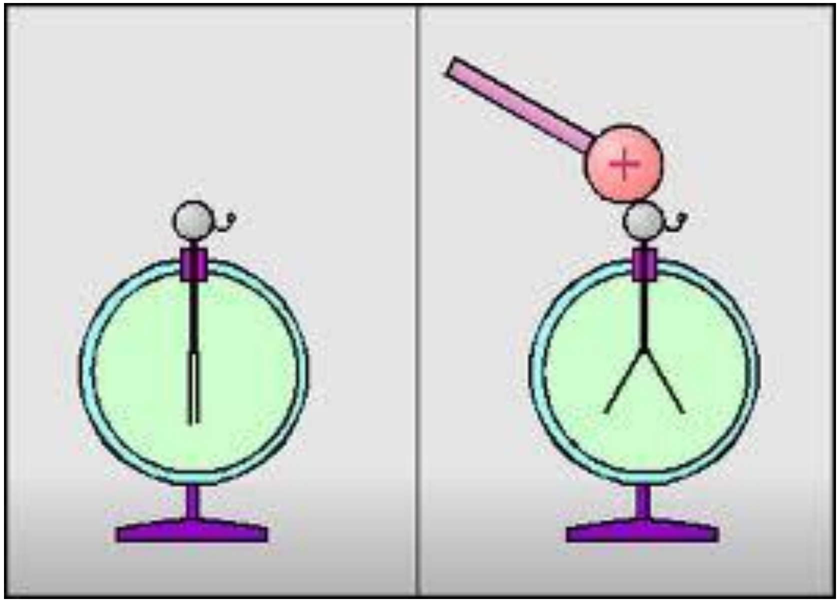
5. lecture

Electrostatics

Electric field, Gauss' law



Experimental background (observations)

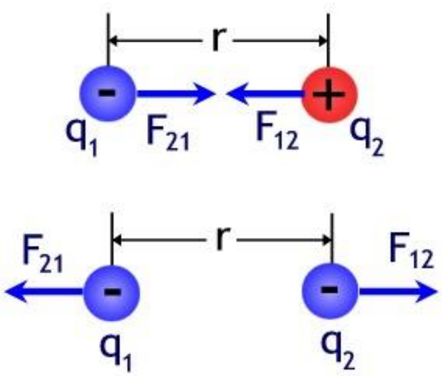
Observation (experience):

- Simple experiments (glass-rod & leather, etc.)
- Charge transfer
- Charge induction
- Charge separation

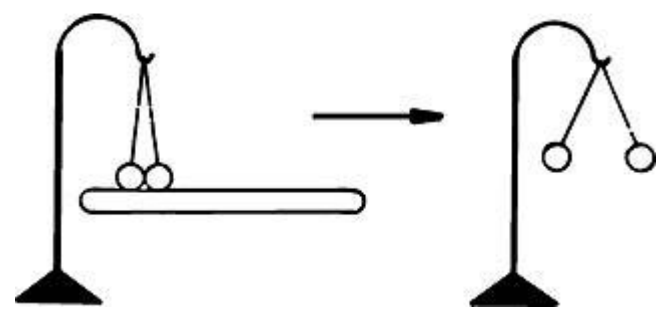
Conclusions:

- charges: + & –
- electron: e^- & proton: p^+
- unit: Coulomb [C]
- $q_e = 1.6 \cdot 10^{-19}$ C

(electron and proton, unit, q_e value: measurement)



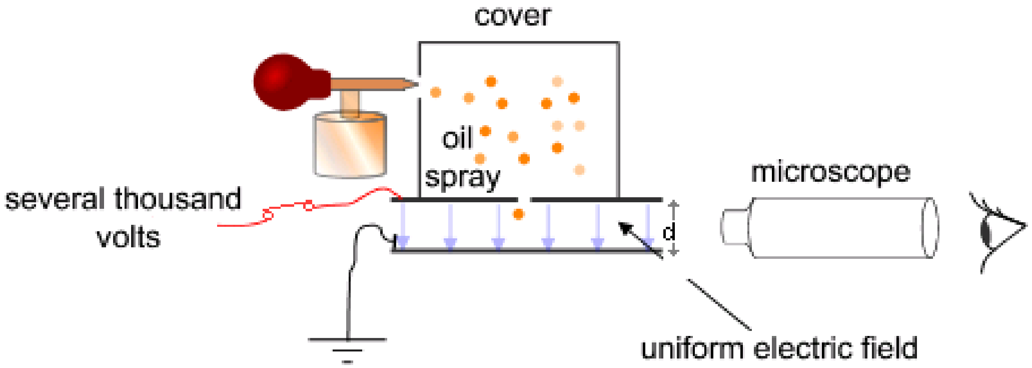
$$|q_e| = |q_p|$$



The Millikan experiment

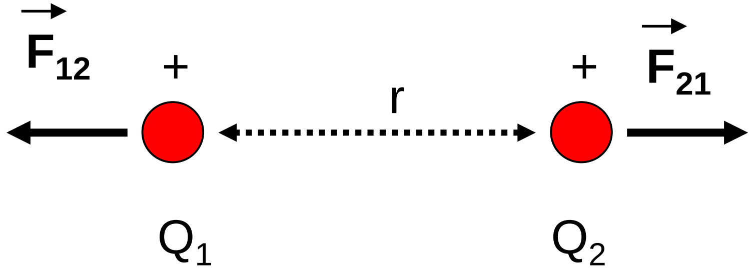
Millikan (1910)

