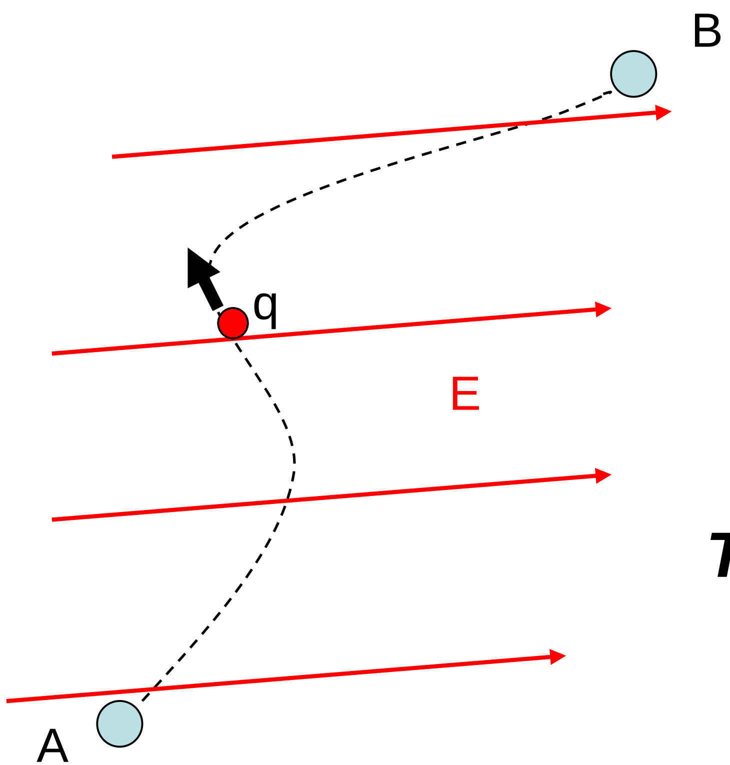
6. lecture

Electric potential

The electric potential difference



$$W_{\text{field}} = \int_A^B \vec{F} d\vec{s} = \int_A^B q\vec{E} d\vec{s} = q \cdot \int_A^B \vec{E} d\vec{s}$$

$$W_{\text{person}} = -q \cdot \int_A^B \vec{E} d\vec{s} = \Delta U$$

(Change in potential energy: ΔU)

The electric potential difference:

$$\boxed{\Delta V = -\int_A^B \vec{E} d\vec{s}}$$

unit : [V]

$$\boxed{\Delta V = \sum_i \vec{E}_i \bullet \Delta \vec{s}_i}$$

$$W_{\text{field}} = -q\Delta V \quad \text{and} \quad W = \Delta E_k$$

Electric potential of a point mass

$$\Delta V = -\int_{r_1}^{r_2} \vec{E} d\vec{s} = -\int_{r_1}^{r_2} k\frac{Q}{r^2} dr = kQ\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

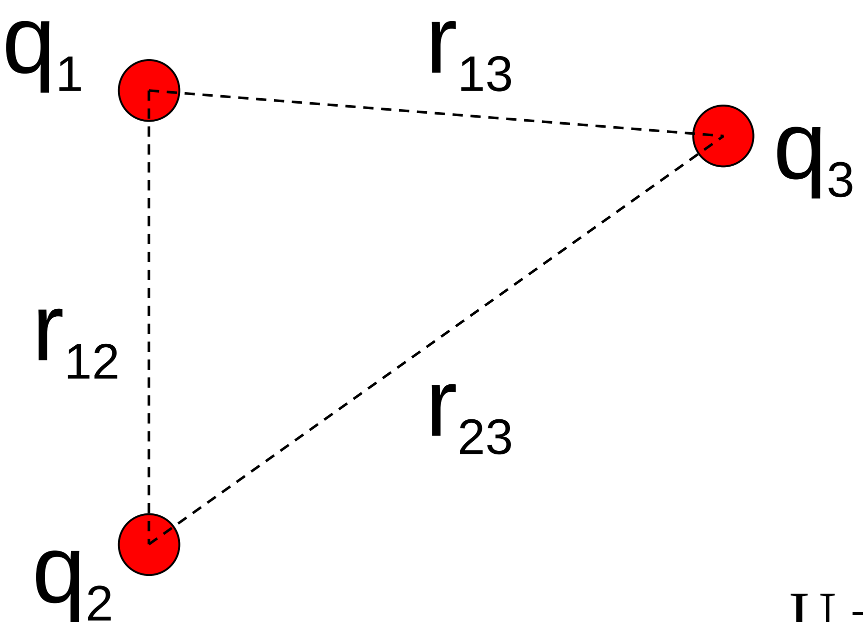
$$r_1 = \infty \quad \text{és} \quad r_2 = r$$

