Spectrum analysis of high frequency signals

Physics laboratory practice

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Chapter 1

Introduction and historical background

The aim of this laboratory practice is to introduce the widely used Fourier transform spectral analysis and the basics of the heterodyne measurements.[1]

The aim of telecommunication is the transfer as much information as possible between two points with the highest possible fidelty. It is particularly challenging today when the communication channels (the free space) is full of electromagnetic radiation. A well developed method for this problem is to transmit the information on different frequencies, the so-called frequency division multiplexing or FDM. This explains the presence of radio channels with different frequencies , where each channel transmits with a different carrier frequency. The broadcast information itself is encoded into the signal as a modulation of the carrier. We herein introduce only the two most well known, the AM (amplitude modulation) and FM (frequency modulation).

It can be proven that the amount of transmitted information and the bandwidth for a given signal (Δf) are proportional to each other. We consider a signal as an example with a constant amplitude, frequency, and phase. Its Fourier transform is a Dirac-delta function, i.e. it has a zero bandwidth. However, the information carried by this signal is also zero. For one of the simplest data transmitting, the Morse coding, one has to switch-on/off the signal, which as a result will not be a signal with a constant frequency and its Fourier transform is also different from a Dirac-delta function. In contrast, a signal with a carrier frequency of 100 MHz and modulated (either amplitude, frequency, or phase) at a speed of 1 kHz can transmit information at a rate of 1 kbit/sec. It is no coincidence that in daily speech *bandwidth* is used as a term for the amount of transmitted information, whereas this term was essentially introduced for the width of the frequency spectrum of a signal. In this note, we refer to bandwidth in this historically original sense.

Transmitting human voice requires about 20 kHz bandwidth (in a mono broadcasting). In practice, the radio channels are separated by about 100 kHz to avoid interference (consider e.g. the FM radios around 87,5-108 MHz). Another consideration which limits the amount of transmitted information is the bandwidth vs noise. The most common thermal or Johnson-Nyquist noise has a white frequency spectrum, i.e. the noise power reads:

$$P_{\text{JN-noise}} = 4k_{\text{B}}T\Delta f \tag{1.1}$$

where k_{B} is the Boltzmann constant, T is the absolute temperature and Δf is the bandwidth[2]. This means that the amount of noise power is proportional to the bandwidth. In telecommunication, for the received power the following is used: $P_{\text{rec\_noise}} = k_{\text{B}}T\Delta f$. The factor 4 difference is due to the fact the only half of the incoming voltage (fourth of the power) can be detected on the working resistor. This means that at 300 K temperature, the noise power

[1] We appreciate any comments, typos etc. when sent to f.simon@eik.bme.hu.

[2] For a good reference, see http://en.wikipedia.org/wiki/Johnson-Nyquist_noise.

reads: $P_{\mathrm{rec\_noise}}(300\mathrm{K}) = k_{\mathrm{B}}T\Delta f = 4.1 \cdot 10^{-21}\,\mathrm{J} \cdot \Delta f$, which give about $1\,\mathrm{nV}$ for a 50 Ohm resistor for a bandwidth of $1\,\mathrm{Hz}$. It is $-174\,\mathrm{dBm} \cdot 1\,\mathrm{Hz}$ [3]

The fundamental problem of measurement techniques is therefore to achieve the best signal-to-noise ratio for a given bandwidth for several different carrier frequencies. To achieve this, the carrier signal is modulated with the signal which holds the information. The information is recovered after reception by demodulating the signal in a frequency selective manner around the carrier frequency. In the following we introduce the different modulation methods and how the frequency selective measurement is achieved.

[3] Defintion of the dBm unit: $P[\mathrm{dBm}] = 10 \cdot \log_{10}(P/1\mathrm{mW})$

Chapter 2

Theoretical and technical background

2.1 Basics of signal modulation

The most general form of the carrier as a harmonic oscillation reads: $\psi(t) = A_\mathrm{c} \cdot \exp\left[\mathrm{i}\left(2\pi f_\mathrm{c} t + \phi_\mathrm{c}\right)\right]$. All three parameters, (A_c, f_c, ϕ_c), can be modulated (f_c denotes the frequency of the carrier signal). This is denoted as amplitude- (AM), frequency- (FM), and phase-modulation (PM), respectively. We discuss herein the AM modulation only. The amplitude modulated signal is shown in Fig. 2.1.

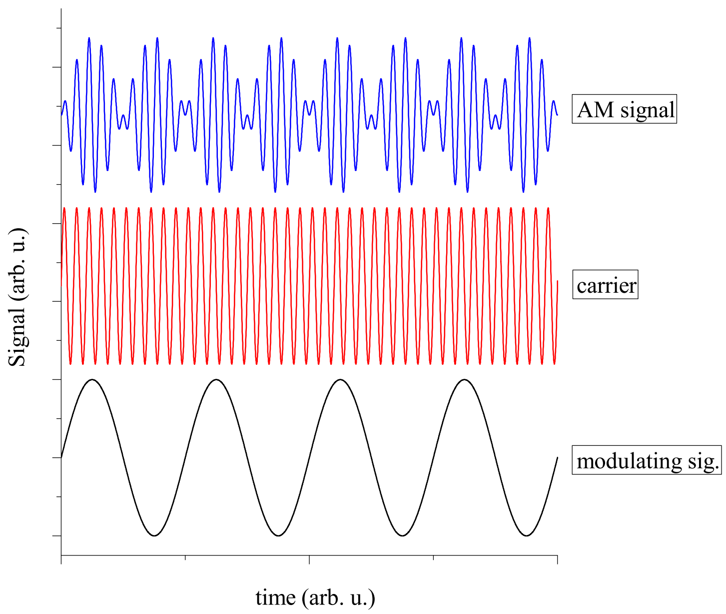

Figure 2.1: Schematics of amplitude modulation.

