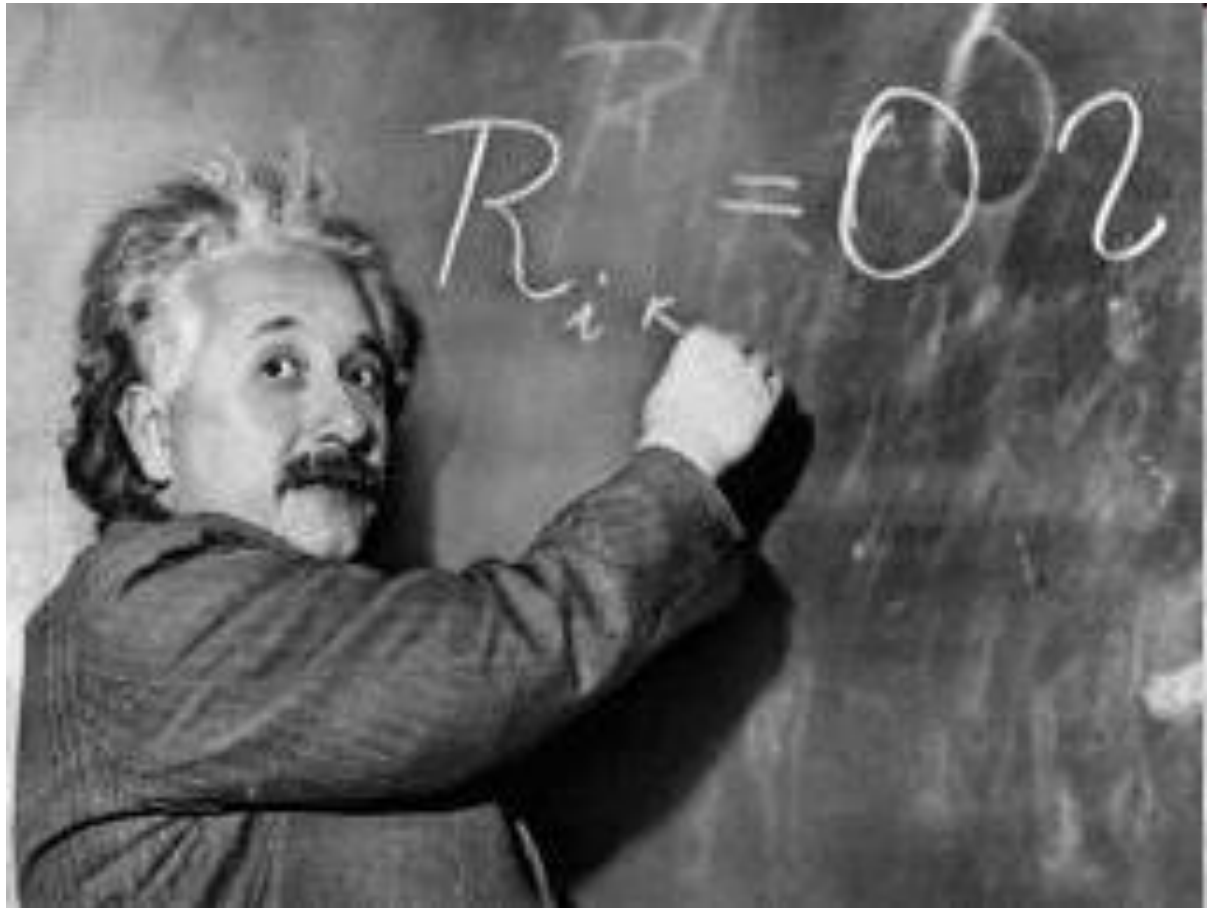


Modern fizika alkalmazásai a mérnöki gyakorlatban

4. előadás

A speciális relativitás elmélete II.



Transzformációk

Elforgatás az $x - y$ síkon:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Elforgatás a z tengely körül:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Galilei transzformáció:

$$x' = x - vt$$

és

$$t' = t$$

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} 1 & -v \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

???