$$\downarrow$$

$$\boxed{V(r) = k\frac{Q}{r}} \quad \text{és} \quad V(r = \infty) = 0$$

The potential energy of q in the filed of Q : $U(r) = qV = k\frac{Qq}{r}$

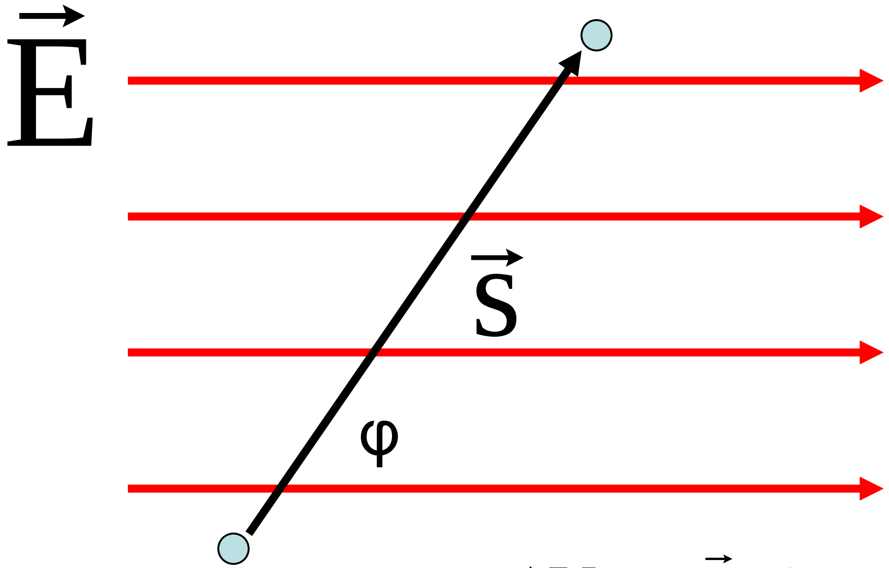
The electrostatic energy of charged particles



$$U = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$

$$U = \sum_{i,j=1}^{N} \frac{1}{2} k \frac{q_i q_j}{r_{ij}} \quad \ldots \quad i \neq j$$

Uniform electric field



$$\Delta V = -\vec{E}\cdot\vec{s} = -E\cdot s\cdot\cos\varphi$$

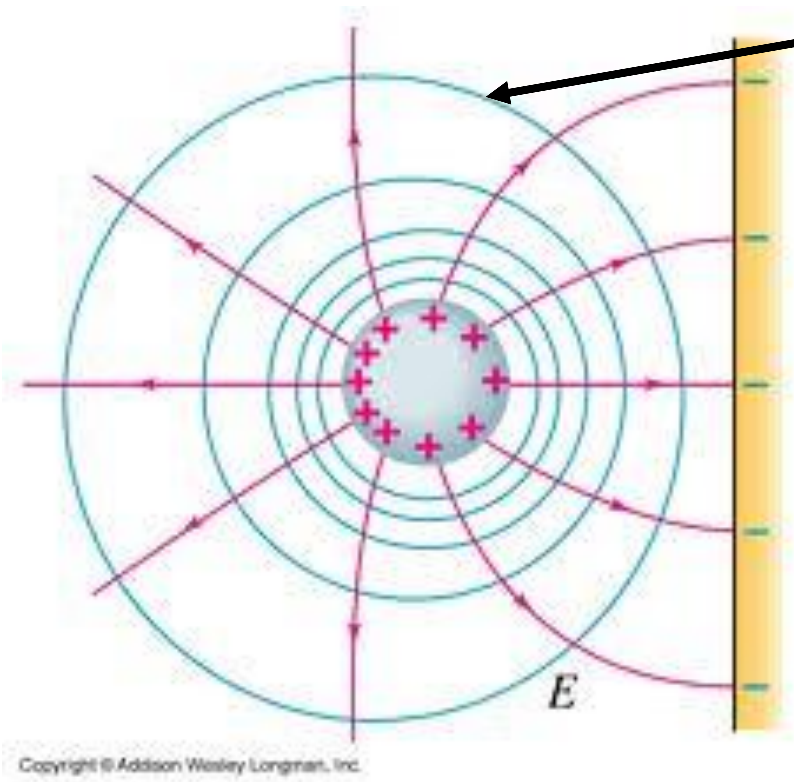
$$\Delta V = -\vec{E}\cdot\vec{s} = -(E_x s_x + E_y s_y + E_z s_z)$$

$$d\vec{s} = dx\,\vec{i}$$

$$dV = -E_x dx \Rightarrow -\frac{dV}{dx} = E_x$$ similarly : $$-\frac{dV}{dy} = E_y$$ and $$-\frac{dV}{dz} = E_z$$

