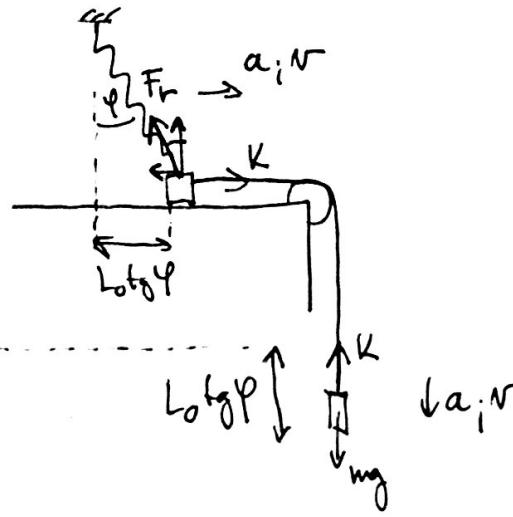
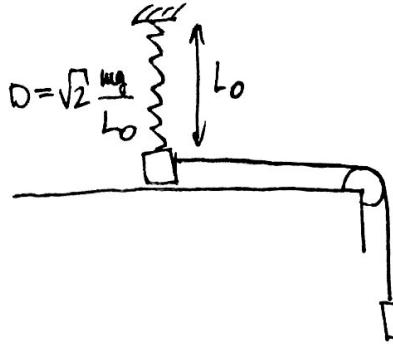


F1

a) N maximális, ha $a=0$.

$$\left. \begin{array}{l} mg = K \\ F_r \cdot \sin \varphi = K \\ F_r \cdot \cos \varphi = mg \end{array} \right\} \Rightarrow \sin \varphi = \cos \varphi \rightarrow \varphi = 45^\circ$$

Energiamegosztás: $mg \cdot L_0 \operatorname{tg} 45^\circ = 2 \cdot \frac{1}{2} m v_{\max}^2 + \frac{1}{2} D \left(\frac{L_0}{\cos 45^\circ} - L_0 \right)^2$

$$mg L_0 = mv_{\max}^2 + \frac{1}{2} \cdot \frac{\sqrt{2}mg}{L_0} \cdot L_0^2 (\sqrt{2}-1)^2$$

$$v_{\max}^2 = g L_0 \left(1 - \frac{\sqrt{2}}{2} (3-2\sqrt{2}) \right) = \frac{6-3\sqrt{2}}{2} g L_0$$

$$v_{\max} = \sqrt{\frac{6-3\sqrt{2}}{2} g L_0} \approx 0,94 g L_0$$

b) maximális h esetén $N=0$. Energiamegosztás:

$$mg L_0 \operatorname{tg} \varphi = \frac{1}{2} D L_0^2 \left(\frac{1}{\cos \varphi} - 1 \right)^2$$

$$mg L_0 \operatorname{tg} \varphi = \frac{1}{2} \frac{\sqrt{2}mg L_0}{L_0} \frac{(1-\cos \varphi)^2}{\cos^2 \varphi}$$

$$\operatorname{tg} \varphi = \frac{\sqrt{2}}{2} \frac{1-2\cos \varphi + \cos^2 \varphi}{\cos \varphi}$$

$$\sqrt{2} \sin \varphi \cos \varphi = 1 - 2\cos \varphi + \cos^2 \varphi$$

Használjuk fel, hogy

$$\sin \varphi = \frac{h}{\sqrt{L_0^2 + h^2}} ; \cos \varphi = \frac{L_0}{\sqrt{L_0^2 + h^2}}$$

$$\sqrt{2} \cdot \frac{hL_0}{L_0^2 + h^2} = 1 - \frac{2L_0}{\sqrt{L_0^2 + h^2}} + \frac{L_0^2}{L_0^2 + h^2}$$

$$\sqrt{2} h L_0 = L_0^2 + h^2 - 2L_0 \sqrt{L_0^2 + h^2} + L_0^2 \quad / : L_0^2$$

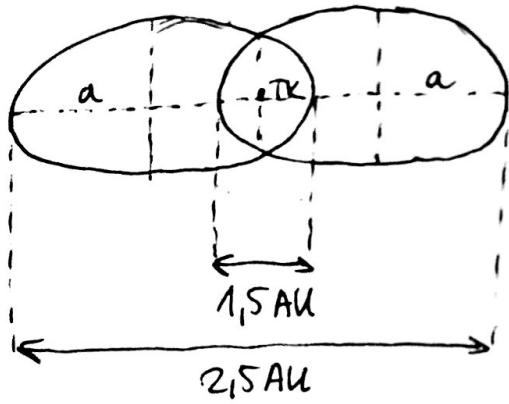
$$\sqrt{2} \frac{h}{L_0} = \left(\frac{h}{L_0}\right)^2 + 2 - 2 \sqrt{1 + \left(\frac{h}{L_0}\right)^2}$$

$$\text{Legyen } \frac{h}{L_0} = x: \quad \sqrt{2} x = x^2 + 2 - 2 \sqrt{1+x^2}$$

Az ítmákatásban megadottak mint próbálgalással: $x = \sqrt{8} \Rightarrow h = \underline{\sqrt{8} L_0}$

F2.