$q_e = 1.6*10^{-19}$ C

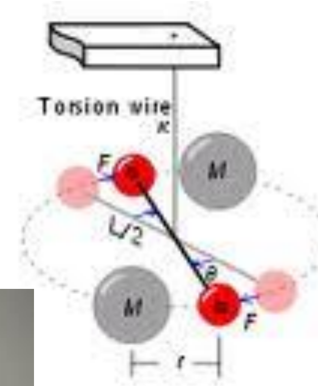


”student version”

Coulomb's law



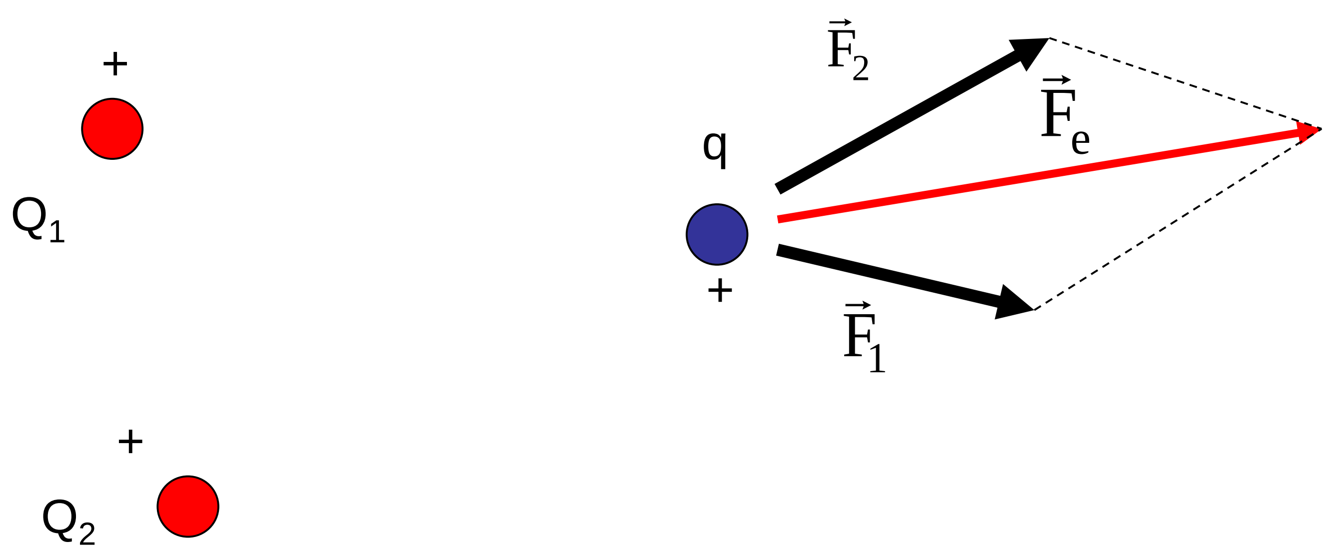
$$\vec{F}_{12} = \vec{F}_{21}$$



Coulomb's law:

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F = k\frac{Q_1 Q_2}{r^2} \;\ldots\ldots\; k = 9 \cdot 10^9 \frac{Nm^2}{C^2}$$

Superposition:

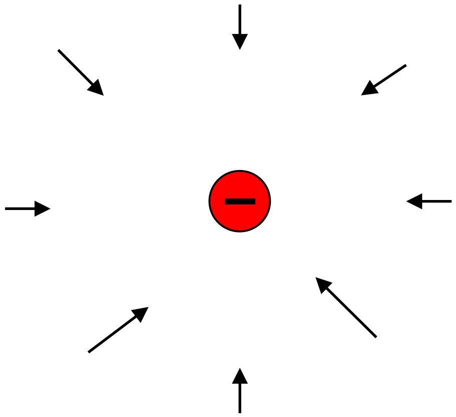


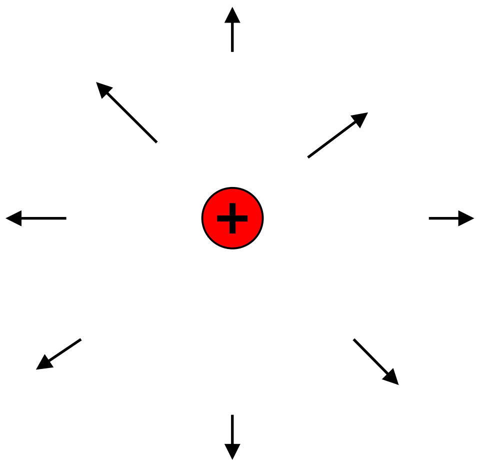
The electric field

$$F = k\frac{Qq}{r^2} = k\frac{Q}{r^2}q = Eq \longrightarrow \vec{F} = \vec{E}q$$

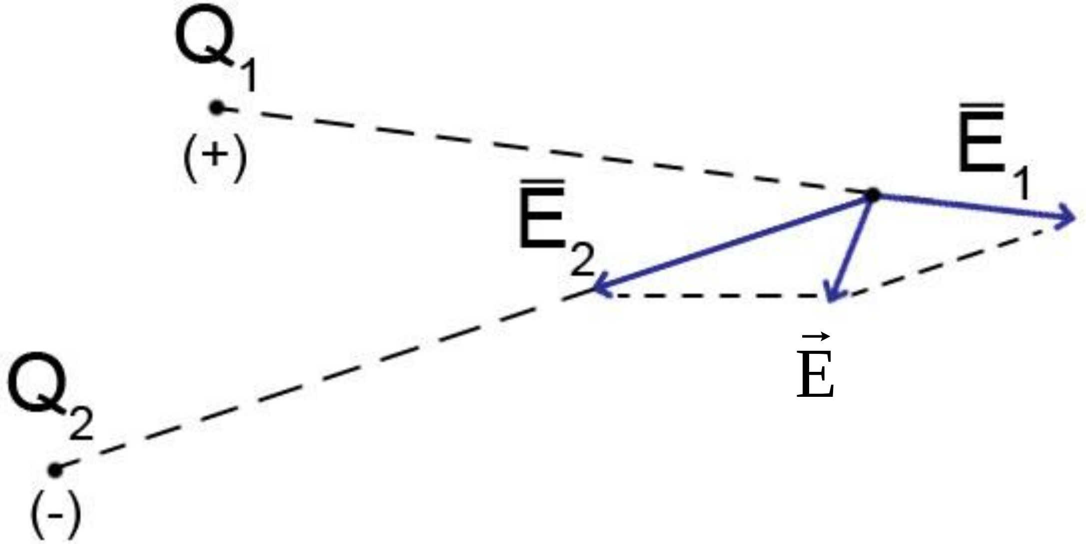
Electric field of a point charge: $\vec{E} = k\frac{Q}{r^2}\vec{e}$ [N/C = V/m]