The AM signal can be considered as a product of the carrier and the modulating signal. If the modulating signal is a pure harmonic wave, it reads: $\psi_\mathrm{m}(t) = A_\mathrm{m} \cdot \exp\left(\mathrm{i}2\pi f_\mathrm{m} t\right)$ (here m index denotes the modulation). According to trigonometric identities:

$$\cos(\omega_1 t)\cos(\omega_2 t) = \frac{\cos\Big(\left(\omega_1 + \omega_2\right) t\Big) + \cos\Big(\left(\omega_1 - \omega_2\right) t\Big)}{2} \tag{2.1}$$

It is clear that the product signal is a sum of two harmonic waves, whose frequency reads: $f_\mathrm{c} \pm f_\mathrm{m}$ (assuming that $f_\mathrm{c} > f_\mathrm{m}$). We present it later how the production is performed technically, which is also called mixing. The demodulation (i.e. obtaining the information) is also obtained by a product (or mixing). The AM signal is multiplied again with the carrier at f_c frequency, the result is the sum of 3 harmonic waves as: f_m, $2f_c \pm f_m$. Of these, clearly the first one is the information carrying signal, the other two higher frequency components are filtered out (by e.g. low-pass filtering). In the real-world communication, the modulating signal is a superposition of several frequencies (such as e.g. voice is). It is however clear that the above considerations remain valid for such a signal, too. This technique (i.e. mixing the modulating signal with the carrier and then mixing it again after reception) is called *mixing* or *heterodyne* detection.

Earlier (about up until 50 years ago) demodulation of AM signals was performed by rectification (the so-called detector method). Its schematics is shown in Fig. 2.2.

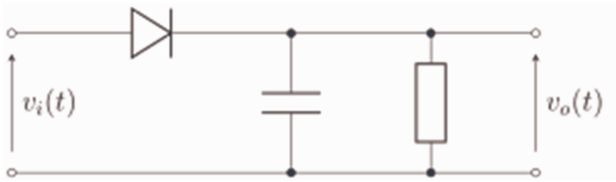

Figure 2.2: Schematics of the detector radio. Input in the left hand-side is rectified and low-pass filtered and arrives to the right hand side output.

This method exploits that rectification of an amplitude modulated signal (and a consecutive low-pass filtering) yields the modulating envelope signal. This technique is also called *envelope detected* reception. The advantage of this method is its simplicity as the detector and loudspeaker does not require external power source (it is of course quite silent) and all its power is drawn from the carrier. A clear disadvantage is that the rectification cannot distinguish between carrier with different frequencies. Today this method is extinct and all detection is performed by heterodyning due to the reasons discussed below.

2.2 Detection of high frequency signals

2.2.1 Frequency selective detection, mixing

The disadvantage of the envelope detection is that it demodulates all carrier signals at once, i.e. if severla of these are present, it is hard to obtain a clear information. This could in principle be fixed if a band selective (or band-pass) filter is present. In practice, it is hard to implement a band-pass filter which is i) tunable, ii) and has a high enough selectivity.

The heterodyne detection does not require band-pass filtering but only a fixed low-pass filter after the mixing. Selecting a specific carrier signal is achieved by using different signal frequencies for the down-converting mixing. This question has been left open yet, how the mixing or multiplication is achieved in practice.

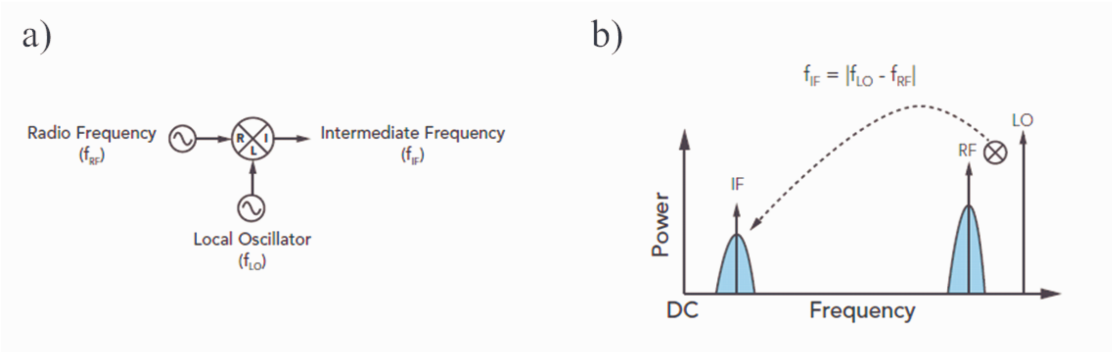

Figure 2.3: a) Mixer symbol with notation of the input-output port: (LO input: local oscillator, RF in/output: radiofrequency signal, IF in/output: intermediate frequency), b) using the mixer as down-converter: IF port is output which has a frequency with the difference of LO and RF.