I. megoldás: TK körül keringenek ellipszispályán, TK a húzópontban van (Kepler I. töréje)

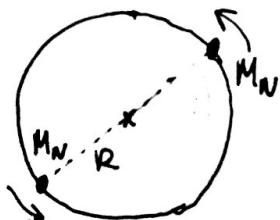


Az ellipszis telj nagytengelye a .

$$2a = \frac{2.5 \text{ AU}}{2} + \frac{1.5 \text{ AU}}{2} = 2 \text{ AU}$$

Kepler III. töréje: $\frac{a^3}{T^2} = \text{áll.}$

Az állandó meghatározásához tekintsünk körpárat:



$$\frac{\gamma M_N}{4R^2} = M_N \cdot \frac{v^2}{R} \rightarrow v = \sqrt{\frac{\gamma M_N}{4R}}$$

$$v = \frac{2\pi R}{T_{\text{kör}}} = \sqrt{\frac{\gamma M_N}{4R}} \Rightarrow \frac{R^3}{T_{\text{kör}}^2} = \frac{\gamma M_N}{16\pi^2}$$

Tehát:

$$\frac{a^3}{T^2} = \frac{\gamma M_N}{16\pi^2} \rightarrow T = a^{3/2} \cdot \sqrt{\frac{16\pi^2}{\gamma M_N}}$$

Föld esetén:

$$\frac{a_F^3}{T_F^2} = \frac{\gamma M_N}{4\pi^2}$$

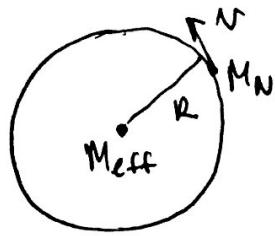
Esetünkben $a_F = a$.

$$\Rightarrow T = a^{3/2} \cdot 4\pi \frac{T_F}{2\pi a^{3/2}} = 2T_F = 2 \text{ év}$$

II. megoldás: TK-ban effektív tömeg:

$$\text{térben: } \frac{\gamma M_N^2}{(2,5 \text{ AU})^2} = \frac{\gamma M_N M_{\text{eff}}}{\left(\frac{2,5 \text{ AU}}{2}\right)^2} \Rightarrow M_{\text{eff}} = \frac{M}{4} \quad (\text{körben is ugyanez az eredmény})$$

M_{eff} hűtői körféjára:



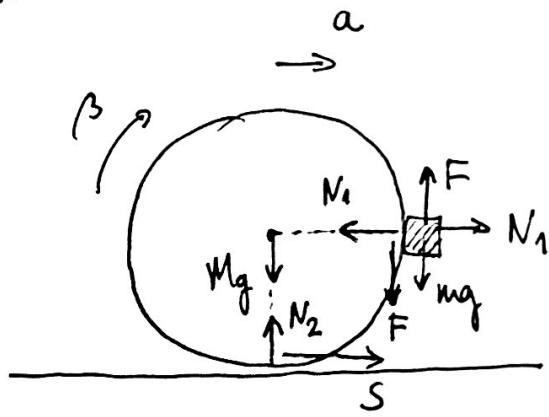
$$\frac{\gamma M_N M_{\text{eff}}}{R^2} = M_N \frac{v^2}{R} \rightarrow v = \sqrt{\frac{\gamma M_{\text{eff}}}{R}} = \frac{2R\pi}{T_{\text{ker}}} \rightarrow$$

$$\rightarrow \frac{R^3}{T_{\text{ker}}^2} = \frac{\gamma M_N}{16\pi^2}$$

innen Kepler III. törvényét is a

Föld keringésére vonatkozó eredmény felhasználva az I. megoldás eredményt kapjuk.

F3.



$$\text{egérre: } mg = F \quad (1)$$

$$N_1 = ma \quad (2)$$

$$\text{albrasra: } Mg + F = N_2 \quad (3)$$

$$S - N_1 = Ma \quad (4)$$

$$(F - S)R = M R^2 \beta ; a = \beta R \quad (5)$$

$$S \leq \mu_0 N_2 \quad (6)$$

$$(5): F - S = Ma \xrightarrow{(1)} mg - S = Ma \quad \left. \begin{array}{l} \\ \end{array} \right\} \Rightarrow mg - S = M \cdot \frac{S}{m+M}$$

$$(2) \xrightarrow{(4)}: S - ma = Ma \rightarrow S = (m+M)a \quad \left. \begin{array}{l} \\ \end{array} \right\} S = mg \cdot \frac{m+M}{m+2M}$$

$$(3): N_2 = (m+M)g$$

$$\xrightarrow{} \quad \quad \quad$$

$$(6): mg \frac{m+M}{m+2M} \leq \mu_0 (m+M)g \Rightarrow \underline{\underline{\mu_0 \geq \frac{m}{m+2M}}}$$

b) Az egér műrője:

$$W = \frac{1}{2}mv^2 + \frac{1}{2}Mv^2 + \frac{1}{2}\varnothing_{\text{TR}}w^2$$