(Measurement of electric field: to measure the force) q: probe charge



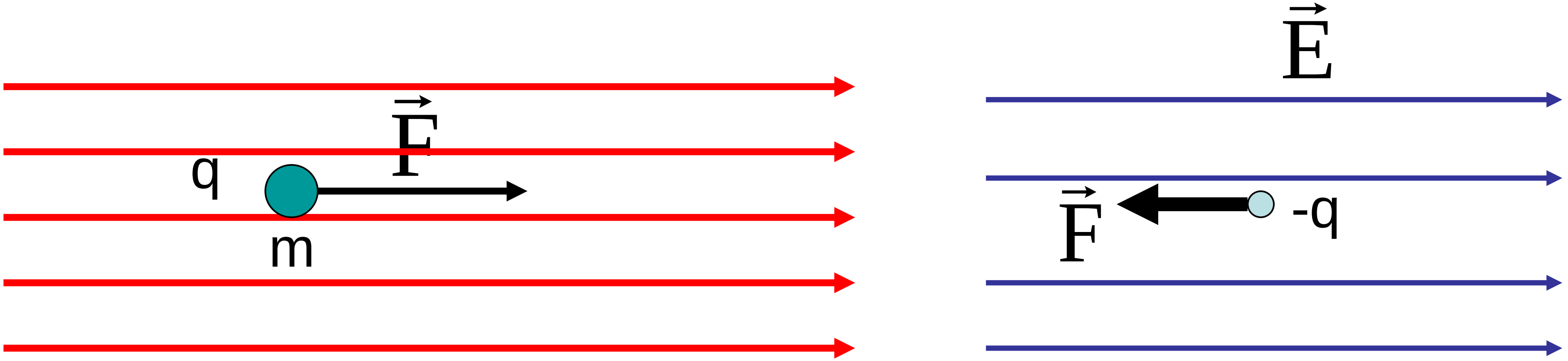


Superposition of electric field



$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

Electric force (Coulomb force) acting on a point charge in uniform electric field



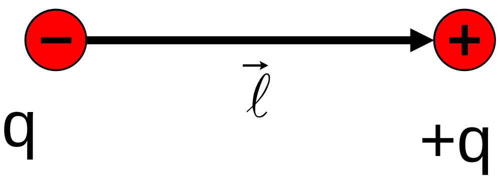
$\vec{E}$ (uniform electric field)

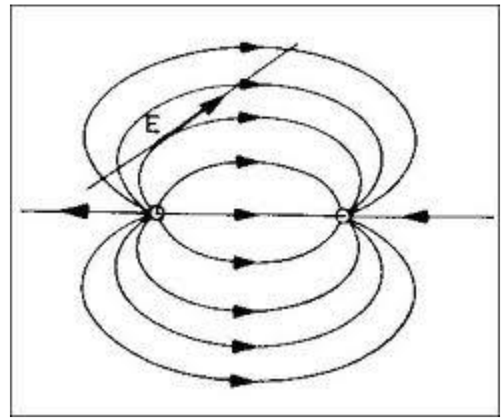
$$\vec{F} = q\vec{E} \quad , \quad \vec{F} = m\vec{a} \Rightarrow \boxed{q\vec{E} = m\vec{a}}$$

$$\boxed{\vec{a} = \frac{q\vec{E}}{m}}$$

The electric dipole

The electric dipole moment: $\vec{p} = q\vec{\ell}$ [Cm]





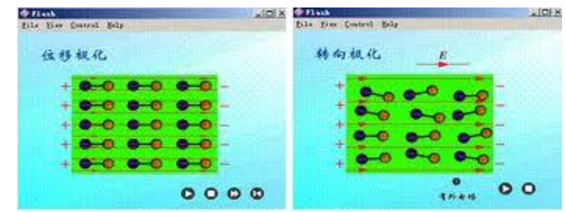
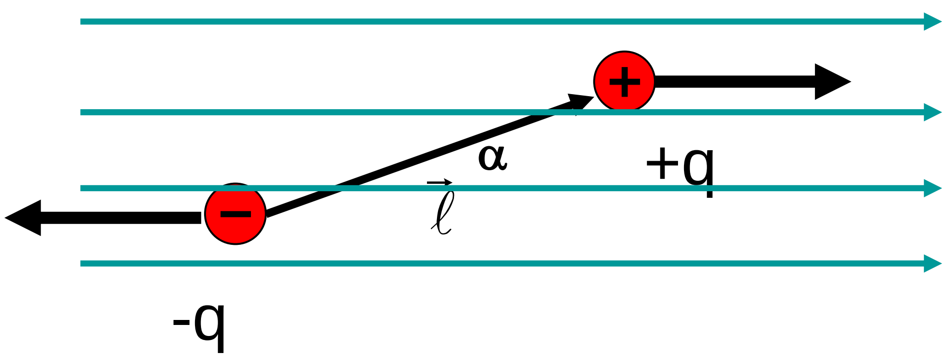


