Lecture 2

System of particles,

Linear momentum, collision,

1. Linear momentum and collision

Def. : Linear momentum $\vec{p} = m\vec{v}$

$$\vec{F} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t} \quad \Longrightarrow \quad \vec{F} = \frac{d\vec{p}}{dt}$$

Def. : Impulse $\vec{I} = \vec{F}_{ave}\Delta t = \vec{p}(t_2) - \vec{p}(t_1)$ $\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)dt = \vec{p}(t_2) - \vec{p}(t_1)$

$\Downarrow$

Average force: $\vec{F}_{ave} = \frac{\vec{I}}{\Delta t} = \frac{\vec{p}(t_2) - \vec{p}(t_1)}{t_2 - t_1}$

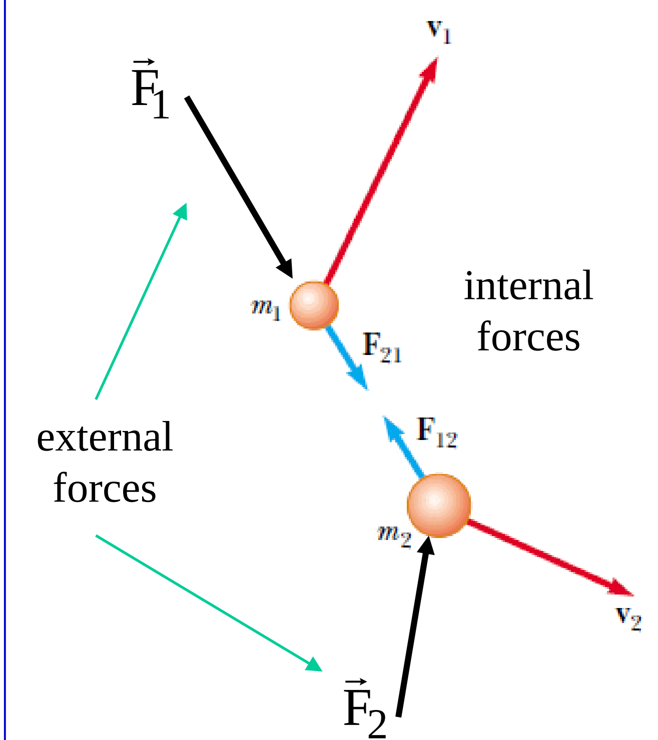
Examples!!!

Conservation of linear momentum I.

$$\vec{F}_1 + \vec{F}_{21} = m_1\vec{a}_1$$

$$\vec{F}_2 + \vec{F}_{12} = m_2\vec{a}_2$$

$$\vec{F}_{net} = \underbrace{\vec{F}_1 + \vec{F}_2}_{\vec{F}_{ext.,net}} + \underbrace{\vec{F}_{12} + \vec{F}_{21}}_{\text{Newton III.}} = m_1\vec{a}_1 + m_2\vec{a}_2$$



$$\vec{F}_{ext.,net} = m_1\frac{\Delta\vec{v}_1}{\Delta t} + m_2\frac{\Delta\vec{v}_2}{\Delta t} \xrightarrow[m_2=\text{const.}]{m_1=\text{const.}} \vec{F}_{ext.,net} = \frac{\Delta(m_1\vec{v}_1)}{\Delta t} + \frac{\Delta(m_2\vec{v}_2)}{\Delta t}$$

$$\vec{F}_{ext.,net} = \frac{\Delta(\vec{p}_1)}{\Delta t} + \frac{\Delta(\vec{p}_2)}{\Delta t} \longrightarrow \vec{F}_{ext.,net} = \frac{\Delta(\vec{p}_1 + \vec{p}_2)}{\Delta t}$$

$$\boxed{\vec{F}_{ext.,net} = \frac{\Delta(\vec{p}_{syst.})}{\Delta t}} \quad *$$

Conservation of linear momentum II.

$$\text{If} \quad \vec{F}_{\text{net, ext.}} = 0 \quad \Rightarrow \quad \vec{p}_{\text{syst.}} = \sum_i \vec{p}_i = \text{const.}$$

