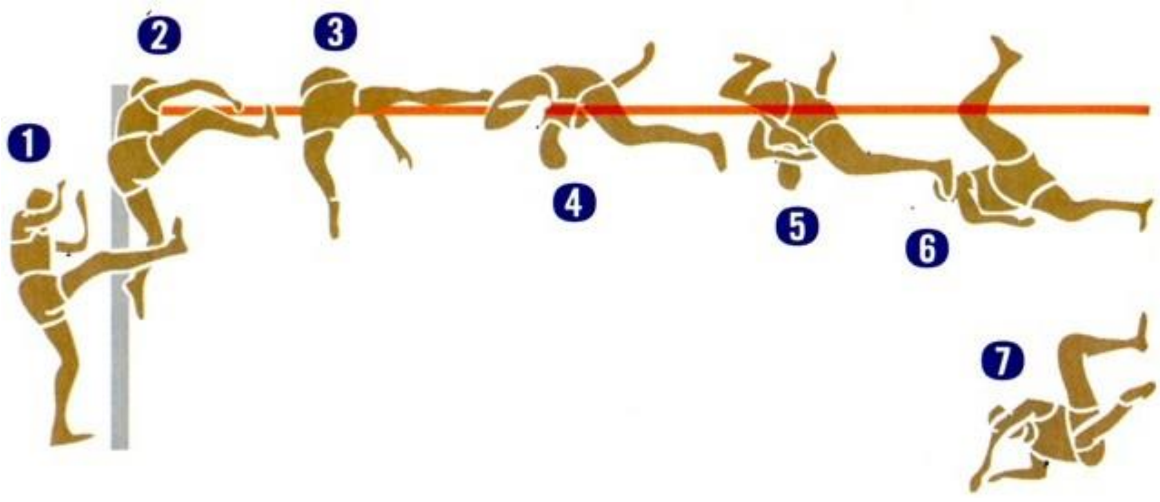
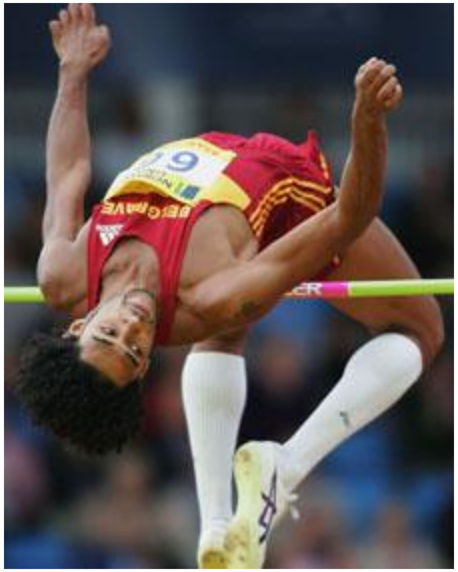
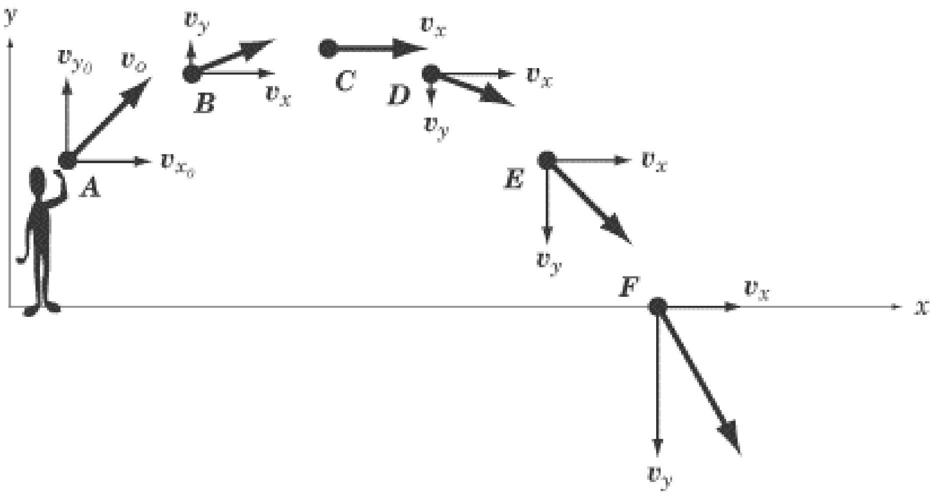
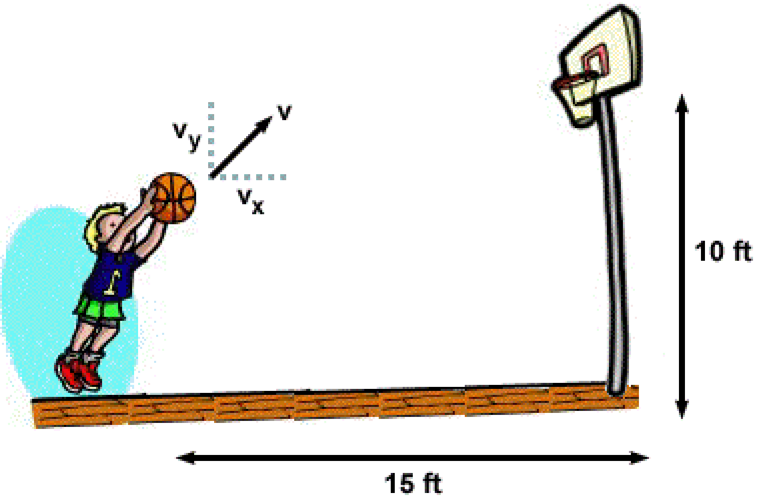
2D (& 3D) motion