Fig. 6-4-5 Flash animation: Polarization of insulators

Torque acting on electric dipole moment:

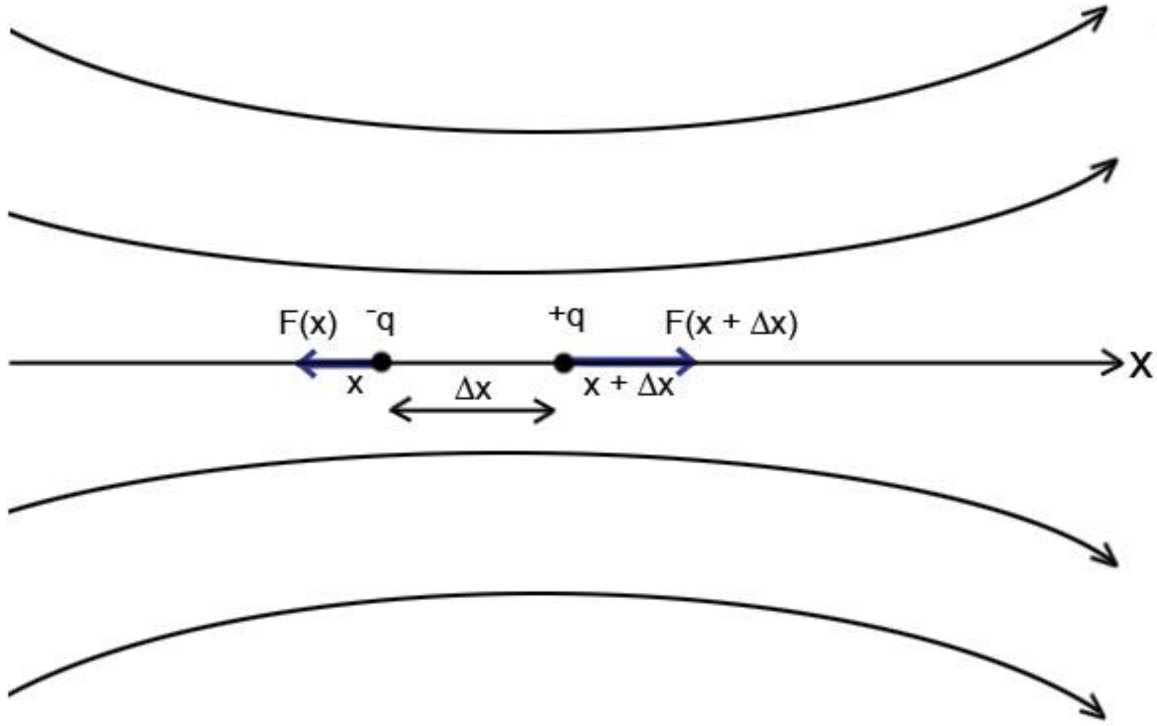


$$\left|\vec{\tau}\right| = pE\sin\alpha = \left|\vec{p}\times\vec{E}\right|$$

$$W = \int_0^{\Theta} pE\sin\alpha Es = pE(1-\cos\Theta)$$

The potential energy: $U = -\vec{p}\vec{E}$

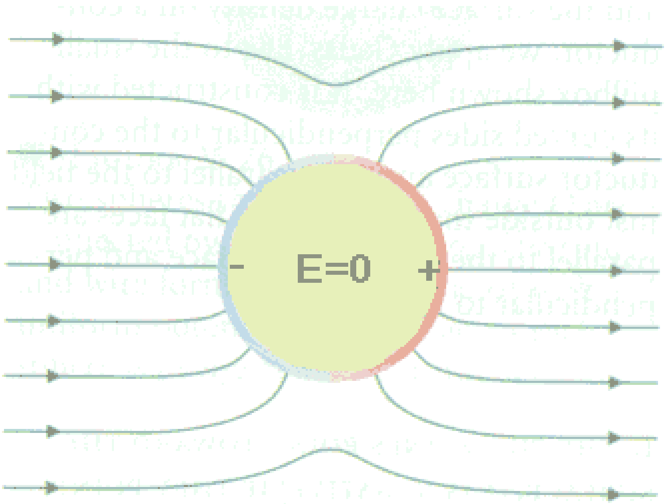
Electric dipole in not homogeneous electric field

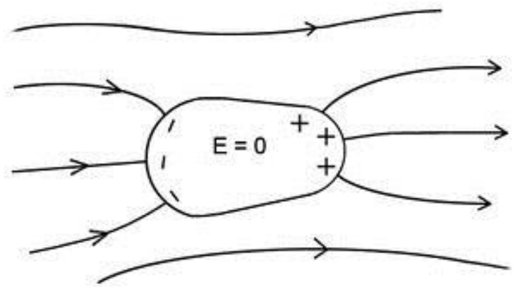


$$F = q\left[E(x+\Delta x) - E(x)\right]$$

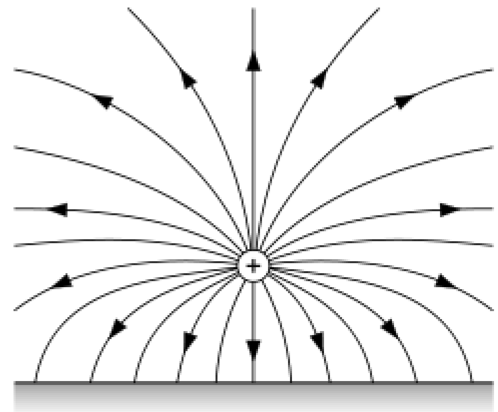
$$F = q\frac{dE}{dx}\Delta x = \frac{dE}{dx}p$$