$$\boxed{\vec{E} = -\mathrm{grad}V(r)}$$

Equipotential surface



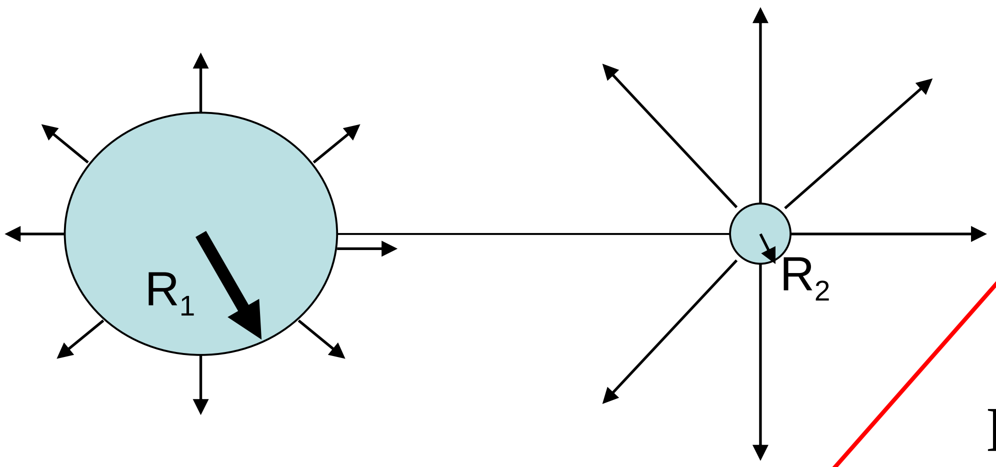
$$U(x, y, z) = \text{const.}$$

$$\vec{E} = -\mathrm{grad}V(r)$$

$$\vec{E} \perp U(\vec{r})$$

An example: the electric field close to a peak

$$V(r) = k\frac{Q}{r}$$



$$V_1 = k\frac{Q_1}{R_1} = k\frac{Q_2}{R_2} = V_2$$

Spherical symmetry :

$$E_1 = k\frac{Q_1}{R_1^2} \quad \text{és} \quad E_2 = k\frac{Q_2}{R_2^2}$$

Conductive spheres are connected by a thin wire

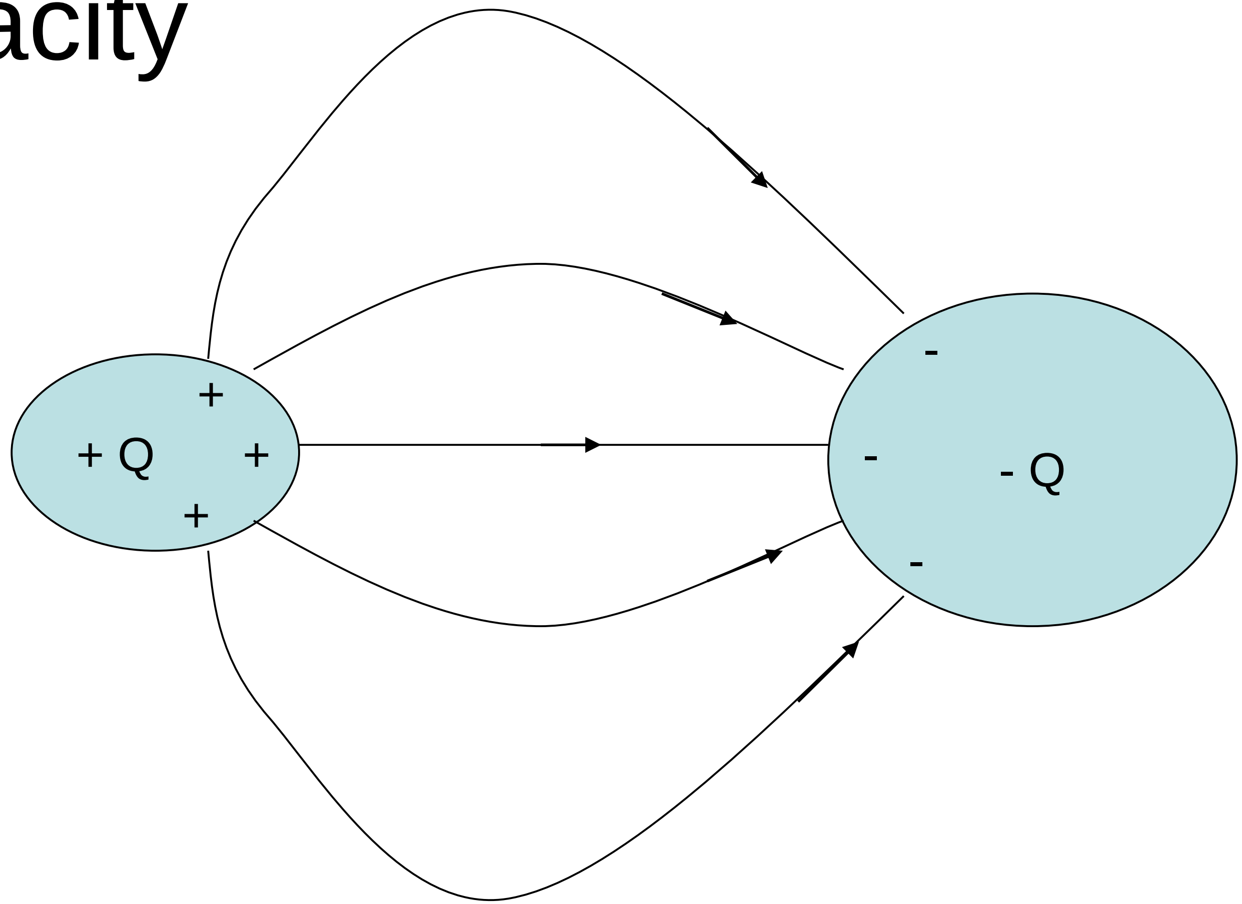
$$\boxed{\frac{E_2}{E_1} = \frac{R_1}{R_2}}$$