The unit multiplying the signal is the *mixer*. This is semiconducting device whose current-voltage characteristics is strongly non-linear. As a result, it yields the product of the two input signals. The mixer schematics, together with the common port notations, is shown in Fig. 2.3a. The mixers can be used for up/down-mixing (or up/down-conversion). For the earlier, the IF is input and RF is output, for the latter RF is input and IF is output. Fig. 2.3b. shows that the frequency spectra of the signals when the mixer is used as a down-converter. The LO has a well defined frequency, whereas the RF is modulated and it has a broader frequency spectrum. The IF port contains the difference of the RF and LO, i.e. its bandwidth matches that of the RF and it also contains the information. When used as an up-converter, the LO is still a signal with a well-defined frequency, the IF port has the modulating, low frequency signal and the RF output contains the amplitude modulated product signal.

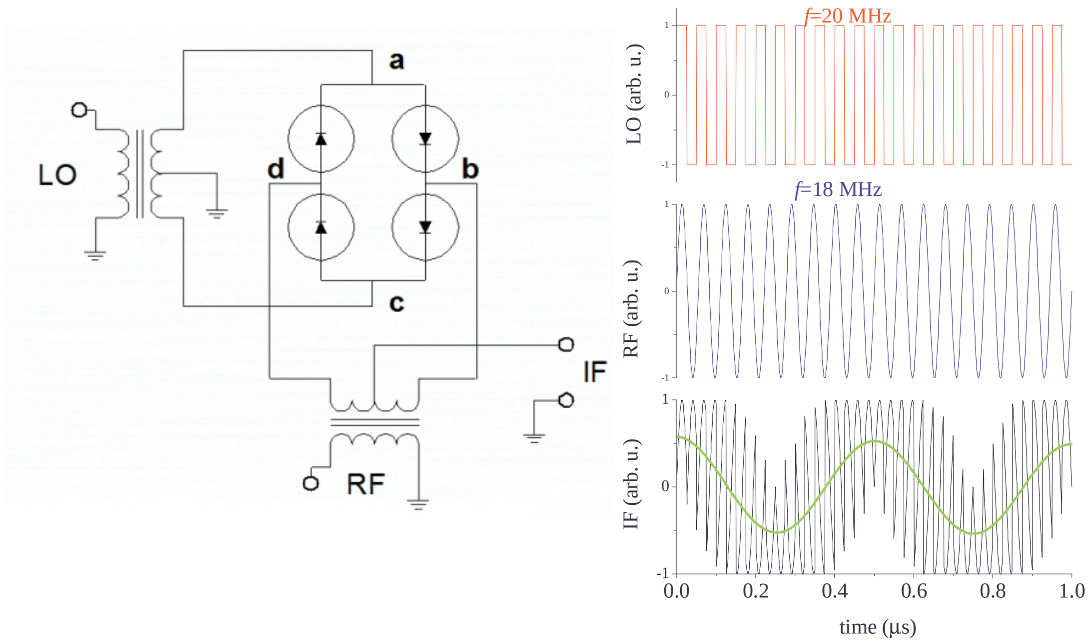

*Figure 2.4: Left panel: A possible realization of a mixer, the so-called (*switching-mixer*). Right panelen: the mixer signal for a given example. LO: 20 MHz signal, RF: 18 MHz signal, the IF output is the modulated signal. The green line is the IF signal after low-pass filtering.*

A possible realization of the mixer is shown in Fig. 2.4. When the LO voltage for the **a-c** points is positive, ($U_{\mathbf{ac}} > 0$), the two diodes on the left are closed and the two diodes on the right are opend. Then, the voltage of point **d** is given by the RF port voltage and point **b** is on the ground. As a result, the RF voltage appears on the IF voltage with the same sign. When ($U_{\mathbf{ac}} < 0$), the two diodes on the left are open thus point **d** is grounded and the two diodes on the right are closed, thus the RF voltage appears with a -1 sign on the IF output.

This behavior is demonstrated in the right panel of Fig.2.4. The switching mixer behaves as if the LO signal would switch the sign of +1 and -1 which multiplies how the RF signal appears on the IF output. The example in the Figure shows an LO with 20 MHz an RF with 18 MHz. The resulting IF output has a 2 MHz and 38 MHz signals. If the latter is low-pass filtered (in our example, we performed a numerical sliding average) then the low frequency component on the IF output is obtained. This description also shows that the magnitude of the IF output is not sensitive for the strength of the LO. The IF voltage in turn, is proportional to the RF (albeit not equal due to losses, which are typically a factor 2, or 6 dB). For typical device the LO power is specified for a given range (it should open the diodes), like 0 dBm to 10 dBm (223 mV to 707 mV for 50 Ohm).