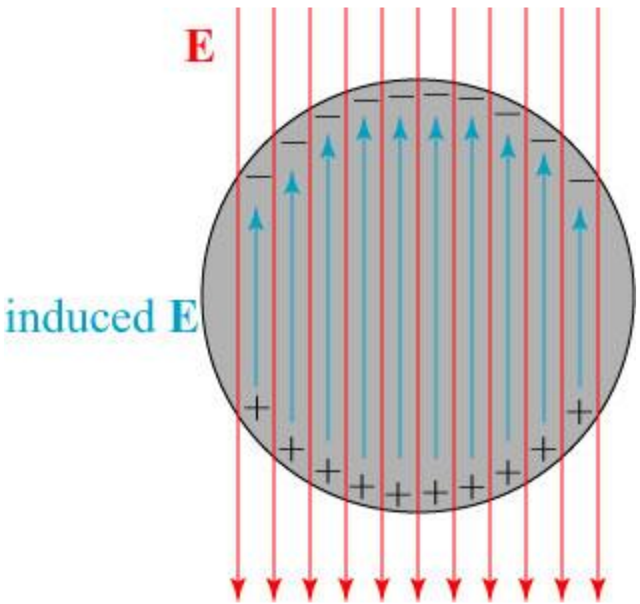
Conductor in electric field

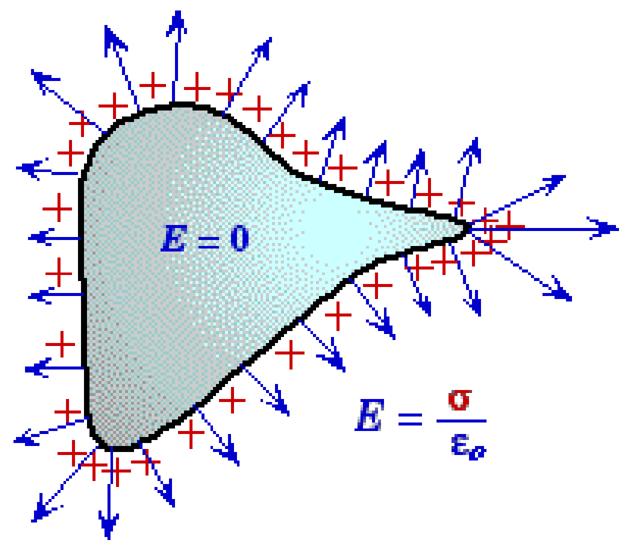




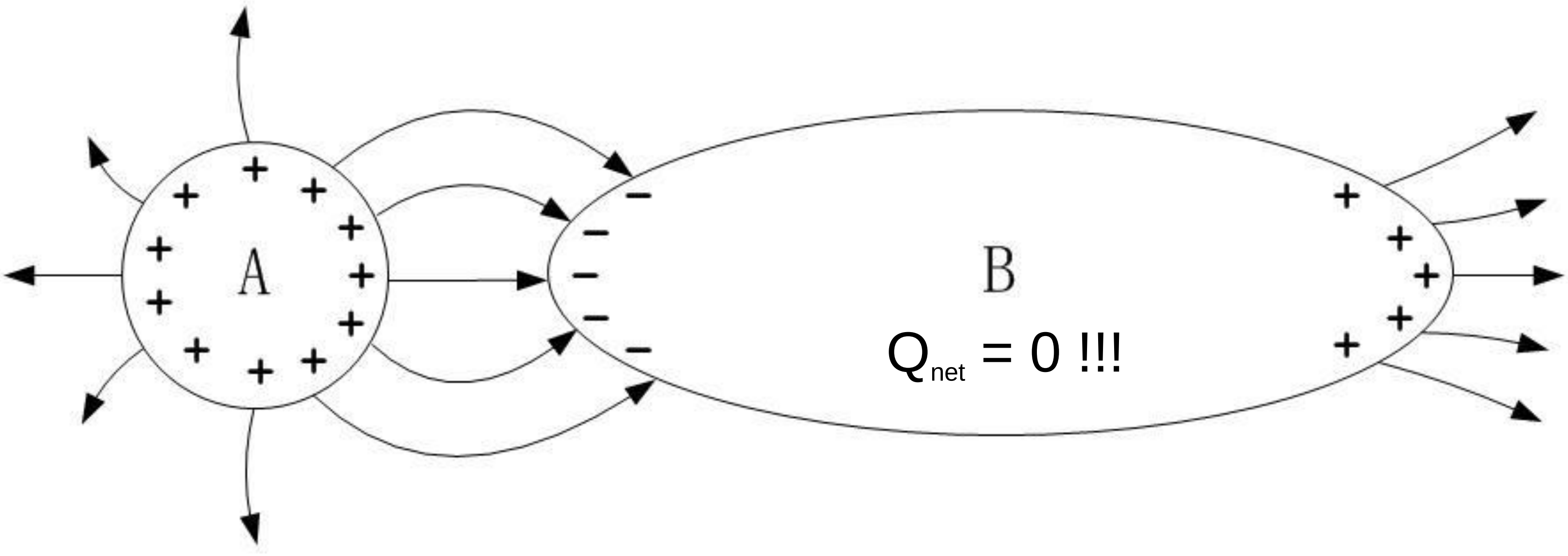
Tükörtöltés!