Def. : Center of mass (center of gravity)

$$\vec{r}_c = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$$

Velocity of the center of mass
(of system of particles)

$$\vec{v}_c = \frac{\Delta \vec{r}_c}{\Delta t} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i} = \frac{\vec{p}_{\text{syst.}}}{M}$$

Acceleration of the center of mass

$$\vec{a}_c = \frac{\Delta \vec{v}_c}{\Delta t} = \frac{1}{M} \frac{\Delta \vec{p}_{\text{syst.}}}{\Delta t}$$

Conservation of linear momentum III.

$$\vec{F}_{ext.,net} = \frac{\Delta(\vec{p}_{syst.})}{\Delta t} \quad *$$

$$\vec{a}_c = \frac{\Delta \vec{v}_c}{\Delta t} = \frac{1}{M}\frac{\Delta \vec{p}_{syst.}}{\Delta t}$$

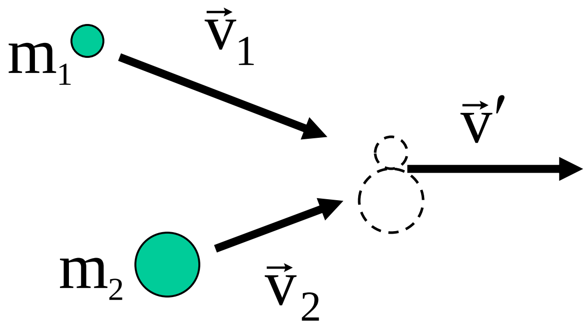
$$\vec{a}_c = \frac{\Delta \vec{v}_c}{\Delta t} = \frac{\sum_i \vec{F}_{ext,\,i}}{\sum_i m_i} = \frac{\vec{F}_{ext,\,net}}{M}$$

If $\vec{F}_{net,\,ext.} = 0 \quad \Rightarrow \quad \vec{a}_c = const. \quad \Rightarrow \quad \vec{v}_c = const.$

If $\vec{F}_{net,\,ext.} = 0$ & $\vec{v}_c = 0 \quad \Rightarrow \quad \vec{r}_c = const.$

Example!!!