Lorentz transzformáció

$$x' = \Gamma(x-vt) \quad \text{és} \quad t' = \Gamma\left(t - \frac{v}{c^2}x\right)$$



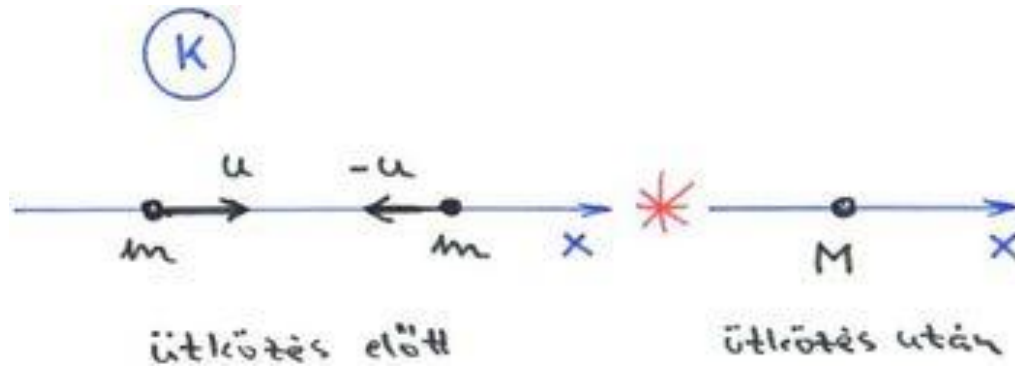
$$x' = \Gamma\left[x - \frac{v}{c}(ct)\right] \quad ct' = \Gamma\left[ct - \frac{v}{c}x\right]$$

$$\beta = \frac{v}{c}$$

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{pmatrix} \Gamma & -\beta\Gamma & 0 & 0 \\ -\beta\Gamma & \Gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Téridő koordináták!!!

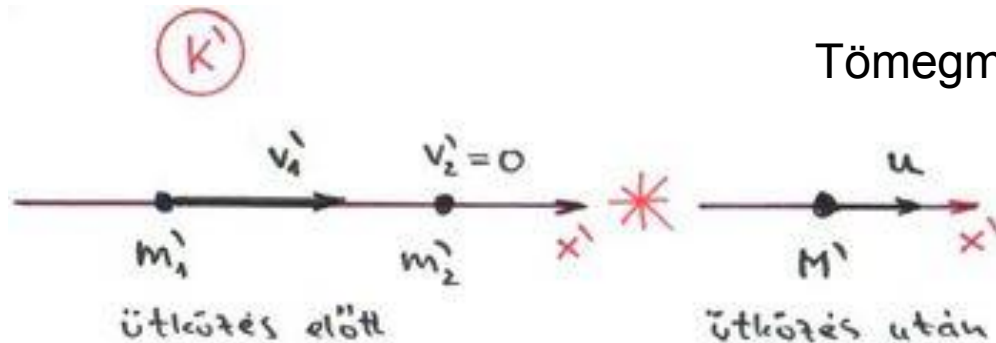
Impulzus-megmaradás I.



$$mu + m(-u) = 0$$

”Kívülről nézve...”

Rögzítsük a K' koordinátarendszerünket a $-u$ sebességgel mozgó tömegponthoz.



Tömegmegmaradás: $m'_1 + m'_2 = M'$ *u

$$m'_1 v'_1 = M' u$$

$$m'_1 (v'_1 - u) = m'_2 u$$

$$\frac{m'_1}{m'_2} = \frac{u}{v'_1 - u} = \frac{1}{\frac{v'_1}{u} - 1}$$

Impulzus-megmaradás II.

$$\frac{m'_1}{m'_2} = \frac{u}{v'_1 - u} = \frac{1}{\frac{v'_1}{u} - 1} \quad (*)$$

$$m'_1 = m'_2 = m \quad \rightarrow \quad 1 = \frac{u}{v'_1 - u} \quad \rightarrow \quad v'_1 = 2u$$

Azonban:

$$v'_1 = \frac{2u}{1 + \frac{u^2}{c^2}} = \frac{2u}{1 + \beta^2}$$

Newtoni tömegfogalom: $m'_1 = m'_2 = m \rightarrow M = 2m$

$$mv'_1 = Mu$$

↑
?

$$m \frac{2u}{1 + \beta^2} \neq 2mu$$

Impulzus-megmaradás III.