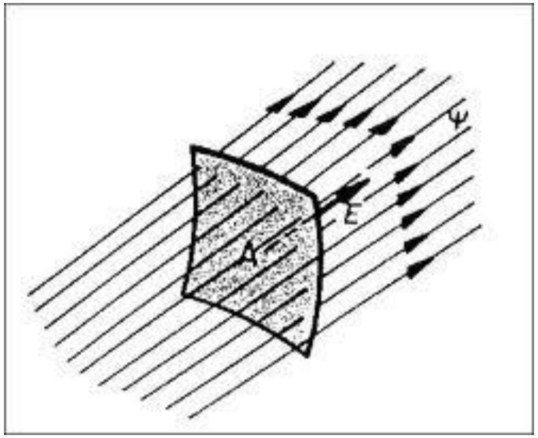
Electric induction



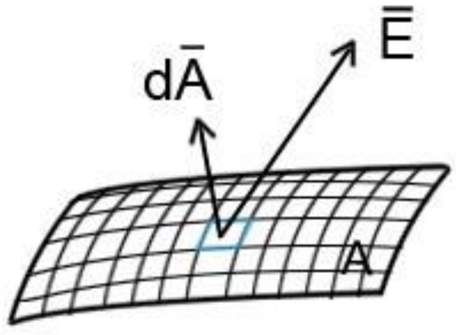
Gauss's law I.

The electric flux: $\Phi_E = \int_A \vec{E} d\vec{A}$

$$\oint_A \vec{E} d\vec{A} = \frac{Q_{net}}{\varepsilon_o}$$



$$\sum_i \vec{E}_i \bullet \Delta\vec{A}_i = \frac{Q_{net}}{\varepsilon_o}$$



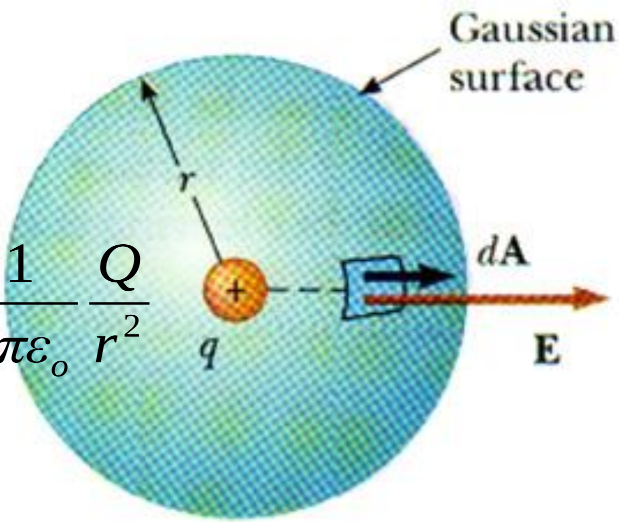
$$\Phi_E = \int_A \vec{E}(\vec{r}) d\vec{A}$$

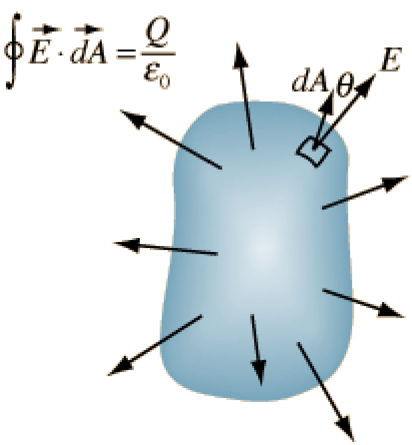
examples

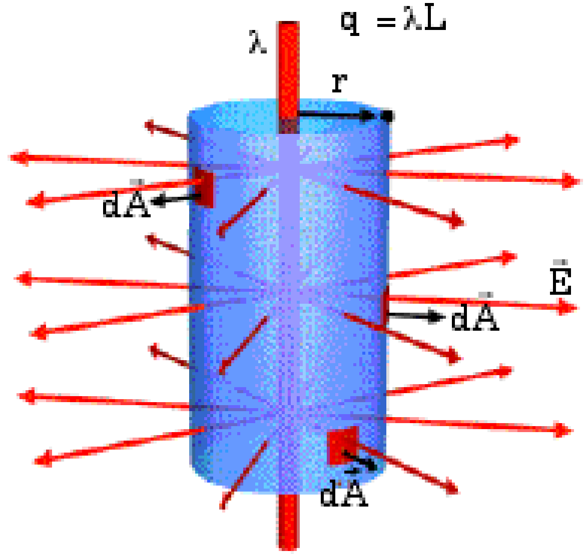
$$\sum_i \vec{E}_i \bullet \Delta\vec{A}_i = \frac{Q_{net}}{\varepsilon_o}$$ ***A Gauss's law II.***

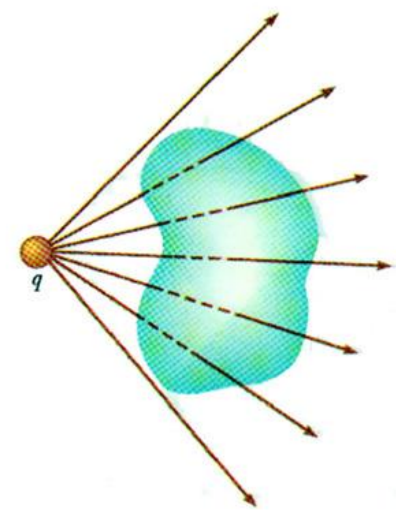
$$\oint_A \vec{E} d\vec{A} = \frac{Q_{net}}{\varepsilon_o}$$