Completely inelastic collision



$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}'$$

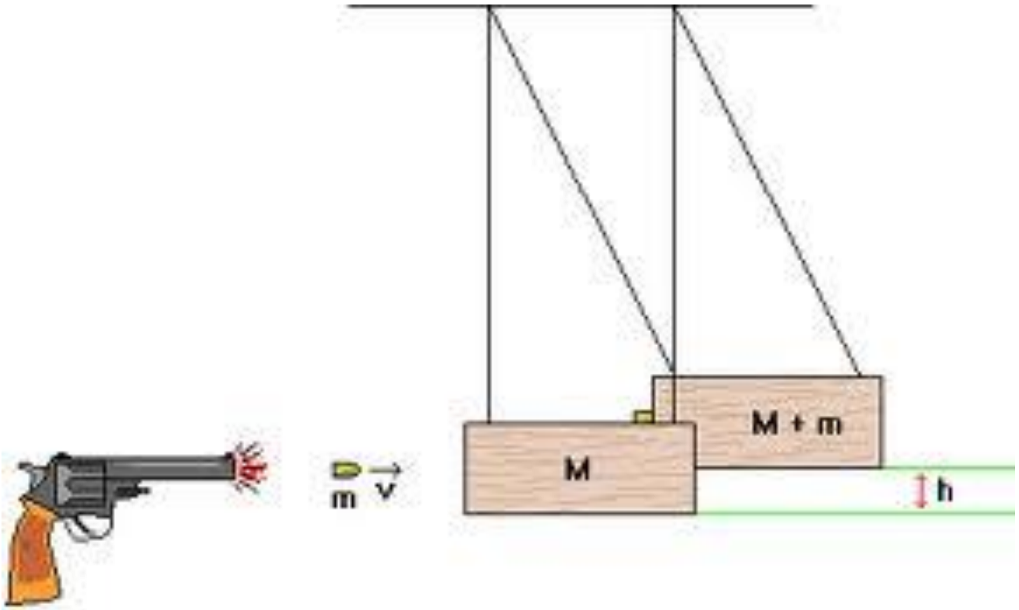
Conservation of linear momentum

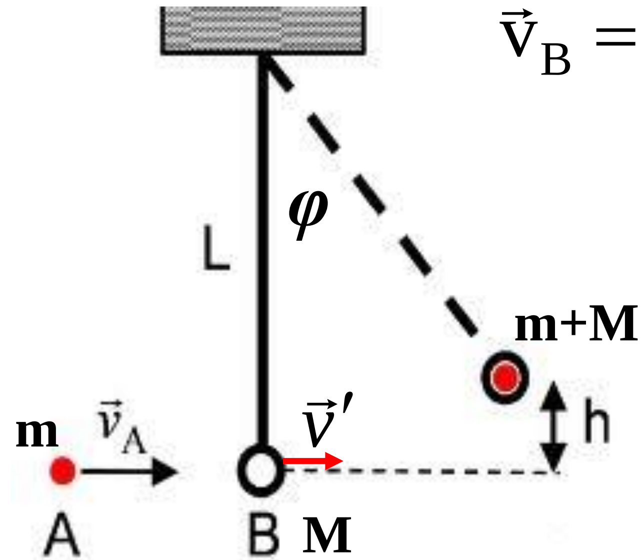
Energy dissipation:

$$\Delta E_k = \frac{1}{2}(m_1 + m_2)v'^2 - \left[\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2\right]$$

(Work of deformation, transfer to heat, etc…)

Completely inelastic collision: an example → ballistic pendulum





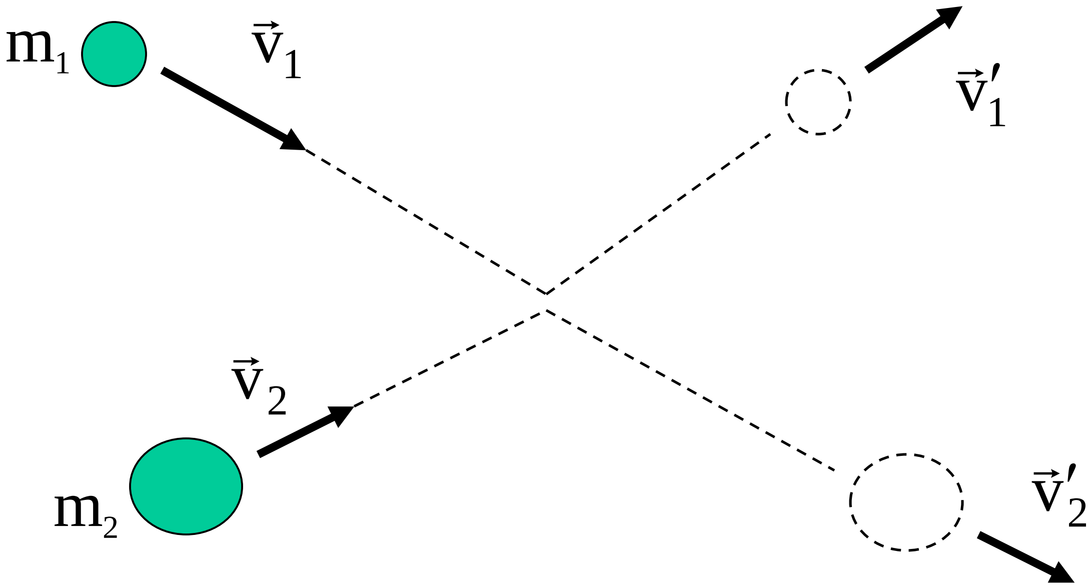
$$\vec{v}_B = 0$$

Conservation of linear momentum: $$m\vec{v}_A = (m+M)\vec{v}'$$

After the collision: $$\frac{1}{2}(m+M)v'^2 = (m+M)gh$$

$$v'^2 = 2gh = 2gL(1 - \cos\varphi) \longrightarrow v_A = \frac{m+M}{m}\sqrt{2gL(1-\cos\varphi)}$$

Elastic collision



Conservation of linear momentum:

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

Conservation of energy:

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

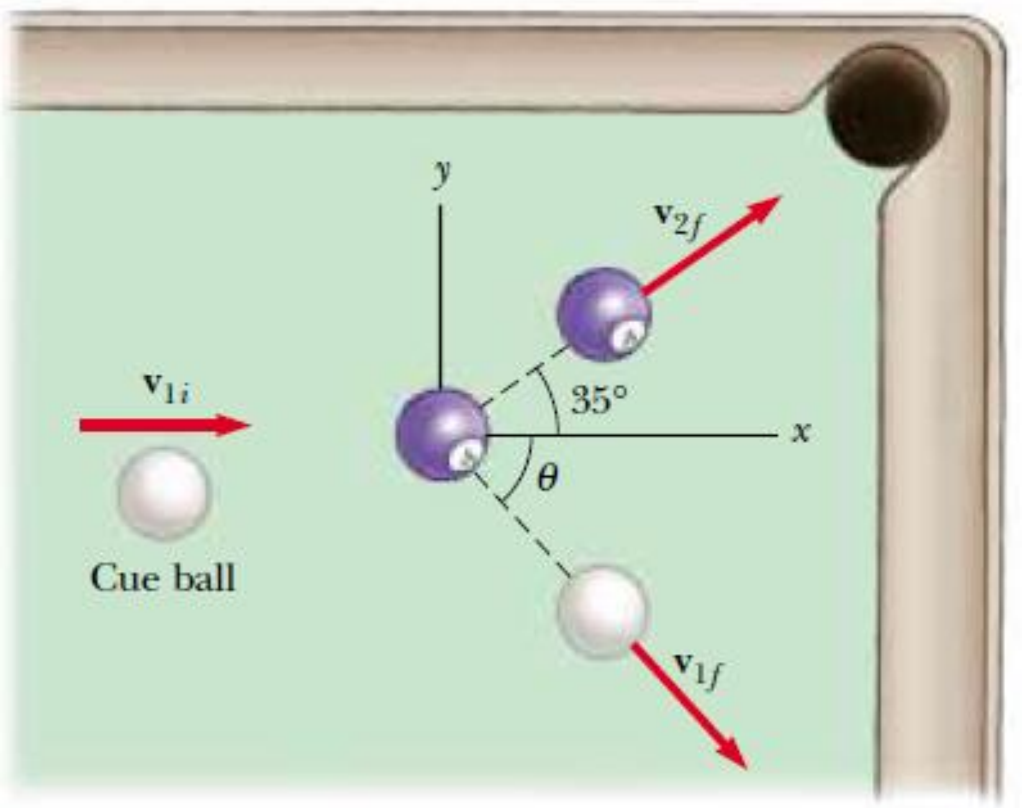
Example:
$v_2=0$, $m_1=m_2$

Example:

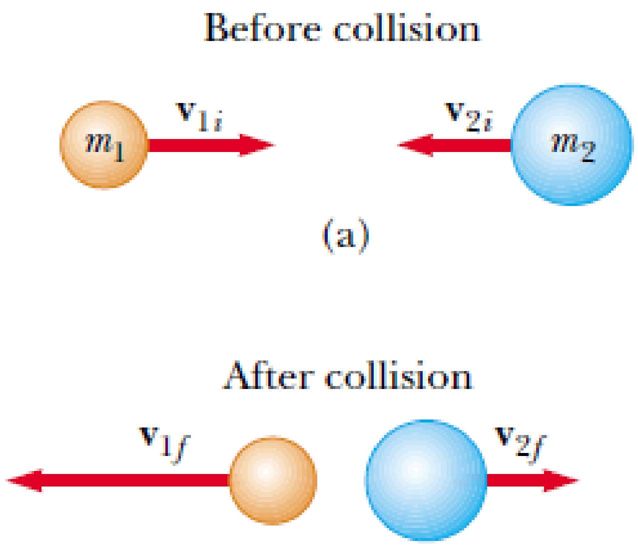
$v_2 = 0$, $m_1 = m_2$

$$\not{m}_1 \mathbf{v}_{1i} = \not{m}_1 \mathbf{v}_{1f} + \not{m}_2 \mathbf{v}_{2f}$$

$$v_{1i}^{\ 2} = v_{1f}^{\ 2} + v_{2f}^{\ 2}$$



Elastic collision in 1D



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 {v_{1i}}^2 + \frac{1}{2} m_2 {v_{2i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2$$

