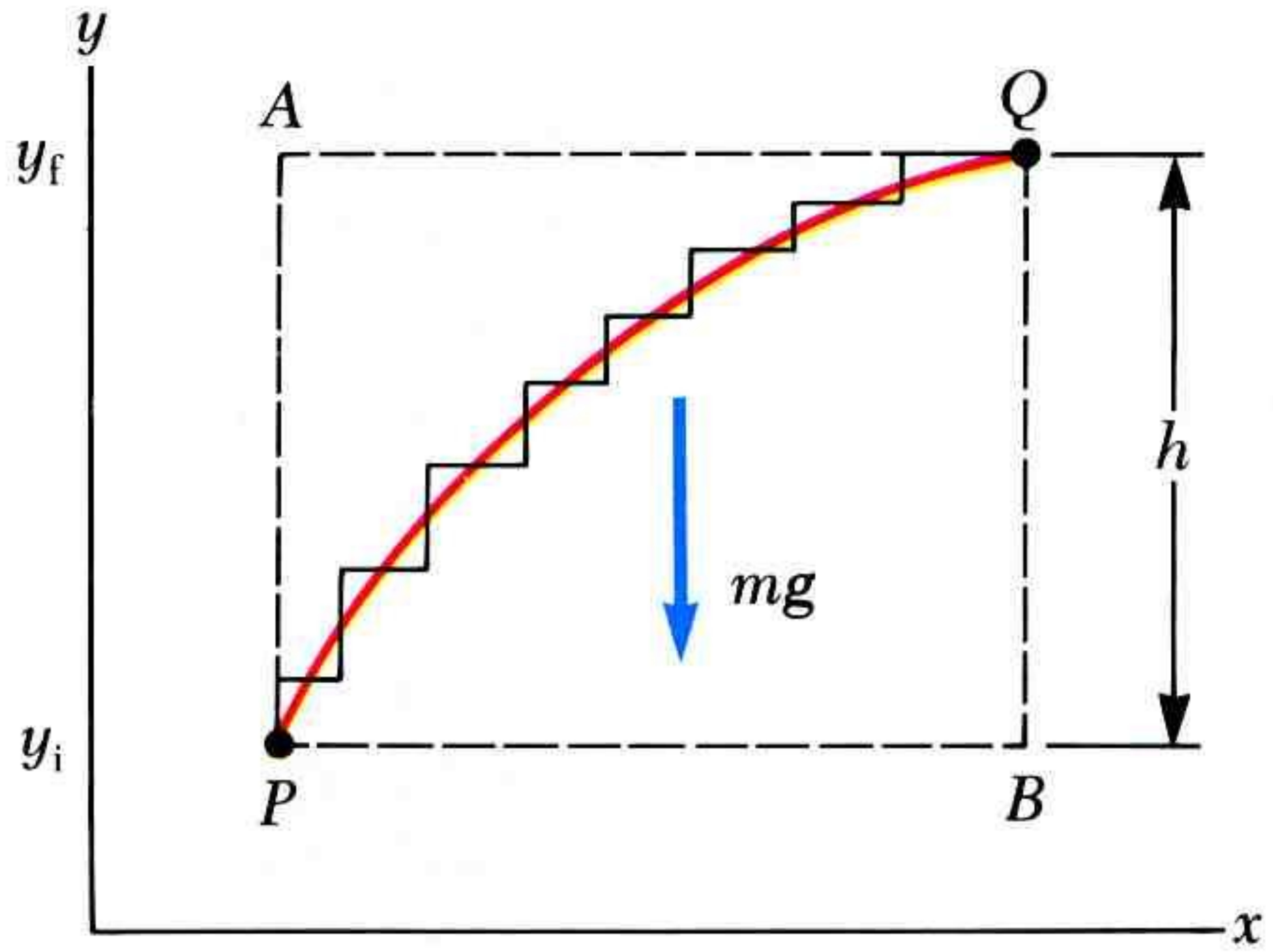
Gravitational potential energy



$$\boxed{U(r) = -G\frac{Mm}{r}}$$



$$\boxed{U = mgh}$$



$$\Delta U = mgh$$