2.2.2 The lock-in amplifier

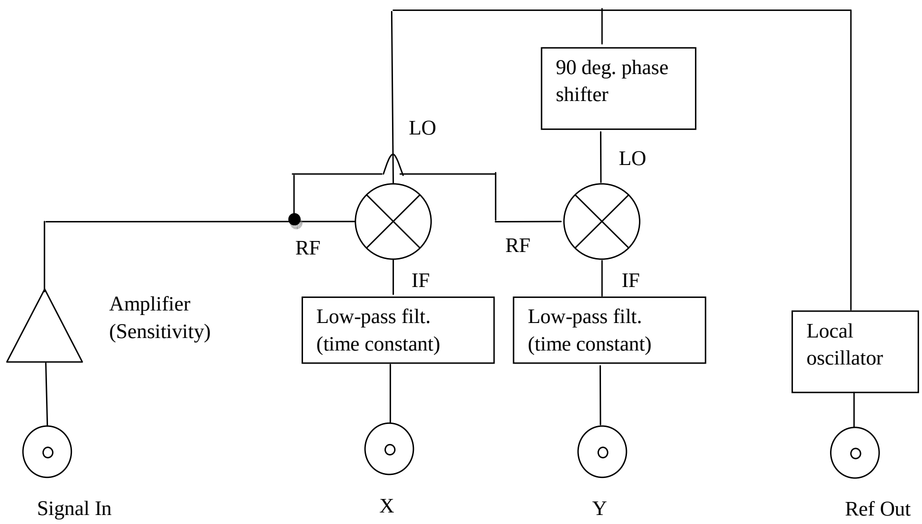

Figure 2.5: Block-diagram of the lock-in amplifier. The layout reflects the control interface of the used Stanford lock-in.

The schematics of a *two channel phase-sensitive rectifier* or *lock-in amplifier* is shown in Fig. 2.5. This essentially consists of a LO source, two down-converting mixers followed by low-pass filtering on the IF. The two-channels mean that both in- and out-of-phase components of the incoming signal are measured. In- and out-of-phase mixing is achieved by using the LO and its 90 degree shifted version for the down-conversion mixing. This is also called *quadrature-detection*. For a single channel lock-in, the incoming signal could have in theory a 90 degree phase to the LO. That case the IF signal would be 0. For the two-channel detection, the modulus of the two channels can be obtained, too: $R = \sqrt{X^2 + Y^2}$. This directly gives the amplitude of the incoming signal, irrespective of its phase to the LO.

We introduce the basics of the lock-in amplifier handling and how it can be used to detect directly the AM broadcast during the laboratory practice.

2.3 Analysis of high-frequency signal, the Fourier transformation

We revisit first the frequency domain decomposition of an arbitrary signal. Mathematically, the decomposition of a time-dependent $F(t)$ signal into different frequency components is give by the Fourier transformation as:

$$f(\omega) = \int_{-\infty}^{\infty} F(t)\mathrm{e}^{-i\omega t}\mathrm{d}t. \tag{2.2}$$

For a real measurement, the Fourier transform function can be approximated only due to the finite measurement time and the discrete nature of the measurement points. We discuss first the effect of finite measurement time on the Fourier transformation.

The finite measurement time is equivalent to multiplying the original time-dependent function with a $W(t)$ window function. Then the product of the window function and the original function is evaluated as :

$$f_W(\omega) = \int_{-\infty}^{\infty} W(t)\cdot F(t)\mathrm{e}^{-i\omega t}\mathrm{d}t, \tag{2.3}$$

where the W(t) is defined as $1/T$ for $|t| < T/2$ and zero besides. It can be shown that the Fourier transform of a product is the convolution of the respective Fourier transformed signals, i.e.:

$$f_W(\omega) = \int_{-\infty}^{\infty} f(\omega')w(\omega-\omega')\frac{\mathrm{d}\omega}{2\pi}, \quad \text{where} \quad w(\omega) = \int_{-\infty}^{\infty} W(t)\mathrm{e}^{-i\omega t}\mathrm{d}t. \tag{2.4}$$

We consider a simple example: let $F(t) = A\cdot\exp(i\omega_0 t)$ a harmonic function whose Fourier transform is a Dirac-delta function: $f(\omega) = A\cdot 2\pi\delta(\omega-\omega_0)$. For a finite measurement time, the Fourier integral gives $f_W(\omega) = A\cdot w(\omega-\omega_0)$ i.e. the Fourier transform of the window function is observed instead of the Dirac-delta function around the ω_0 angular frequency. For the above example of a rectangular window function (see Fig. 2.6a. solid blue curve) its Fourier transform is $w(\omega) = (2/\omega T)\cdot\sin(\omega T/2)$ i.e. $f_W(\omega) = (2/(\omega-\omega_0)T)\cdot\sin((\omega-\omega_0)T/2)$ (see Fig. 2.6b. solid blue curve). It is clear that this Fourier transform is peaked at the desired frequency but it has a finite width and oscillations (or side-lobes) further away which is more apparent when shown on a log plot. This effect is called spectral leakage. The first zeros of the $f_W(\omega)$ function around ω_0 are separated by $4\pi/T$ thus the peak around ω_0 has a characteristic width of $\sim 2\pi/T$. Our first important conclusion is therefore that for a measurement with finite duration, the Fourier transformed signal is broadened which limits the attainable frequency (or spectral) resolution.