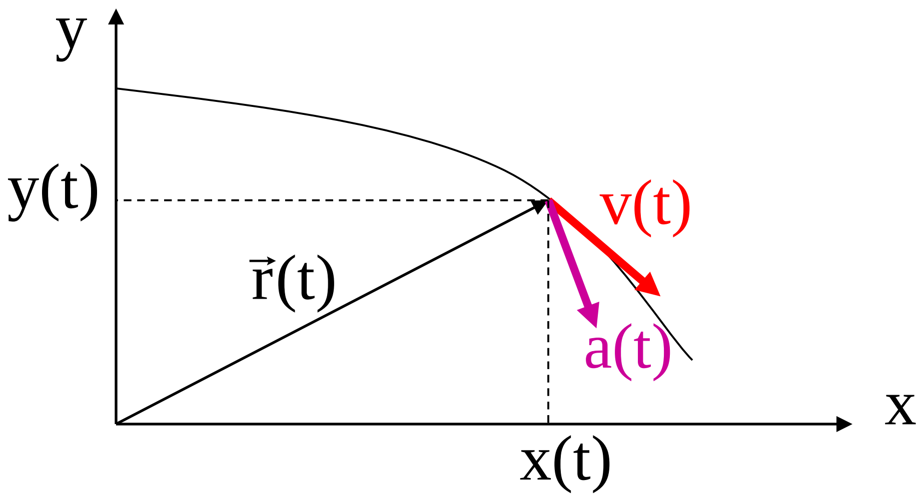

1
2
3
4
5
6
7




y
v_{y_0}
v_o
A
v_{x_0}
v_y
B
v_x
C
v_x
D
v_x
v_y
E
v_x
v_y
F
v_x
v_y
x


v_y
v
v_x
10 ft
15 ft

Two dimensional motion (& 3D motion)



$$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

$$\Downarrow$$

$$v_{ave_x} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

$$\Downarrow$$

$$v_{ave_y} = \frac{y(t_2) - y(t_1)}{t_2 - t_1}$$

Instantaneous velocity:

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t}$$

$$v_x(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$v_y(t) = \lim_{\Delta t \to 0} \frac{y(t + \Delta t) - y(t)}{\Delta t}$$