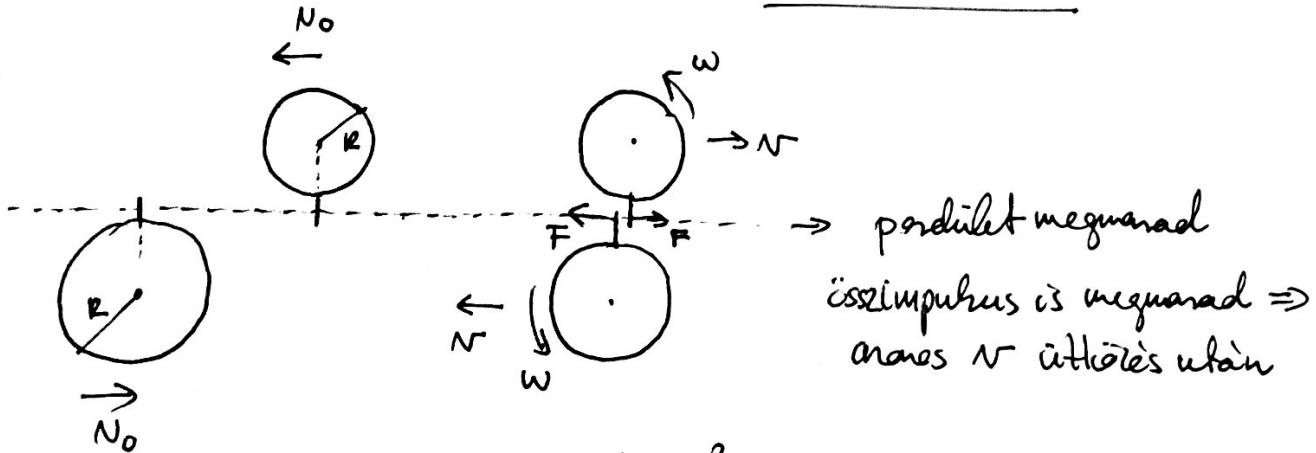
gyorsulások: $a = \frac{s}{m+M} = \frac{m}{m+2M}g \rightarrow v = at = \frac{m}{m+2M}gt$

$$\beta = \frac{a}{R} = \frac{m}{m+2M} \frac{g}{R} \rightarrow w = \beta t = \frac{m}{m+2M} \frac{gt}{R}$$

$$W = \frac{1}{2}m\left(\frac{m}{m+2M}gt\right)^2 + \frac{1}{2}M\left(\frac{m}{m+2M}gt\right)^2 + \frac{1}{2}M\cancel{\beta^2}\left(\frac{m}{m+2M}gt\right)^2 \cdot \frac{1}{R^2} =$$

$$= \left(\frac{m}{m+2M}gt\right)^2 \left(\frac{1}{2}m + \frac{1}{2}M + \frac{1}{2}M\cancel{\beta^2}\right) = \underline{\underline{\frac{1}{2} \frac{m^2}{m+2M} g^2 t^2}}$$

F4.



eredetileg megnarad

összimpulusz is megnarad \Rightarrow
azaz v áthorás után

egy keretben: $mv_0R = -mvR + \frac{1}{2}mR^2\omega \rightarrow v = \frac{R\omega}{2} - v_0 \quad (1)$

energiamegnaradás: $2 \cdot \frac{1}{2}mv_0^2 = 2 \cdot \frac{1}{2}mv^2 + 2 \cdot \frac{1}{2} \cdot \frac{1}{2}mR^2\omega^2 \rightarrow$

$$\rightarrow 2v_0^2 = 2v^2 + R^2\omega^2 \quad (2)$$

(1) és (2): $2v_0^2 = 2 \cdot \left(\frac{R^2\omega^2}{4} - v_0R\omega + v_0^2 \right) + R^2\omega^2$

$$0 = \frac{R^2\omega^2}{2} - 2v_0R\omega + R^2\omega^2$$

$$\frac{3}{2}R^2\omega^2 = 2v_0R\omega \quad (R\omega \neq 0)$$

$$\underline{\underline{\omega = \frac{4}{3} \frac{v_0}{R}}}$$

(1): $v = \frac{2}{3}v_0 - v_0 = -\frac{1}{3}v_0$
(felvett irányú elhatárolás)