$$\frac{v'_1}{u} = \frac{2}{1 + \beta^2} \quad \text{Beírjuk (*)-be:} \quad \frac{m'_1}{m'_2} = \frac{1}{\frac{v'_1}{u} - 1} = \frac{1}{\frac{2}{1 + \beta^2} - 1} = \frac{1 + \beta^2}{1 - \beta^2}$$

$$\beta_1 = \frac{v'_1}{c} = \frac{2u/c}{1 + \beta^2} = \frac{2\beta}{1 + \beta^2}$$

$$1 - \beta_1^2 = 1 - \frac{4\beta^2}{(1 + \beta^2)^2} = \frac{1 + 2\beta^2 + \beta^4 - 4\beta^2}{(1 + \beta^2)^2} = \frac{(1 - \beta^2)^2}{(1 + \beta^2)^2}$$

$$\frac{m'_1}{m'_2} = \frac{1}{\sqrt{1 - \beta_1^2}}$$



$$m'_1 = \frac{m'_2}{\sqrt{1 - (v'_1/c)^2}}$$

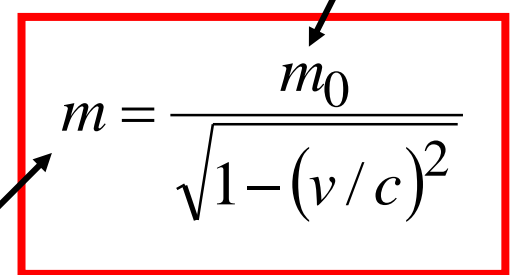
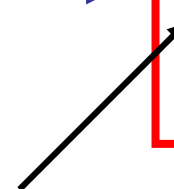


$$m = \frac{m_0}{\sqrt{1 - (v/c)^2}}$$

Nyugalmi tömeg



Relativisztikus tömeg



Relativisztikus impulzus (relativisztikus tömeg nélkül !!!)



$$\text{K'-ben: } a = \frac{F}{m} = \text{const.}$$

$$mdv_o = Fd\tau$$

K'-ben:

$$mdv_o = Fd\tau = F \sqrt{1 - \frac{v^2}{c^2}} dt$$

$$dv : dv_o = \frac{dl}{dt} : \frac{dl_o}{d\tau}$$

$$dl = dl_o \sqrt{1 - \frac{v^2}{c^2}} \quad \text{és} \quad dt = \frac{d\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dv = dv_o \left(1 - \frac{v^2}{c^2} \right)$$

$$\frac{dv}{dt} = \left(1 - \frac{v^2}{c^2} \right)^{3/2} \frac{F}{m}$$

$$\frac{m}{\left(1 - v^2 / c^2 \right)^{3/2}} \frac{dv}{dt} = F$$

$$\frac{dp}{dt} = F$$

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

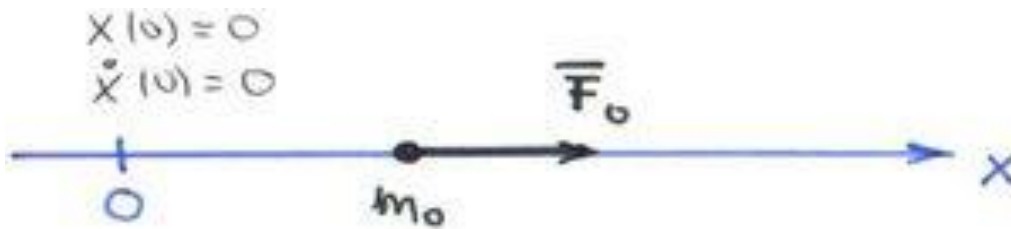
Relativisztikus mozgásegyenlet

$$\dot{\vec{p}} = \vec{F}$$

ahol

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Példa: tömegpont mozgása állandó erő hatására I.



$$m_0 \ddot{x} = F_0$$

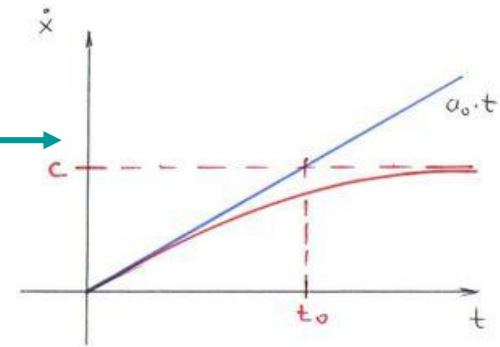
$$\ddot{x} = \frac{F_0}{m_0} \equiv a_0$$

Példa: tömegpont mozgása állandó erő hatására II.