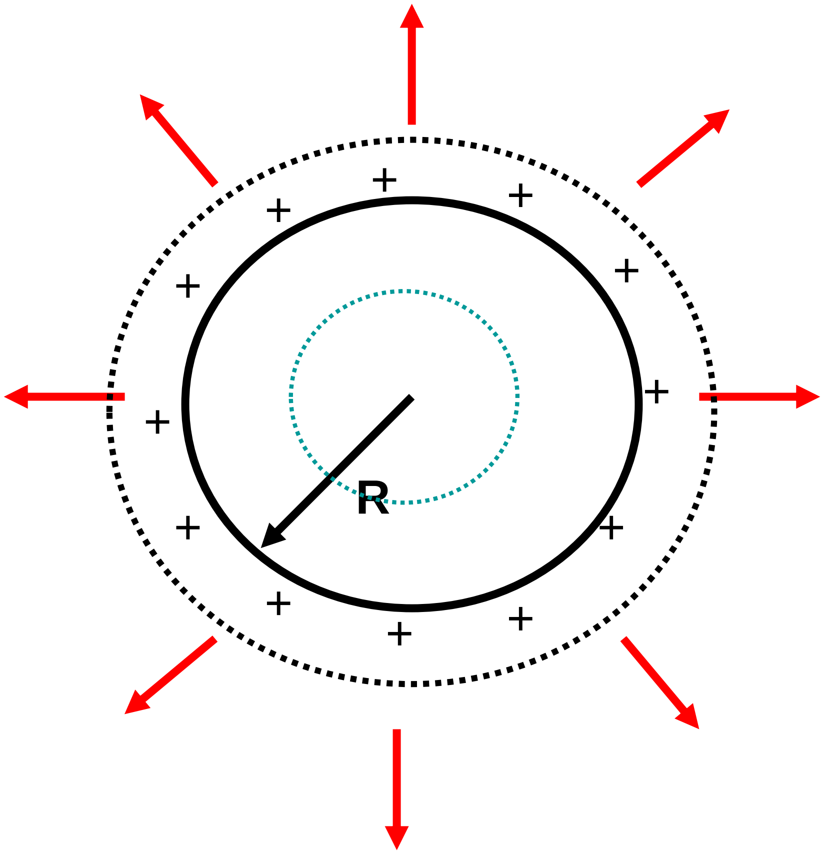
$$4\pi r^2 E = \frac{Q}{\varepsilon_o} \rightarrow E = \frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$$











Conducting sphere

$$\sum_i \vec{E}_i \bullet \Delta\vec{A}_i = \frac{Q_{net}}{\varepsilon_o}$$

r < R : E = 0

r > R :

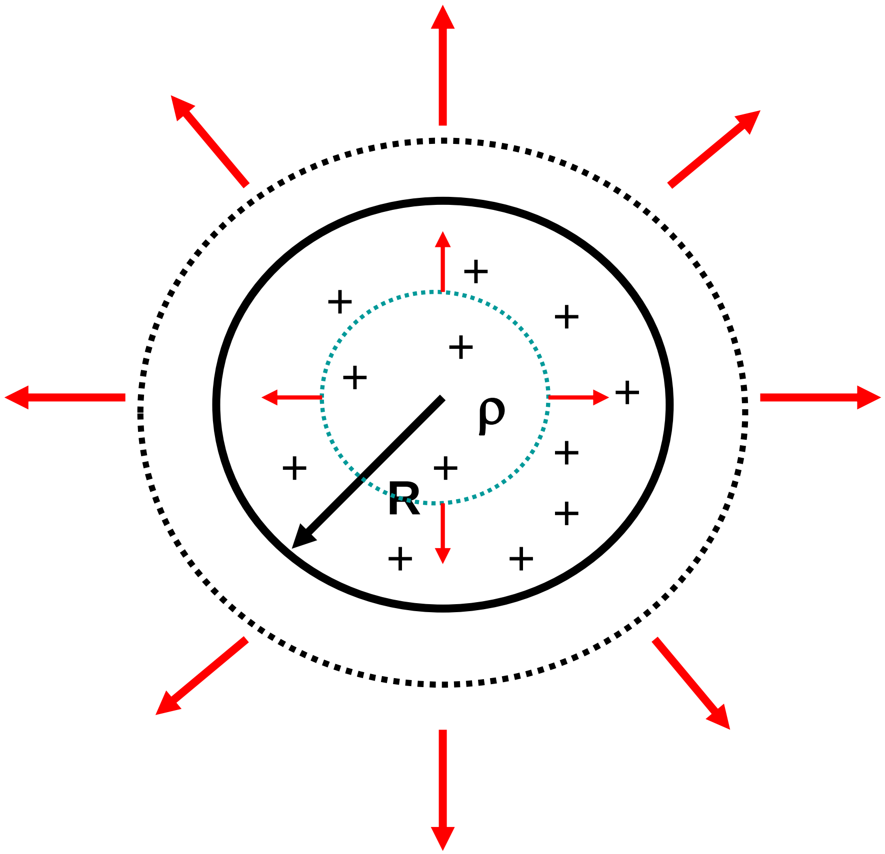
$$4r^2\pi E = \frac{Q}{\varepsilon_o}$$

~ Electric field of a point charge: (outside)

$$\vec{E} = k\frac{Q}{r^2}\vec{e}$$

Electric field of a uniformly charged sphere

σ = const.