It is worth noting that other window functions can be selected [1], e.g. $W(t) = \cos^2(t\pi/T)$ the so-called Hanning-window. This means that the signal near the interval ends of the $|t| < T/2$ measurement is down-weighted (see Fig. 2.6a., dashed red curve). This results in a broader Fourier transform of the harmonic signal (i.e. the frequency resolution is worse) but the oscillations around ω_0 are much smaller, i.e. the spectral leakage is lowered (see Fig. 2.6b. dashed red curve).

As a next step, we consider the effect of discrete data sampling. This means that the Fourier integral is approximated by a discrete sum, a method called discrete Fourier transformation (DFT):

$$f_W(\omega) = \sum_{n=0}^{N-1} W(n\cdot\Delta t)F(n\cdot\Delta t)\mathrm{e}^{-i\omega n\Delta t}\Delta t, \tag{2.5}$$

where Δt is the time sampling period, $N = T/\Delta t$ is the number of sampled points. The so-called Nyquist-Shannon sampling theorem states the highest obtainable (angular-) frequency

[1] Selective alternative window functions is also known as apodization.

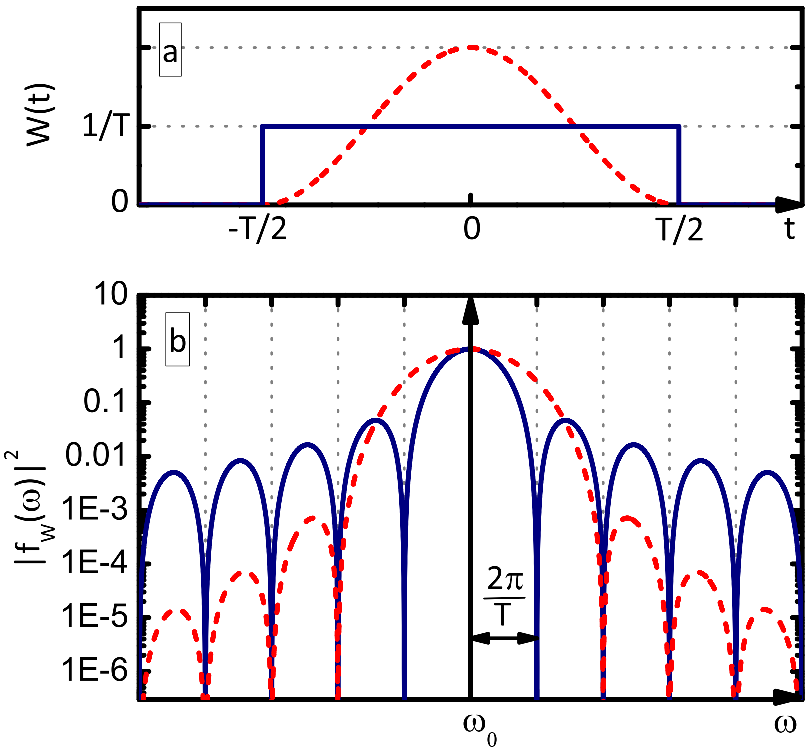

Figure 2.6: a) The $W(t)$ rectangular window function (solid blue curve) and the so-called Hanning window (dashed red curve). b) The amplitude of the Fourier transform of these two window functions (calculated from the real and imaginary parts) and the $F(t) = A \cdot \exp(i\omega_0 t)$ harmonic signal for the above two window functions. Note that the Hanning window results in a broader main peak, i.e the resolution is inferior but the amplitude of the sidelobes is significantly reduced, i.e. the spectral leakage is smaller.

is $\omega_{\max} = 2\pi/2\Delta t$ for a sampling period of Δt. It can be shown that performing the DFT for N measurement points requires $\sim N^2$ operations. However, it can be shown that if N is a power of two ($N = 2^p$), the number of operations can be reduced from N^2 to $N \log_2 N$ with the so-called Fast Fourier Transform (FFT) algorithm. This is a substantial gain for large N's. Most instruments (such as the oscilloscope used in the laboratory practice) evaluate the Fourier transform with the FFT method. For most of the cases, the instruments evaluate the real and imaginary parts but display on the absolute value (or modulus) of the signal. In addition, the result if given as a function of the frequency and not angular-frequency.

The definition of the Fourier transform shows (Eq. (2.2)) that the Fourier transform is a complex quantity. It therefore carries information not only about the frequency of the components but also about their phase. It is clear that if $F(t)$ is an odd function (e.g. sine) then the Fourier transform is purely imaginary. For an arbitrary $F(t)$, the Fourier transform contains both real and imaginary values[2]. In case, the phase is not important and one is interested in the strength of the Fourier components, the *Fourier magnitude* is evaluated as square root of the squared sum of the real and imaginary part.

One question has been left open: we refer to a harmonic signal as cos, sin, or $e^{i\omega_0 t}$. What is the origin and purpose of the complex notation? The brief answer is that the cos or sin function does not contain the *sign* of the frequency of the harmonic signal. At first instance, it might sound surprising that a harmonic signal can have a negative frequency. However, when we think about the mixing down conversion of an RF with a lower frequency than that of the

[2]In this respect, the complex form simply abbreviates the notation.

LO, one obtains an If with a *negative* frequency. It is clearly of importance to distinguish between the IF with a positive of negative frequency.