Average acceleration, instantaneous acceleration

$$\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}$$

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$

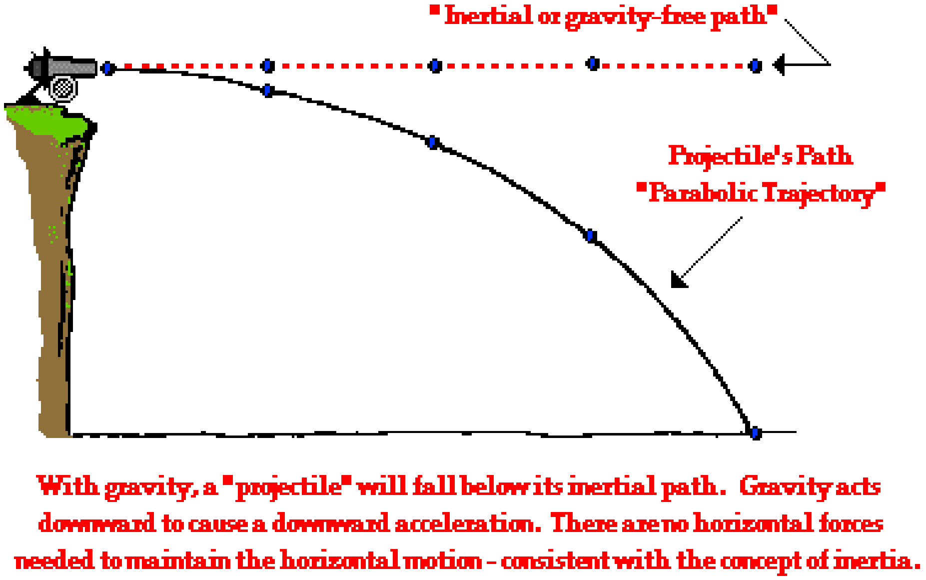
$$a_{ave_x} = \frac{v_x(t_2) - v_x(t_1)}{t_2 - t_1}$$