$$\sum_i \vec{E}_i \bullet \Delta\vec{A}_i = \frac{Q_{net}}{\varepsilon_o}$$

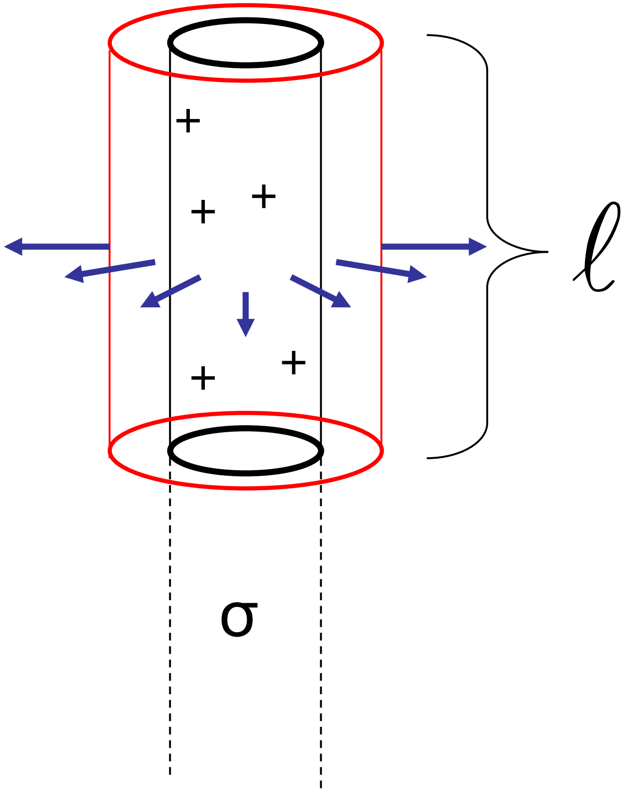
r > R :
$$4r^2\pi E = \frac{Q}{\varepsilon_o}$$

$$\vec{E} = k\frac{Q}{r^2}\vec{e}$$

r < R :
$$4r^2\pi E = \frac{4r^3\pi\rho}{3\varepsilon_o}$$

$$E = \frac{\rho}{3\varepsilon_o} r$$

An infinite (very long) conducting cylinder



$$\sum_i \vec{E}_i \bullet \Delta\vec{A}_i = \frac{Q_{net}}{\varepsilon_o}$$

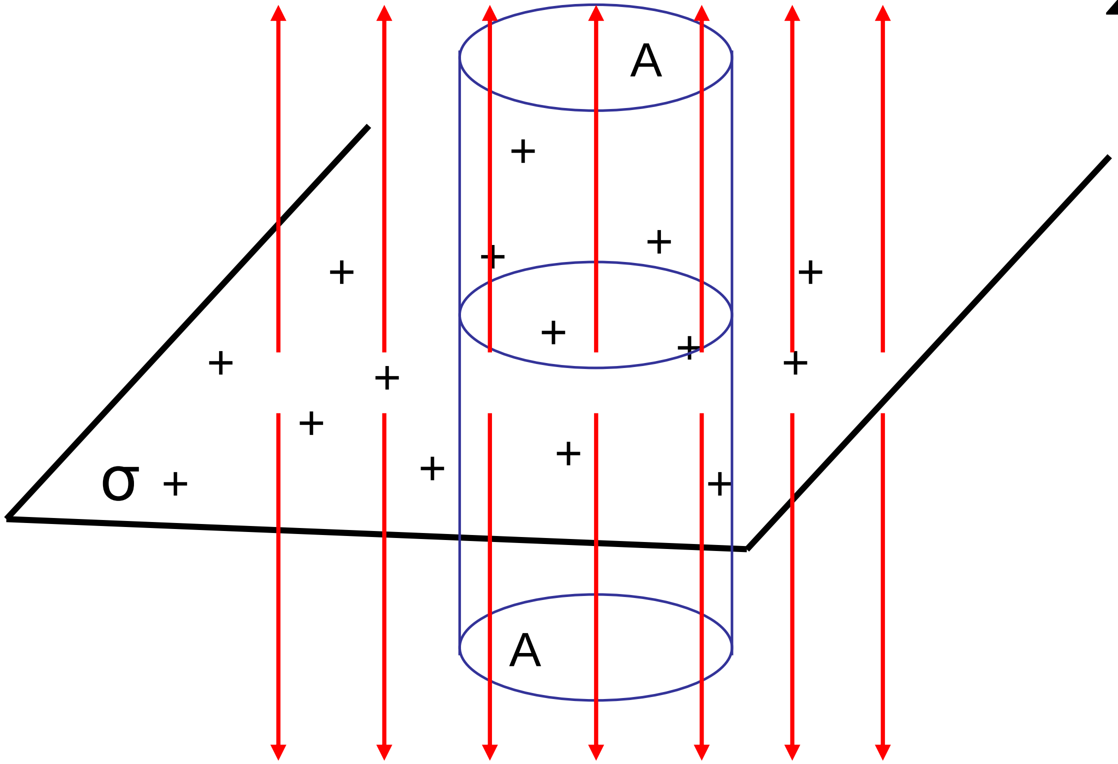
Inside: r < R : E = 0

Outside: r > R :

$$2r\pi\ell E = \frac{\sigma 2R\pi\ell}{\varepsilon_o}$$

$$E = \frac{\sigma R}{\varepsilon_o} \cdot \frac{1}{r}$$

The field of a uniformly charged plane



$$\sum_i \vec{E}_i \bullet \Delta\vec{A}_i = \frac{Q_{net}}{\varepsilon_o}$$

$$2AE = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

Surface charge density: σ [C/m^2]

Charge density

linear: $\lambda = \frac{q}{\ell} \quad \left(\text{or} \quad \lambda = \lim_{\Delta\ell \to 0} \frac{\Delta q}{\Delta \ell} \right)$

surface: $\sigma = \frac{q}{A} \quad \left(\text{or} \quad \sigma = \lim_{\Delta A \to 0} \frac{\Delta q}{\Delta A} \right)$

volume: $\rho = \frac{q}{V} \quad \left(\text{or} \quad \rho = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} \right)$