To demonstrate the loss of the frequency sign, we consider first a $\cos(\omega_{\mathrm{IF}} t)$ signal. Its Fourier transform is real and it contains according to Eq. (2.2) two positive peaks at $\pm\omega_{\mathrm{IF}}$. In the second example, we consider a signal of $\sin(\omega_{\mathrm{IF}} t)$, whose Fourier transform is imaginary and contains two peaks at $\pm\omega_{\mathrm{IF}}$ with opposite sign. As a result, we cannot determine the sign of the frequency in either of these examples. In contrast, when we consider a signal of $\mathrm{e}^{-i\omega_{\mathrm{IF}} t}$, its Fourier transform has a single peak at ω_{IF} (in a sign specific manner)[3].

The real and imaginary parts of the $\mathrm{e}^{-i\omega_{\mathrm{IF}} t}$ expression behave as the projections of the point moving on the unit circle: the frequency sign is defined by the direction of the point movement (clockwise or anti-clockwise). In contrast, the sin and cos functions are insensitive to the movement direction of the point.

We finally consider how the $\mathrm{e}^{-i\omega_{\mathrm{IF}} t}$ complex expression is obtained from the measured real data. The real and imaginary parts are obtained from the quadrature detection (which was introduced for the lock-in): i.e. the incoming signal is split into two and is down-converted with an LO and its 90 degree shifted counterpart. We denote the two sets of time dependent data as $X(t)$ and $Y(t)$, using the notations of Fig. 2.5. We then combine the two as: $\widetilde{X}(t) = X(t) + iY(t)$ to obtain the complex data set. We obtain two sets of data (real and imaginary parts) for the Fourier transform according to Eq. (2.2):

$$\widetilde{x}(\omega) = x(\omega) + iy(\omega) = \int_{-\infty}^{\infty} \widetilde{X}(t)\mathrm{e}^{-i\omega t}\mathrm{d}t, \tag{2.6}$$

thus it is called complex or two-channel Fourier transformation.

The two terms read explicitly as:

$$x(\omega) = \int_{-\infty}^{\infty} X(t)\cos(\omega t)\mathrm{d}t + \int_{-\infty}^{\infty} Y(t)\sin(\omega t)\mathrm{d}t, \tag{2.7}$$

$$y(\omega) = \int_{-\infty}^{\infty} -X(t)\sin(\omega t)\mathrm{d}t + \int_{-\infty}^{\infty} Y(t)\cos(\omega t)\mathrm{d}t. \tag{2.8}$$

the Fourier magnitude is obtained from these two sets of data as: $\sqrt{x(\omega)^2 + y(\omega)^2}$. This quantity is displayed usually on a spectrum analyzer. [4]

2.4 Spectrum analysis

Three methods are known for the spectrum analysis of an incoming signal with unknown frequency decomposition:

1. The so-called "FFT around the DC".
2. The so-called "swept heterodyne spectrum analysis".
3. The so-called hybrid heterodyne-FFT spectrum analysis.

The block diagrams of these methods are shown schematically in Fig.2.7. In the first method, the spectrum is obtained by a direct FFT of the incoming signal(see Fig. 2.7a.). This method is used mostly for audio signals around DC, as the spectrum starts from DC and ends

[3] That is it has not peak at $-\omega_{\mathrm{IF}}$.

[4] For a so-called vector network analyzer (or VNA), both components are displayed, most usually for a frequency swept operation.

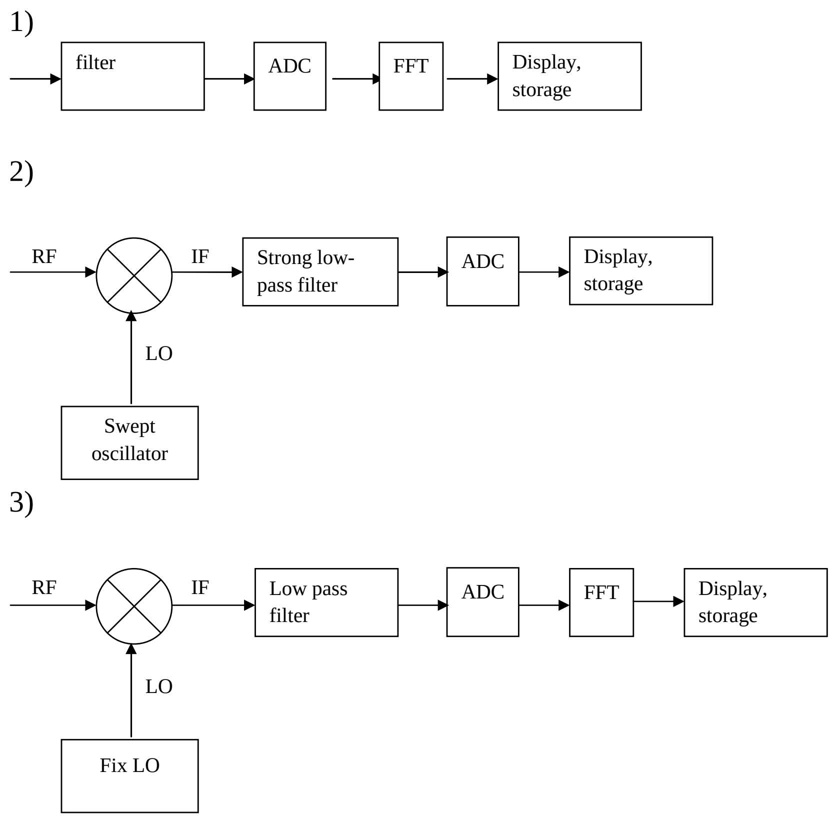

Figure 2.7: Schematics of the three types of spectrum analysis.

at the Nyquist frequency. This is clearly not an optimal method when we are interested in e.g. a 10 kHz broad frequency range with good resolution around 100 MHz.