$$a_x(t) = \lim_{\Delta t \to 0} \frac{v_x(t + \Delta t) - v_x(t)}{\Delta t}$$

$$a_{ave_y} = \frac{v_y(t_2) - v_y(t_1)}{t_2 - t_1}$$

$$a_y(t) = \lim_{\Delta t \to 0} \frac{v_y(t + \Delta t) - v_y(t)}{\Delta t}$$

Motion with uniform acceleration



$$\vec{a} = \text{const.}$$

$$\vec{r}(t) = \vec{r}_o + \vec{v}_o \cdot t + \frac{1}{2}\vec{a} \cdot t^2$$

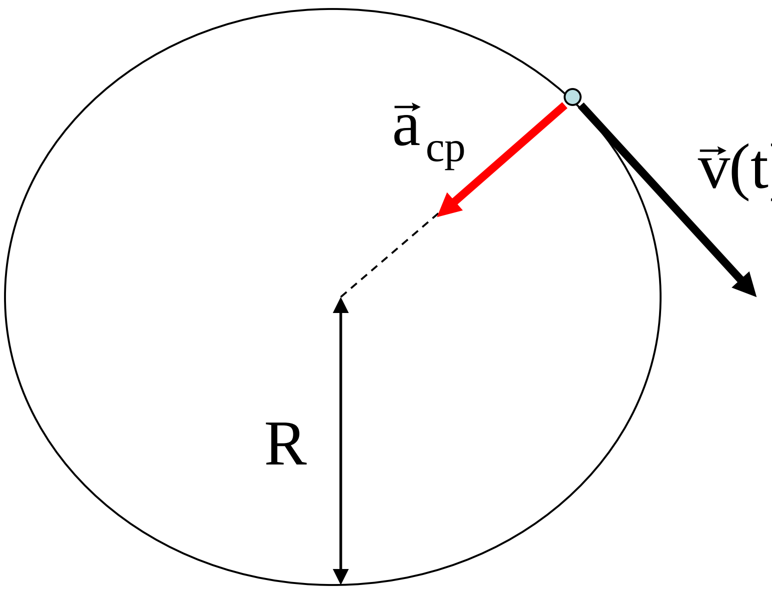
⇓

$$x(t) = x_o + v_{ox} \cdot t + \frac{1}{2} a_x t^2$$ ← horizontal motion

⇓

$$y(t) = y_o + v_{oy} \cdot t + \frac{1}{2} a_y t^2$$ ← vertical motion

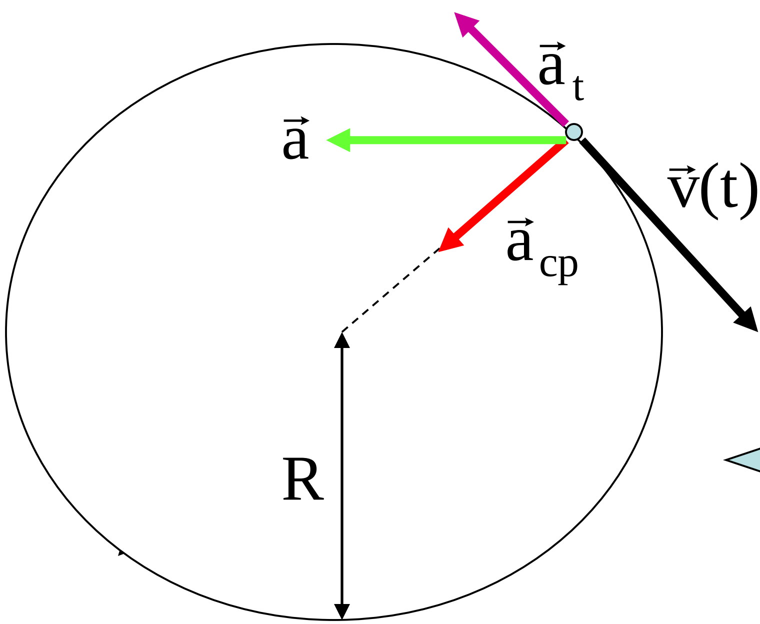
Centripetal acceleration