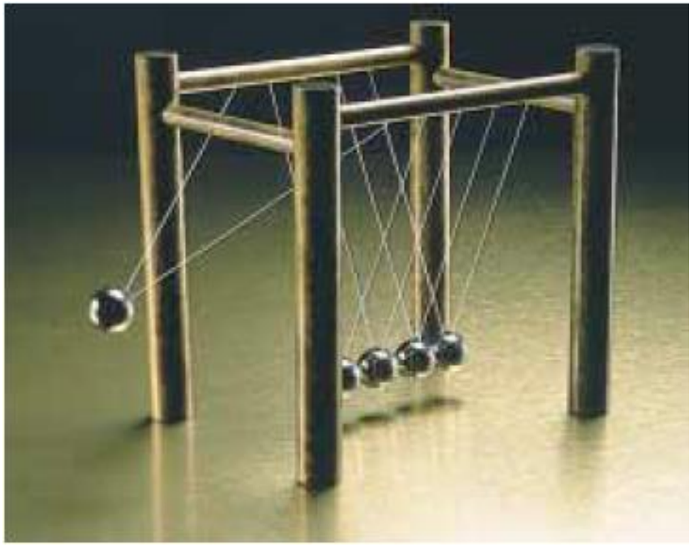
$$m_1 ({v_{1i}}^2 - {v_{1f}}^2) = m_2 ({v_{2f}}^2 - {v_{2i}}^2)$$

$$m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i})$$

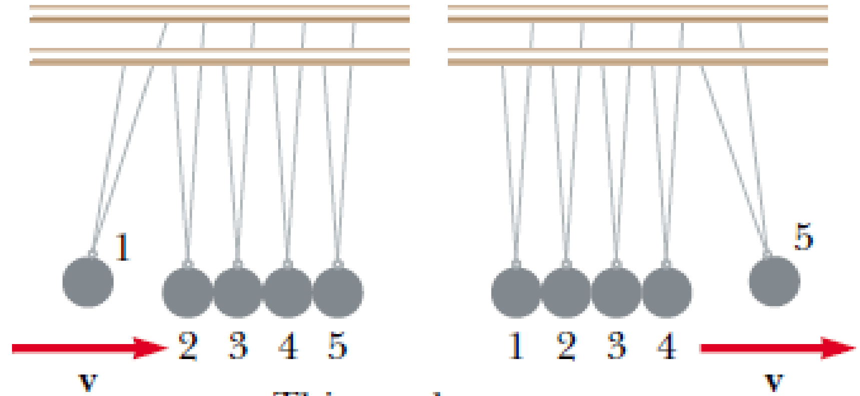
$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i})$$

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$	$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$

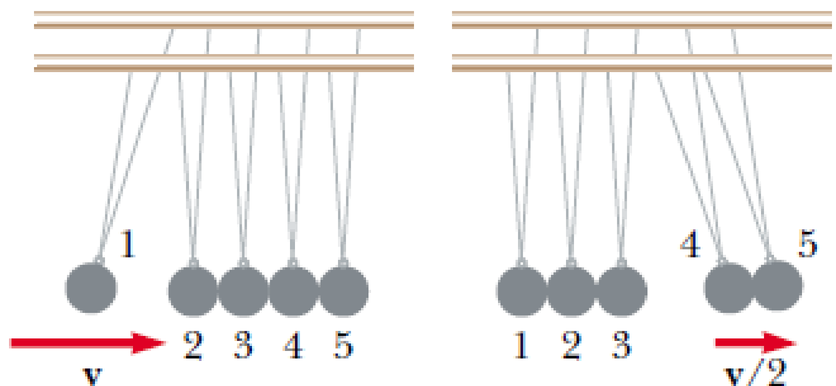


”Stress reliever”



This can happen.

(b)



Can this happen?