In the second method, the local oscillator frequency is continuously swept, the resulting IF signal is low-pass filtered (see Fig. 2.7a.) which results in the spectrum of the incoming signal. An advantage of this method is that it allows to study the frequency spectrum in a more or less real time manner. Its disadvantage is that the frequency of the swept oscillators is inaccurate and that only one frequency component is measured at once. We note that the setup shown in Fig. 2.7. shows a single channel measurement, it has to be amended with a second channel to yield the Fourier magnitude spectrum (this amendment is not shown).

The third method combines the advantages of the first two. Therein, a fixed frequency oscillator is used as an LO to down-convert the signal first to an IF which is then digitized and an FFT is performed. This allows to study the vicinity of an arbitrary LO frequency with any resolution. In addition the nature of the Fourier transformation provides a simultaneous measurement of all frequency components (also known as multiplexing or multiplex advantage). An additional advantage is the frequency accuracy of the LO, i.e. the resulting frequency spectrum is calibrated. Its disadvantage is that FFT is relatively processor demanding, thus a real time measurement is limited when very high frequency resolution (i.e. N is large, around a million) is required.

The quadrature detection is also employed for the heterodyne-FFT (not shown in the schematics) to yield the sign of the IF frequency and as a result, the absolute frequency of the RF. This is explained as follows: let the LO=100 MHz and RF=99.9 MHz. The IF output of a mixer is a 0.1 MHz signal, whose sign is unknown. As a result, this measurement cannot determine whether the incoming RF was 99.9 MHz or 100.1 MHz. In turn, for quadrature heterodyne mixing, the IF sign is clearly determined, i.e. the RF frequency is obtained. Modern spectrum-analyzers, e.g. the Tektronix DPO/MSO mixed domain oscilloscope performs this.

We employ a simpler setup in the laboratory practice due to limitations of the instrumentation. We first downconvert the incoming signal with an LO and mixer and then detect the IF with an oscilloscope. Given that the studied FM radio broadcast is in the 87.5-108 MHz range, we use an LO of 87.5, thus avoiding the problem of image frequencies.

Chapter 3

Measurements

Important: a voltage larger than 1 V on the mixer ports can damage it! Always take care for this limit. meghibásodásához vezethet!

3.1 AM signal demodulation with a lock-in amplifier

1. Connect the coil antenna to the input of *CH1* input of the SR844 lock-in! Set the lock-in freuqency to 540 kHz, input impedance 50 Ohm. Connect the loud-speaker to the CH1 output! Study the change in the detected voice as a function of the lock-in frequency, time constant, sensitivity, and CH1 output type (X, R, v. R[dBm]). Record the most optimal setting!

2. Explain the the logbook why the lock-in performs demodulation of the AM broadcast! Also explain the observed dependences on the LI frequency, time constant etc.!

3. The CH1 output type R gives directly the voltage which is induced in the coil antenna. We know that the broadcasting antenna (Solt) emits 2 MW power, its distance is approximately 80 km from the lab. The tangential component of the magnetic field in the plane of emission (in the far-field regime) for a distance of r is: $H = \frac{I_0}{2\pi r}$. The connection between the antenna current, I_0, and emitted power, P_{rad}, is: $P_{\text{rad}} = I_0^2 Z_{\text{rad}}/2$, where $Z_{\text{rad}} = 73\,\Omega$ is the so-called radiation impedance of the antenna. Compare the calculated the measured voltages which is induced in the coil antenna!

4. Perform a Fourier transform of the lock-in *CH1* output in type R with the oscilloscope for the $< 20\,\text{kHz}$ range, while listening to it! Note how the Fourier spectrum changes as a function of the LI time constant and filter slope.

3.2 The modulated signals and the Fourier transform spectrum analysis

5. Set e.g. 100 kHz signal on the Siglent generator and explain the FT spectra if the signal is i) sine, ii) square, or iii) ramp! By means of storing the data, explain how these depend on the oscilloscope time base and the apodization function (rectangular, Hanning, flattop)!

6. Set a 100 kHz sine (and later a square wave) which is amplitude modulated with a frequency of 1-20 kHz. Use the ”Mod” button of the generator (*AM Freq*: 1-20 kHz, *AM depth*: 100 %). Study the signals and their FT if the modulation waveform is Sine or

Square! Store both FT spectra and explain the position, shape, number and magnitude of the sidebands!

7. Form the AM signal with the help of HP 10534A mixer! For this, connect a 100 kHz or 1 MHz (unmodulated) sine signal from the generator to the L mixer input (around 0.5-1 V_{RMS}), set the X input X a 1-100 kHz 0.1 V_{RMS} modulating signal from the other output of the generator, and connect the R to the oscilloscope input! Study the resulting signals in both time- and frequency-domains! Note what happens if the modulating signal is a square wave! (In ideal case only two sidebands should appear, however, the carrier and third harmonics will appear, too due to the log scale and the so-called intermodulation mixing effect (not discussed herein).