R: radius

a_{cp}: centripetal acc. $$a_{cp} = \frac{v^2}{R}$$

v=const. $\vec{a}_{cp} \perp \vec{v}(t)$



$\vec{a} = \vec{a}_{cp} + \vec{a}_t$ where $a_t = \frac{\Delta v}{\Delta t}$

$\vec{a}_{cp} \perp \vec{a}_t$

$\Downarrow$

v≠const

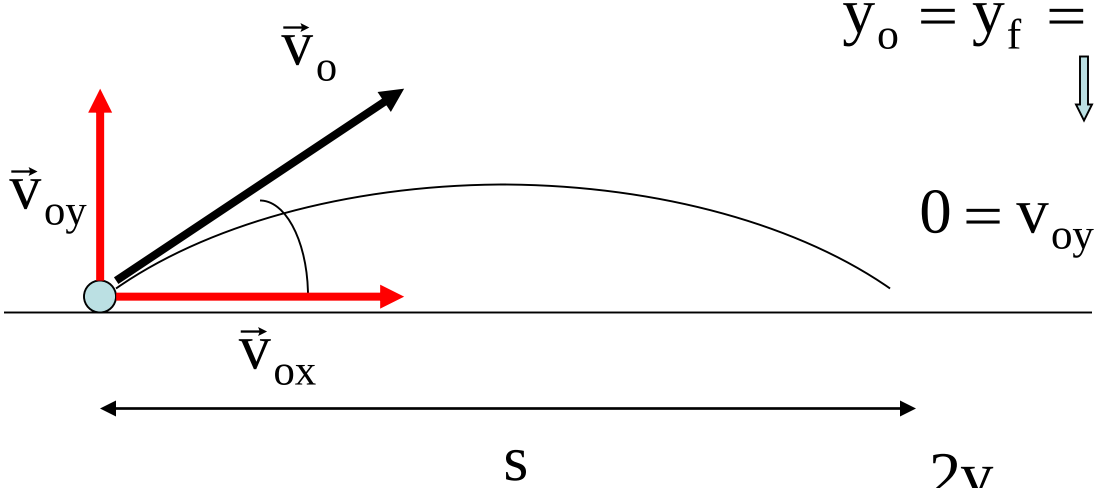
$$a = \sqrt{a_{cp}^2 + a_t^2}$$

Projectile motion

$$y(t) = y_o + v_{oy}t + \frac{1}{2}a_y t^2$$

$$v_{ox} = v_o\cos\Theta$$

$$v_{oy} = v_o\sin\Theta$$



$$y_o = y_f = 0$$

$$0 = v_{oy}t + \frac{1}{2}a_y t^2$$

$$t = 2\frac{v_{oy}}{g}$$

$$t = \frac{2v_{oy}}{a} = \frac{2v_o\sin\Theta}{g}$$

$$s = \frac{v_o^2}{g}\sin(2\Theta)$$ ← horizontal range