(c)

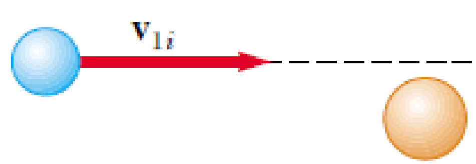
Collision in 2D

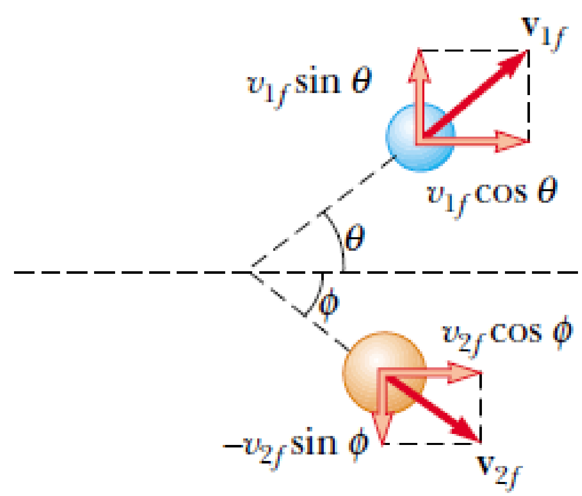
Conservation of linear momentum:

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Example:





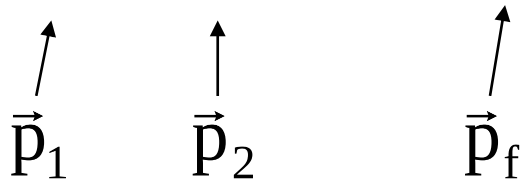
x: $$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

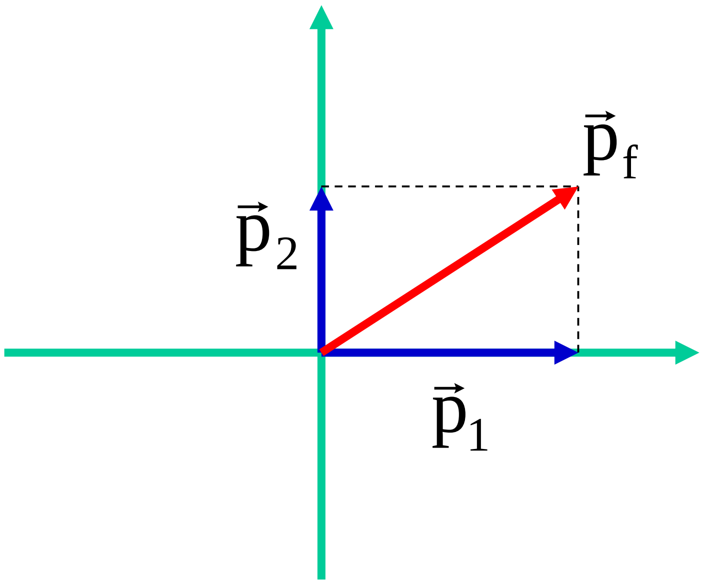
y: $$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

