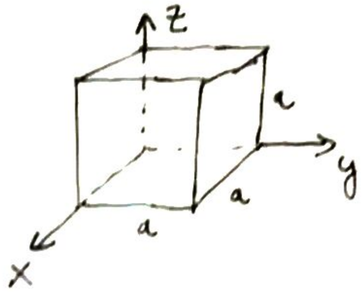
F1.



A lapok normálvektorai:

(x;y) sík: $\underline{n}_1 = -\hat{\underline{z}}$; ezzel párhuzamos lap: $-\underline{n}_1$

(x;z) sík: $\underline{n}_2 = -\hat{\underline{y}}$; —II— : $-\underline{n}_2$

(y;z) sík: $\underline{n}_3 = -\hat{\underline{x}}$; —II— : $-\underline{n}_3$

a) $\underline{E} = E_0 \hat{\underline{z}}$

alsó lapon: $\Psi_1 = \underline{E}\cdot\underline{n}_1\cdot a^2 = -E_0 a^2$; felső lapon: $\Psi_1' = \underline{E}(-\underline{n}_1)a^2 = E_0 a^2$

az oldalsó lapokon: $\Psi = 0$, mert pl. $\Psi_{xz} = \underline{E}\,\underline{n}_2\cdot a^2 = -E_0 a^2\cdot \underbrace{\hat{\underline{y}}\hat{\underline{z}}}_{=0}$

b) $\underline{E} = \frac{E_0}{\sqrt{3}}(\hat{\underline{x}}+\hat{\underline{y}}+\hat{\underline{z}})$

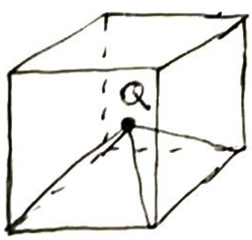
alsó lap: $\Psi_1 = \underline{E}\,\underline{n}_1 a^2 = -\frac{E_0}{\sqrt{3}}a^2$; felső lap: $\Psi_1' = \underline{E}(-\underline{n}_1)a^2 = \frac{E_0}{\sqrt{3}}a^2$

oldalsó lapok:

$\Psi_2 = \underline{E}\,\underline{n}_2 a^2 = -\frac{E_0}{\sqrt{3}}a^2$; $\Psi_2' = \underline{E}(-\underline{n}_2)a^2 = \frac{E_0}{\sqrt{3}}a^2$

$\Psi_3 = \underline{E}\,\underline{n}_3 a^2 = -\frac{E_0}{\sqrt{3}}a^2$; $\Psi_3' = \underline{E}(-\underline{n}_3)a^2 = \frac{E_0}{\sqrt{3}}a^2$

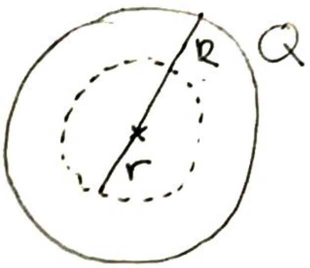
c)



A szimmetria miatt egy lapon átmenő fluxus értéke a Q körüli $r = \frac{\sqrt{3}}{2}a$ (testátló fele) sugarú gömbfelszínen áthaladó fluxus $\frac{1}{6}$-ával egyenlő:

$\Psi_{gömb} = \frac{Q}{\varepsilon_0}$ (Gauss-törvény) $\rightarrow$ $\Psi_{lap} = \frac{1}{6}\Psi_{gömb} = \frac{Q}{6\varepsilon_0}$

F2.



a) ha $0 \leq r \leq R$:

Gauss-törvény: $E\cdot 4r^2\pi = \frac{1}{\varepsilon_0}\cdot Q(r)$

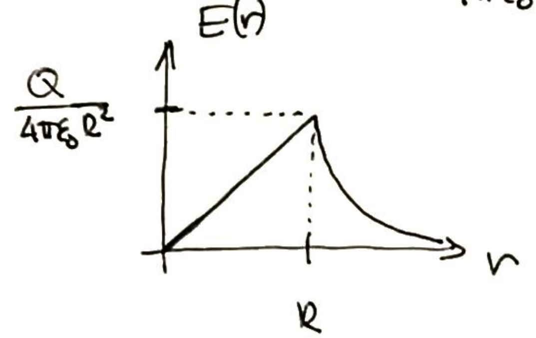
$$E \cdot 4r^2\pi = \frac{1}{\varepsilon_0} \cdot Q \cdot \frac{\cancel{\frac{4}{3}} r^3 \cancel{\pi}}{\cancel{\frac{4}{3}} R^3 \cancel{\pi}}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{r}{R^3} = \frac{kQ}{R^3} \cdot r$$

ha $R \leq r$:

Gauss-törvény: $E \cdot 4r^2\pi = \frac{1}{\varepsilon_0} Q$

$$E(r) = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} = \frac{kQ}{r^2}$$