$$\frac{d}{dt} \left[\frac{m_0 \dot{x}}{\sqrt{1 - (\dot{x}/c)^2}} \right] = F_0 \quad \longrightarrow \quad \left[\frac{m_0 \dot{x}}{\sqrt{1 - (\dot{x}/c)^2}} \right]_0^{\dot{x}} = F_0 t$$

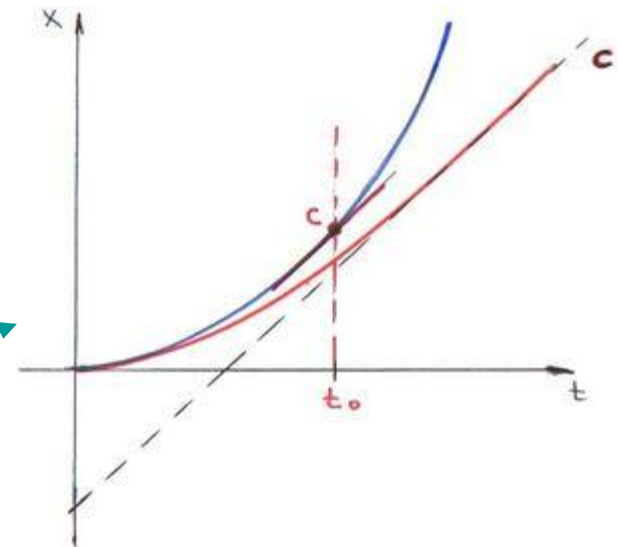
$$\dot{x} = a_0 t \sqrt{1 - (\dot{x}/c)^2} \quad \longrightarrow \quad \dot{x} = \frac{a_0 t}{\sqrt{1 + (a_0 t/c)^2}}$$

$$\lim_{t \rightarrow \infty} \dot{x} = c$$



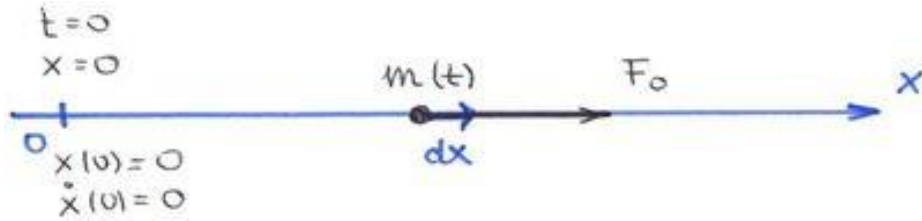
$$[x]_0^t = \int_0^t \frac{a_0 t}{\sqrt{1 + (a_0 t/c)^2}} dt = \left[\frac{c^2}{a_0} \sqrt{1 + (a_0 t/c)^2} \right]_0^t$$

$$x(t) = \frac{c^2}{a_0} \left(\sqrt{1 + (a_0 t/c)^2} - 1 \right)$$



A tömeg-energia ekvivalencia

$$W = \int_0^x F dx$$



$$\frac{dp}{dt} = F \rightarrow p = mv = \frac{m_0 v}{\sqrt{1-(v/c)^2}}$$

$$W = \int_0^x F dx = \int_0^x \frac{dp}{dt} dx = \int_0^p \frac{dx}{dt} dp = \int_0^p v dp = \int_0^v v \frac{dp}{dv} dv \rightarrow W = [pv]_0^v - \int_0^v p dv$$

$$W = [pv]_0^v - m_0 \int_0^v \frac{v}{\sqrt{1-(v/c)^2}} dv \rightarrow W = m_0 \frac{v^2}{\sqrt{1-(v/c)^2}} - m_0 \left[-c^2 \sqrt{1-(v/c)^2} \right]_0^v$$