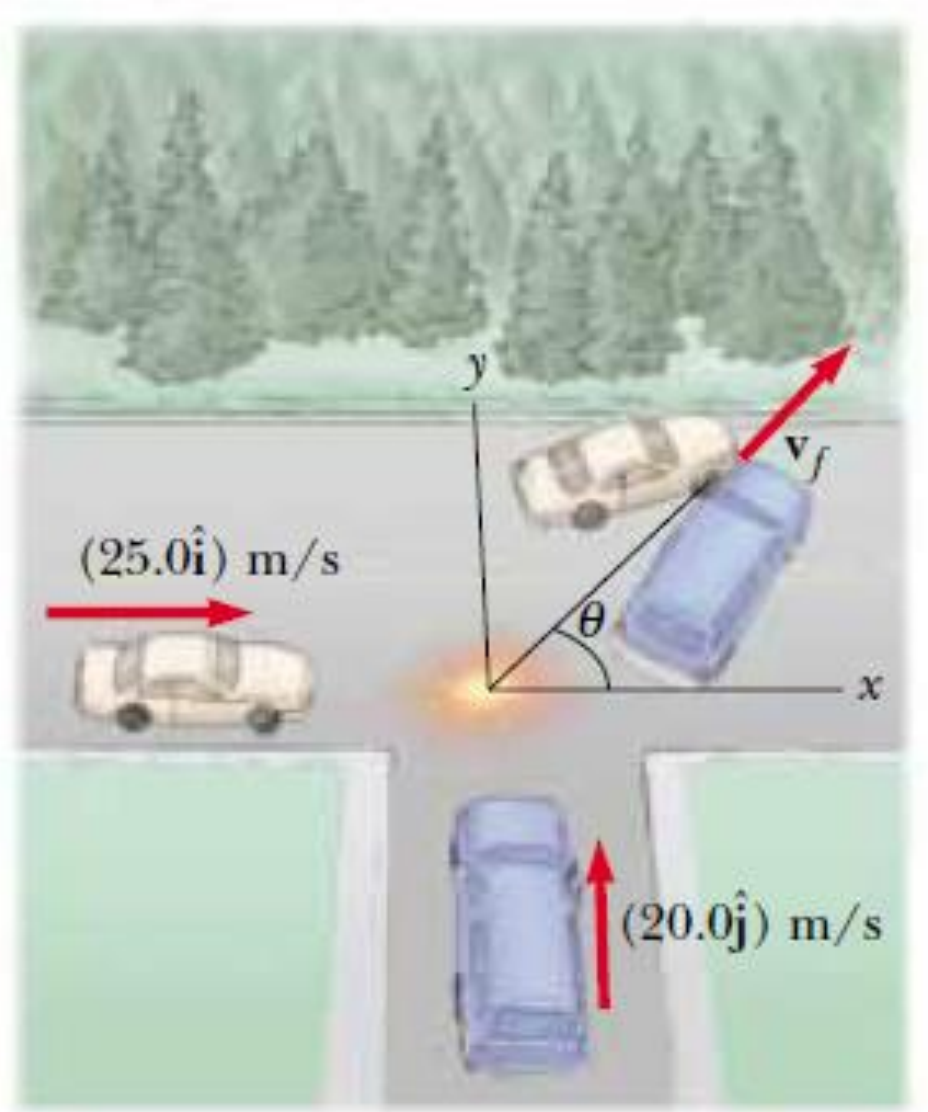
If the collision is elastic: $$\frac{1}{2} m_1 {v_{1i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2$$

Completely inelastic collision in 2D: example → collision of cars

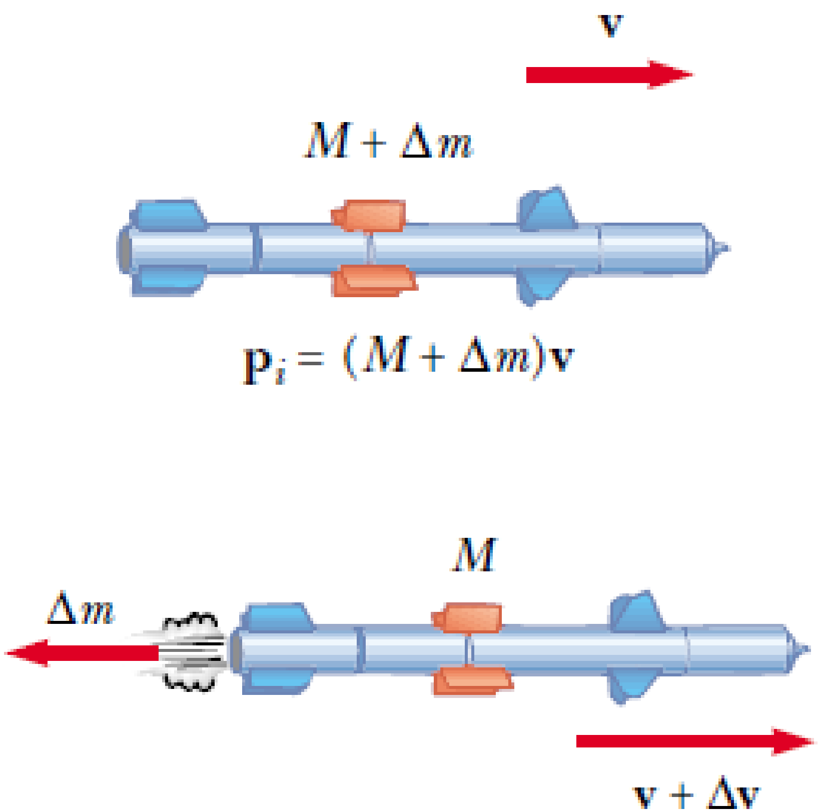
$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}'$$







Rocket propulsion



$$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e) \longrightarrow M\,\Delta v = v_e\,\Delta m$$

$$\boxed{v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)}$$