Surface of arrangement → equipotential surface → V = const.

Lightning rod !!!

Capacity, capacitors, dielectrics

Capacity

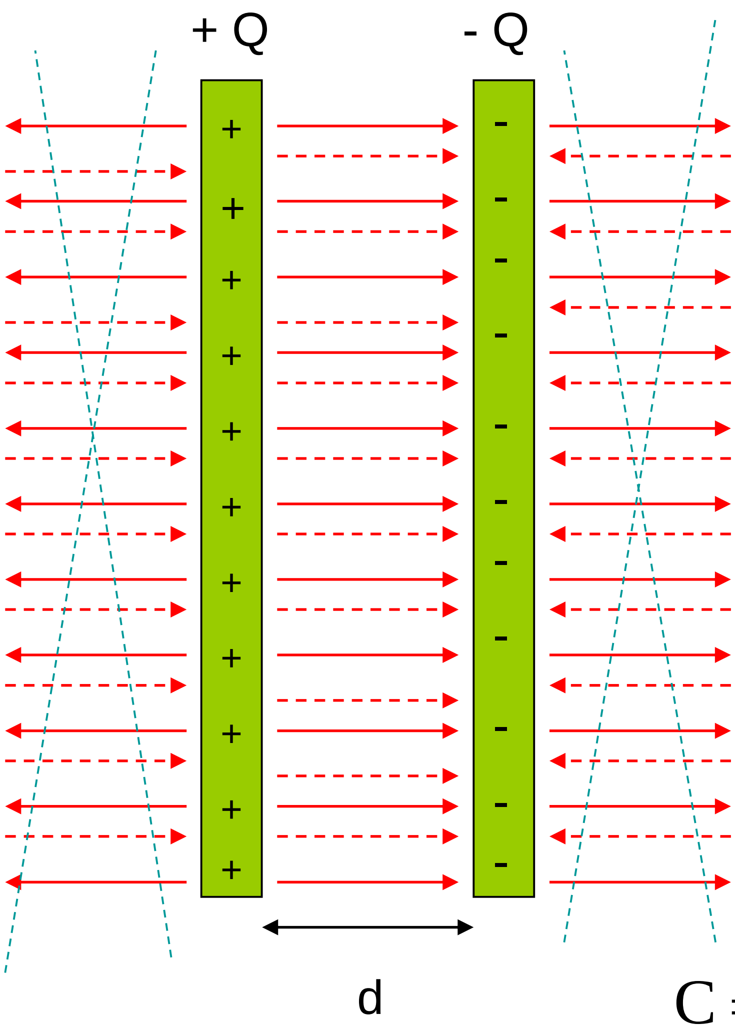


Capacity:

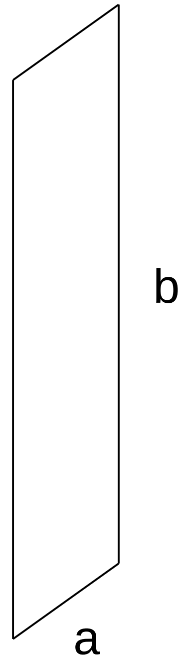
$$C = \frac{Q}{V} \qquad \left[\frac{C}{V} = F \right]$$

Capacitors:

1. Parallel plate capacitor



$$C = \frac{Q}{V}$$



d << a, b

The electric field of uniformly charged (σ = const.) plate:

$$E = \frac{\sigma}{2\varepsilon_0}$$

Electric field between the plates:

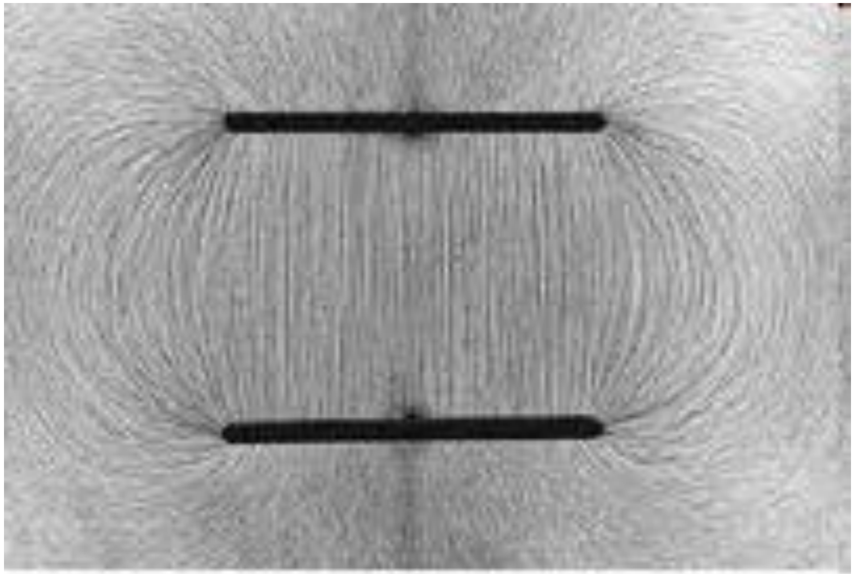
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Electric potential (difference) of plates:

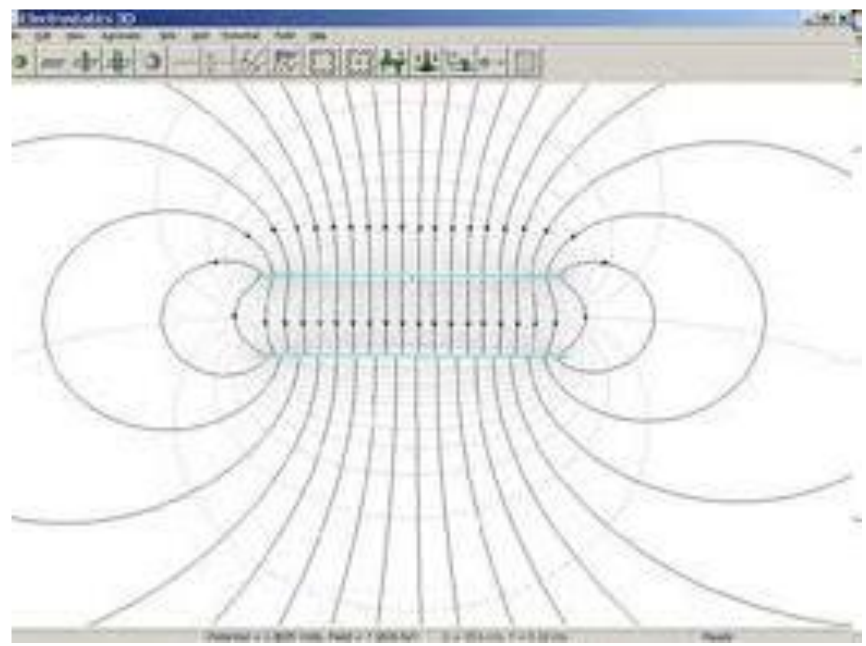
$$V = E \cdot d$$

The capacity:

$$\boxed{C = \frac{\varepsilon_0 A}{d}}$$

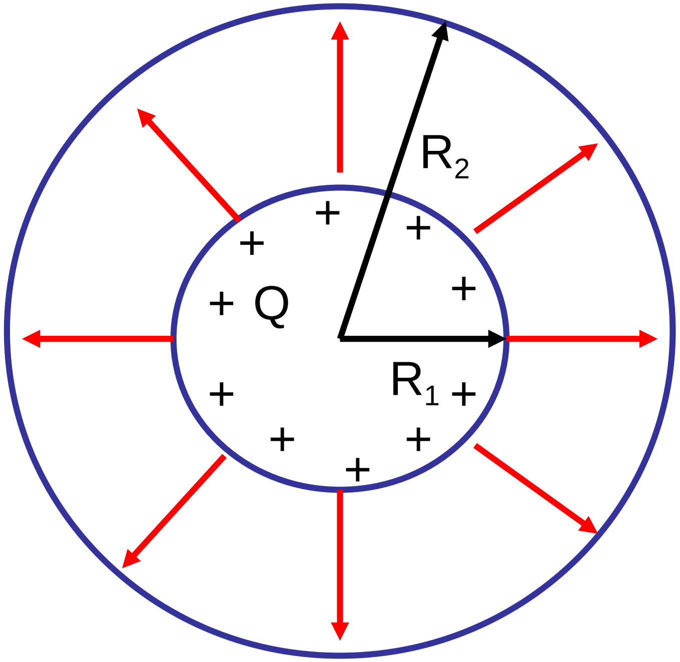


(c)



2. The spherical capacitor

Electric field strength between shells: $E = k\frac{Q}{r^2}$



We have seen:

$$\Delta V = -\int_{r_1}^{r_2} \vec{E} d\vec{s} = -\int_{r_1}^{r_2} k\frac{Q}{r^2} dr = kQ\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Electric potential:

$$V = kQ\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$C = \frac{Q}{V}$$

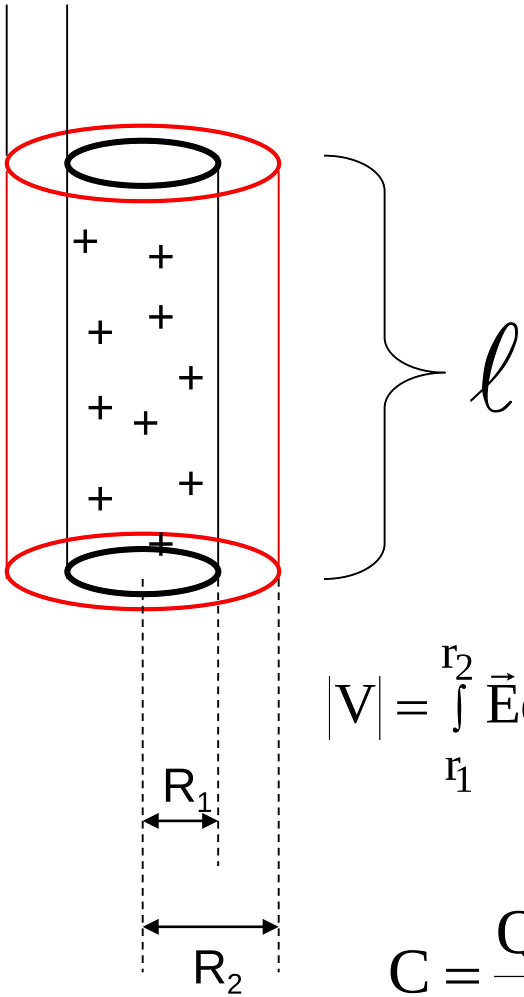
$$\boxed{C = \frac{4\pi\varepsilon_o R_1 R_2}{R_2 - R_1}}$$

Capacity of a conducting sphere:

$R_2 \to \infty$ (R_1=R):

$C=4\pi\varepsilon_o R$

3. The cylindrical capacitor



Electric field strength between cylinders :

$$E = \frac{\sigma R}{\varepsilon_o} \cdot \frac{1}{r} \qquad (R = R_1)$$

$$Q = R_1 2\pi\ell\sigma \rightarrow E = \frac{Q}{2\pi\varepsilon_o\ell}\frac{1}{r}$$

Electric potential:

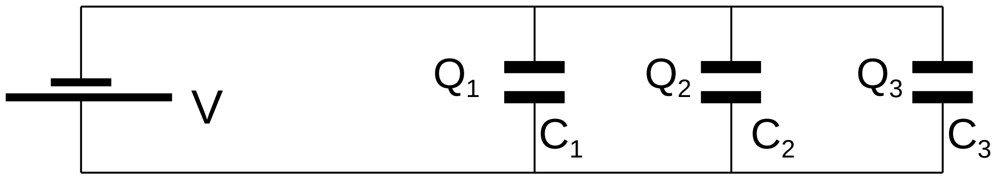
$$|V| = \int_{r_1}^{r_2} \vec{E}d\vec{s} = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_o\ell}\frac{1}{r}dr = \frac{Q}{2\pi\varepsilon_o\ell}[\ell n(r)]_{R_1}^{R_2} = \frac{Q}{2\pi\varepsilon_o\ell}\ell n(\frac{R_2}{R_1})$$

$$C = \frac{Q}{V}$$

$$\boxed{C = \frac{2\pi\varepsilon_o\ell}{\ell n\left(\frac{R_2}{R_1}\right)}}$$

Combination of capacitors:

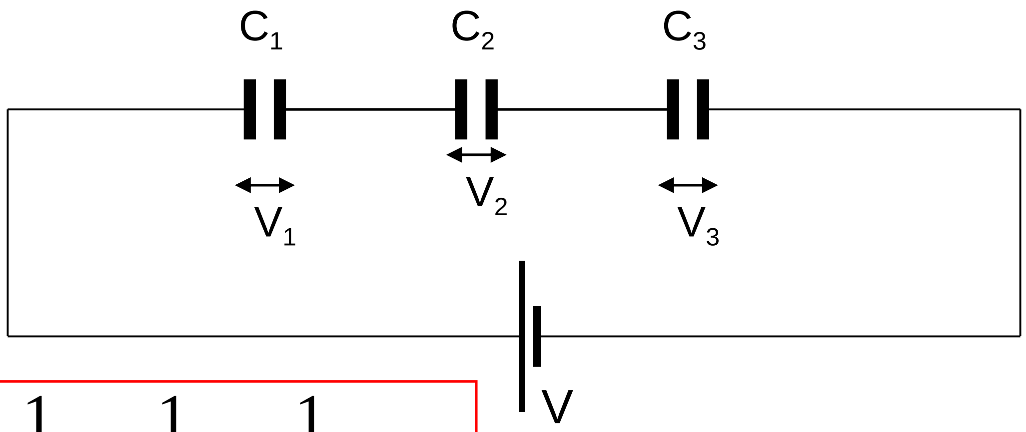
1. Parallel combination:



$$Q = Q_1 + Q_2 + Q_3 \longrightarrow C_e V = C_1 V + C_2 V + C_3 V$$

$$\boxed{C_e = C_1 + C_2 + C_3 + \ldots}$$

2. Series combination :



$$V = V_1 + V_2 + V_3$$

$$\frac{Q}{C_e} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\boxed{\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$

The energy stored in charged capacitor:

$$dW = VdQ \rightarrow W = \int_0^Q V(q)dq = \int_0^Q \frac{q}{C}dq \boxed{= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}VQ = \frac{1}{2}CV^2}$$

The energy density of electric field: $\varepsilon_E = \dfrac{\text{energy}}{\text{volume}} = \dfrac{W}{V}$

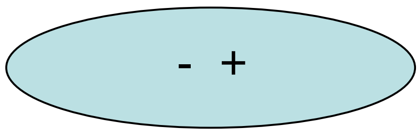
The energy of a parallel plate capacitor:

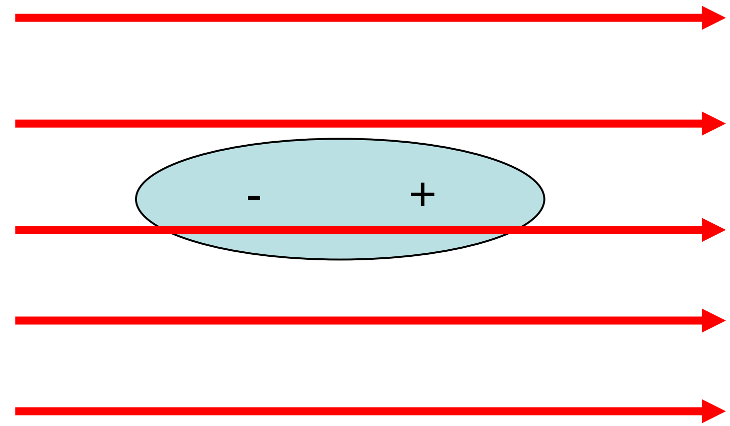
$$W = \frac{1}{2}CV^2 = \frac{1}{2}\underbrace{\left(\frac{\varepsilon_o A}{d}\right)}_{C}\underbrace{(Ed)^2}_{V=Ed} = \frac{1}{2}\varepsilon_o E^2 Ad$$

$$\boxed{\varepsilon_E = \frac{1}{2}\varepsilon_o E^2}$$

Dielectrics

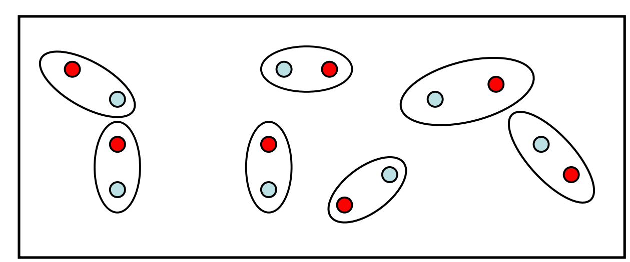
Non polar dielectics:



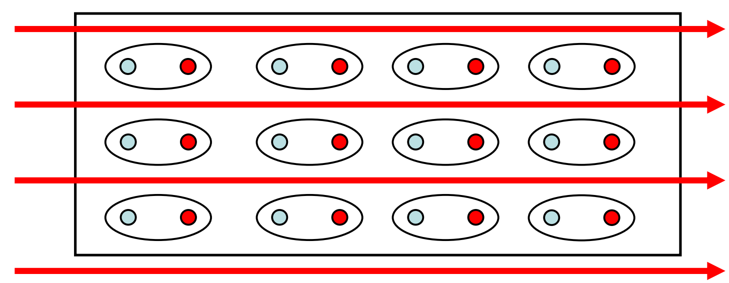


Polar dielectics:

Without external field:



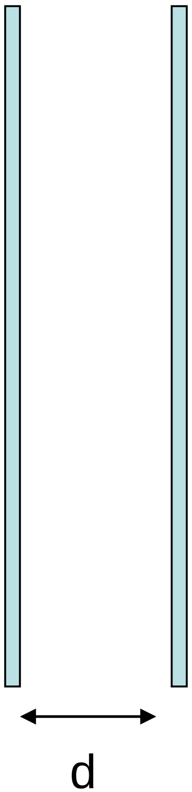
In electric field:



Dielectrics

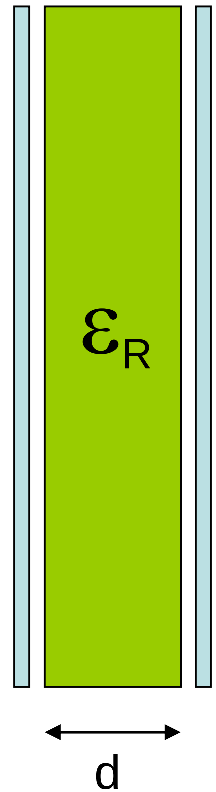
Substance:	dielectic. Const. (ε_R)	electric strength (10^6 V/cm)
Vacuum	1	-
Dry air	1.00059	3
Water	80	-
Glass	4 – 6	13
Mika	5	3000

Dielectrics

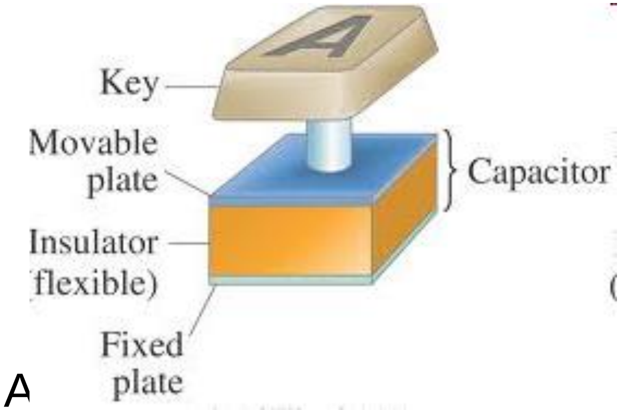


A

$$C_o = \frac{\varepsilon_o A}{d}$$



A



$$C = \frac{\varepsilon_o \varepsilon_R A}{d} = \varepsilon_R C_o$$

Application: electronics, sensors, to measure displacement, etc.

Examples

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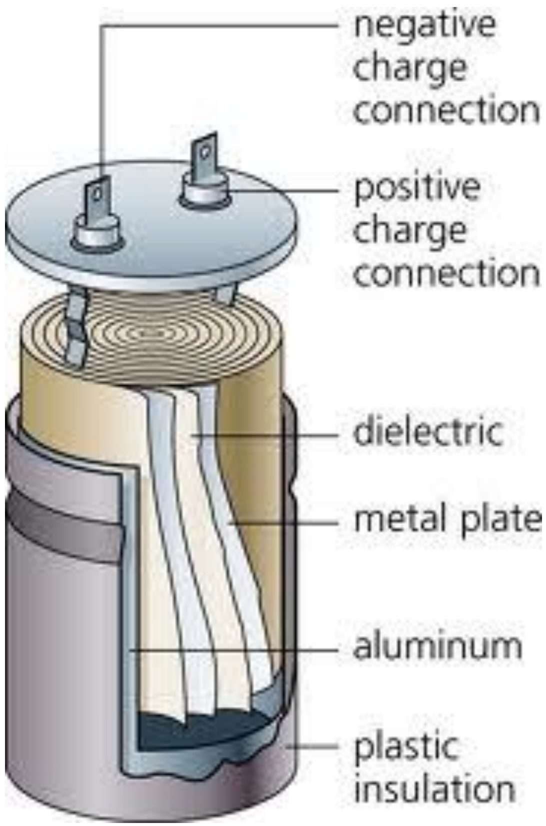

”Parallel-plate capacitor”

Variable capacitor