8. Down-convert the AM signal (which is done at point 6) with the HP mixer. Set the mixer in- outputs alone according to the above information! Let the carrier wave 1 MHz and the AM modulation frequency 20 kHz. Set the downconverting LO to 1 MHz. Search for the down-converted signal around DC below 50 kHz! (Imperfections in the oscilloscope sampling and mixer distortions, some unwanted signals will appear, too, but the spectrum is dominated by the desired signal. It is common for the unwanted signals that their position is independent of the modulation frequency.)

3.3 The FM radio broadcasts

9. Study the FM radio broadcasts in the 87.5-108 MHz range! For this, use the lock-in *Ref out* at 87.5 MHz as LO, the mixer RF input is the antenna output (turn its built-in amplifier on), let the IF output be connected to the oscilloscope! Set the scope such that a 25 MHz range is diplayed in FT mode. Identify the FM channels and compare the result with Table A. in the Appendix (this contains some ”odd one out” on purpose). Measure the relative power ratios for the channels which are broadcast from the same position. Compare this with the table.

Acknowledgements

We thank the comments by Dr. Ferenc Fülöp, Bence Bernáth, and Bence Márkus. The acquisition software was written by Bence Bernáth, Bence Márkus prepared the table of the FM radio broadcasts.

Suggested literature

David M. Pozar: Microwave Engineering (4th Ed.)
E. Brigham: Fast Fourier Transform and Its Applications

Appendix A

FM radio broadcasts around Budapest

AM: Kossuth radio at 540 kHz (Solt), separation from Budapest about 80 km (strongest AM emitter in Europe, second strongest in the world).

Table A.1: List of FM radio broadcasts near Budapest (and a few "odd ones out").

f (MHz)	P (kW)	Radio name	Radio position
87.60	nincs adat	Kontakt Rádió	Terézváros
88.10	1	InfóRádió	Nagyvárad tér, SOTE épület
88.80	1	Rádió C	Széchenyi-hegyi adótorony
89.00	0.986	MR2 Petõfi Rádió	Debrecen
89.50	77.6	Music FM	Széchenyi-hegyi adótorony
90.20	8.3	MR1 Kossuth Rádió	Nagykanizsa
90.30	0.4	Tilos Rádió	Gellért-hegy, Citadella rádióadó állomás
90.90	2	90.9 Jazzy	Sashegy adótorony
91.40	1.2	Dankó Rádió	Debrecen
92.10	2.2	Klasszik Rádió	Gellért-hegy, Citadella rádióadó állomás
92.90	5	Klub Rádió	Sashegy adótorony
94.20	1	Mária Rádió	Sashegy adótorony
94.80	77.6	MR2 Petõfi Rádió	Széchenyi-hegyi adótorony
95.90	25.2	MR1 Kossuth Rádió	Pécs
96.70	37.1	MR2 Petõfi Rádió	Komádi
96.80	1	Rádió 17	XVII. Kerület, Rákosmente
97.50	50.1	MR1 Kossuth Rádió	Tokaj
98.00	5	Civil Rádió	Hármashatárhegy, Felso-Kecske-hegy
98.20	7.5	MR2 Petõfi Rádió	Tiszavasvári
99.50	3	Rádió Q	Sashegy 2
99.70	1.4	MR1 Kossuth Rádió	Debrecen
100.30	1	Lánchíd Rádió	Nagyvárad tér, SOTE épület
100.50	67.6	Class FM	Kabhegy
100.80	79.4	MR Dankó Rádió	Széchenyi-hegyi adótorony
102.00	22.4	Class FM	Sopron
102.10	0.741	Magyar Katolikus Rádió	Széchenyi-hegyi adótorony
102.70	30.2	MR2 Petõfi rádió	Kékes
103.30	81.3	Class FM	Széchenyi-hegyi adótorony
103.90	0.82	Juventus Rádió	Széchenyi-hegyi adótorony
104.80	0.3	Budaörs Rádió	Budaörs
105.30	81.3	MR3 Bartók Rádió	Széchenyi-hegyi adótorony
105.90	2	Gazdasági Rádió	Gellért-hegy, Citadella rádióadó állomás
106.20	23.4	RTVS Rádio Regina BB	Besztercebánya, Száraz-hegy
107.80	83.2	MR1 Kossuth Rádió	Széchenyi-hegyi adótorony
110.30	2	Rádió 7	Kõbánya, Határ úti adótorony

The HP 10534A radio frequency mixer. The L port of the mixer is always the LO input. Drive this with not larger than 0.5-1 V_{RMS}, a higher value can damage it! The R (usually denoted as RF) and X (usually denoted as IF) ports can be either in- or outputs. When the mixer is used for down-conversion, the R port is input and X is output. For up-conversion, X is input and R is output. The L and R ports can be driven with a 50 kHz-150 MHz signal, whereas X can be between DC-150 MHz. The X and R are always proportional to each (although unequal) after some conversion loss (typically a factor 2 in voltage, or 6 dB).

The voltage on the X and R ports should not exceed 0.2 V_{RMS} but as a rule of thumb a 1/5th of the LO voltage is preferred!