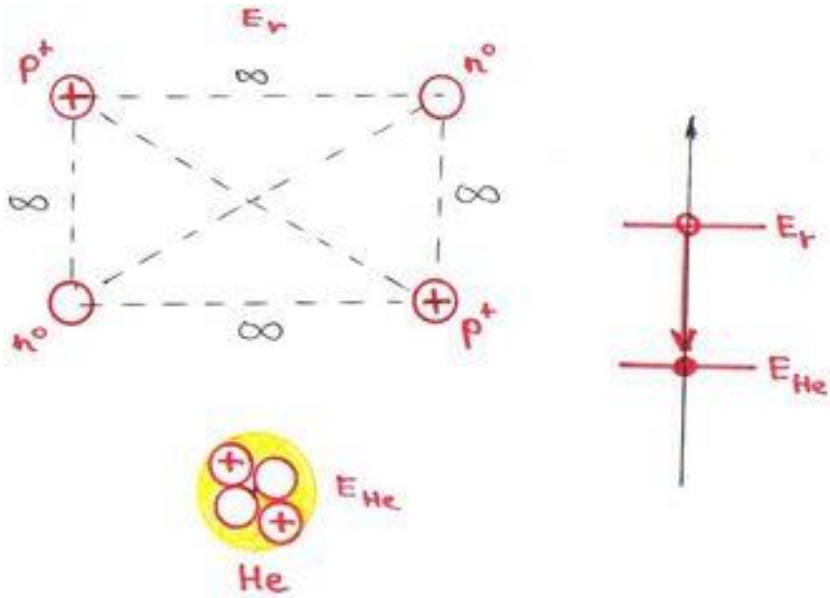
$$W = m_0 \frac{v^2}{\sqrt{1-(v/c)^2}} + m_0 c^2 \sqrt{1-(v/c)^2} - m_0 c^2$$

$E = mc^2$

$$\Delta E_k = W = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}} - m_0 c^2$$

$$\frac{v}{c} \ll 1 \rightarrow E_k = \frac{1}{2} m v^2 + \dots$$

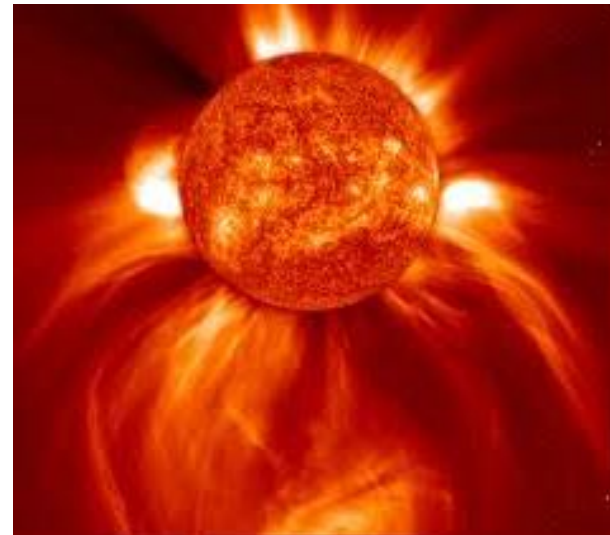
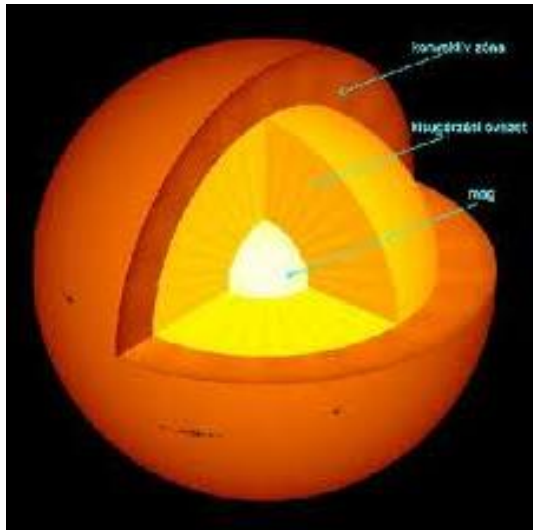
A tömegdefektus



$$2m_p + 2m_n > m_{\text{He}}$$

$$\Delta E = 28 \text{ MeV}$$

$$1 \text{ mól} \rightarrow 10^{11} \text{ J}$$



Az energia, mint a sebesség és az impulzus függvénye

Klasszikus fizika: $E = \frac{1}{2}mv^2$

$p = mv$ \rightarrow $E = \frac{p^2}{2m}$

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-(v/c)^2}}$$

$$p = \frac{m_0v}{\sqrt{1-(v/c)^2}}$$

Belátható: $E^2 - p^2c^2 = m_0^2c^4$

$E = \sqrt{m_0^2c^4 + p^2c^2}$

 $\rightarrow E_k = \sqrt{m_0^2c^4 + p^2c^2} - m_0c^2 \approx \frac{p^2}{2m}$ ha $p \ll m_0c$