$$x(t) = v_{ox}t = v_o\cos\Theta t$$

$$s = v_{ox}t = v_o\cos\Theta t = v_o\cos\Theta \cdot \frac{2v_o\sin\Theta}{g}$$

Horizontal range & launching (elevation) angle

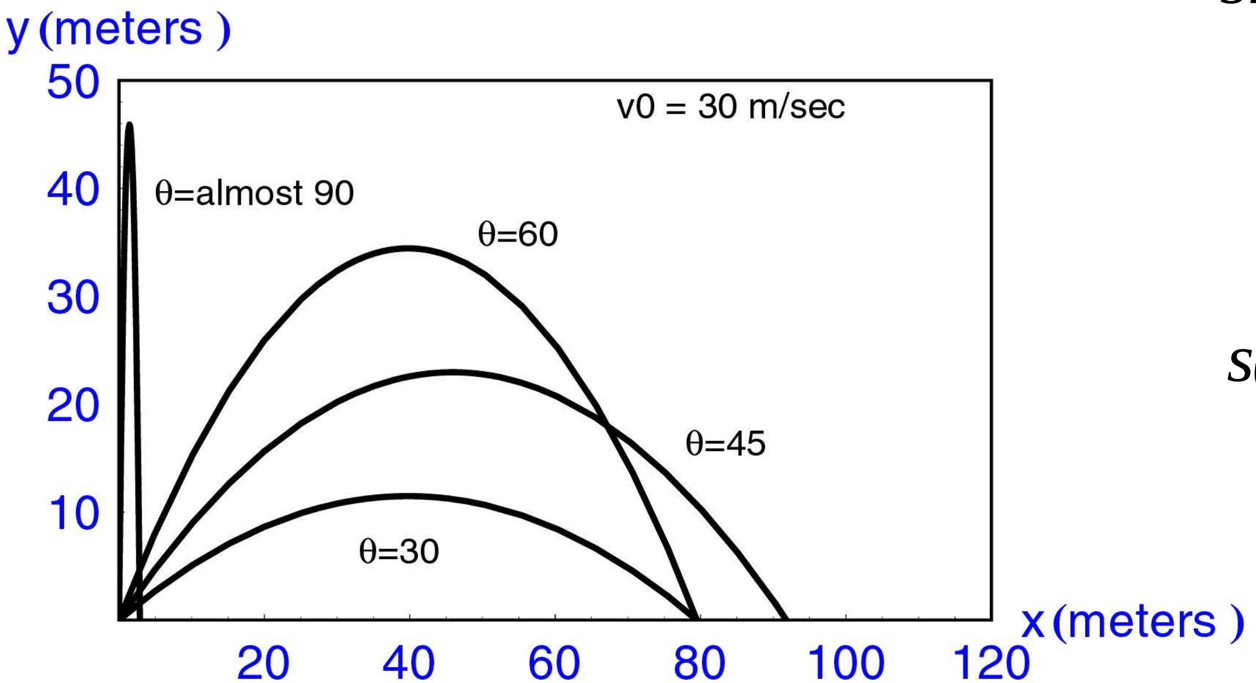
$$s = \frac{v_o^2}{g}\sin(2\Theta)$$

$$s_{max} = \frac{v_o^2}{g} \leftarrow \Theta = 45^o$$

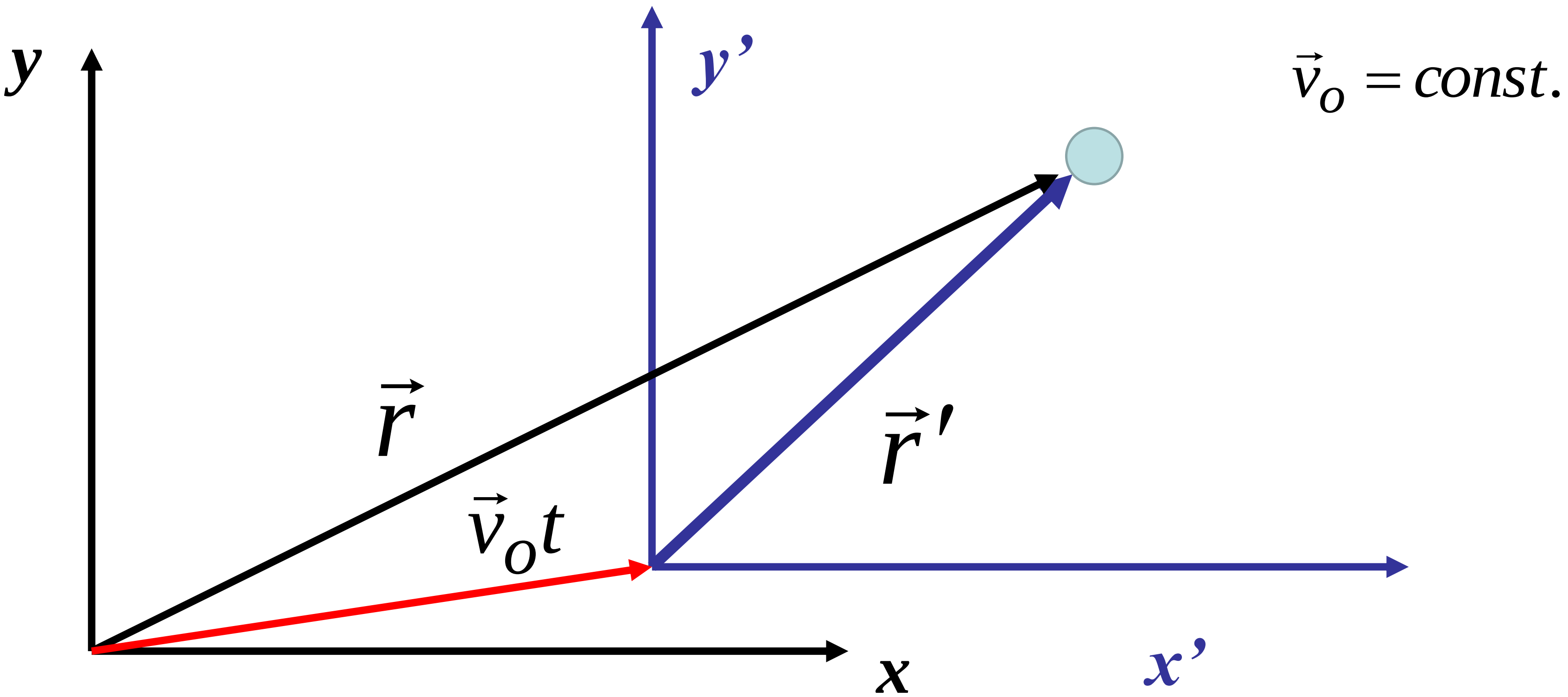
$$\sin\alpha = \sin\left(180^o - \alpha\right)$$

$$\downarrow$$

$$s(\Theta) = s(90^o - \Theta)$$



Relative velocity & relative acceleration



$$\vec{r} = \vec{r}\,' + \vec{v}_o t \quad \longrightarrow \quad \vec{v} = \vec{v}\,' + \vec{v}_o$$

$$\vec{a} = \vec{a}\,'$$