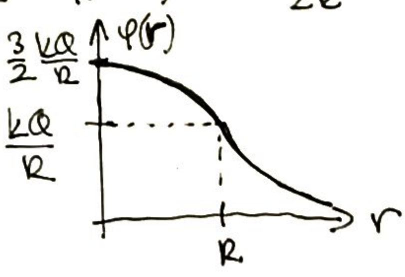
b) $\varphi(r \to \infty) = 0$

ha $R \leq r$: $\varphi(r) = \frac{kQ}{r}$ (ponttöltés)

ha $0 \leq r \leq R$: $\varphi(r) = \underbrace{\frac{kQ}{R}}_{\substack{\text{munkavégzés} \\ \infty\text{-ből a gömbfelületig}}} + \underbrace{\frac{E(R)+E(r)}{2}(R-r)}_{\substack{\text{gömbön belül} \\ \text{lineáris a térerősség}}} =$

$$= \frac{kQ}{R} + \frac{1}{2}\left(\frac{kQ}{R^2} + \frac{kQ}{R^3} r\right)(R-r) = \frac{kQ}{R} + \frac{1}{2} \frac{kQ}{R^3}(R+r)(R-r) =$$

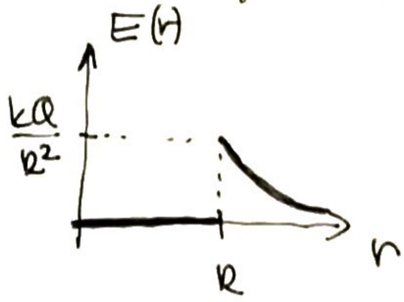
$$= \frac{kQ}{R}\left(1 + \frac{1}{2} \frac{R^2-r^2}{R^2}\right) = \frac{kQ}{2R^3}\left(2R^2+R^2-r^2\right) = \frac{kQ}{2R^3}\left(3R^2-r^2\right)$$



c) A fémgömbön belül nincs elektrosztatikus tér (leárnyékolás)

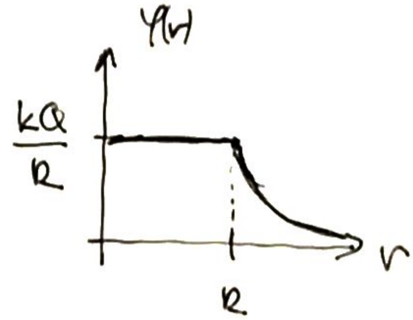
ha $0 \leq r \leq R$: $E = 0$

ha $R \leq r$: $E = \frac{kQ}{r^2}$



ha $R \leq r$: $\varphi(r) = \frac{kQ}{r}$

ha $0 \leq r \leq R$: $\varphi(r) = \text{áll.} = \frac{kQ}{R}$



F3.

$R = 2\,\mathrm{cm}$

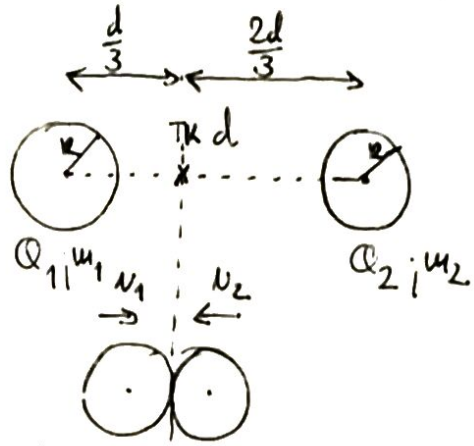
$Q_1 = -10\,\mathrm{nC}$

$Q_2 = 20\,\mathrm{nC}$

$m_1 = 10\,\mathrm{g}$

$m_2 = 5\,\mathrm{g}$

$d = 12\,\mathrm{cm}$



A homogén töltéseloszlás, szigetelőgömbök miatt ponttöltéseknek foghatjuk fel.

Lendületmegmaradás: $0 = m_1 v_1 - m_2 v_2 \rightarrow$

Energiamegmaradás: $\rightarrow v_2 = v_1 \cdot \frac{m_1}{m_2}$

$$-\frac{kQ_1Q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{kQ_1Q_2}{2R}$$

$$-\frac{kQ_1Q_2}{d} + \frac{kQ_1Q_2}{2R} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \cdot v_1^2 \frac{m_1^2}{m_2^2}$$

$$2kQ_1Q_2\left(\frac{1}{2R} - \frac{1}{d}\right) = v_1^2\left(1 + \frac{m_1}{m_2}\right) m_1 \rightarrow v_1 = \sqrt{\frac{2kQ_1Q_2}{m_1\left(1+\frac{m_1}{m_2}\right)}\left(\frac{1}{2R} - \frac{1}{d}\right)} =$$

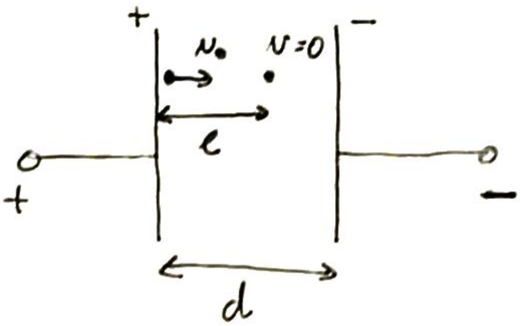
$$= 0{,}045\,\frac{\mathrm{m}}{\mathrm{s}} \;;\; v_2 = 2v_1 = 0{,}09\,\frac{\mathrm{m}}{\mathrm{s}}$$

F4.

$d = 10\,mm$

$U = 12\,V$

$E_0 = 9\,eV$



$q = 1{,}6 \cdot 10^{-19}\,C$

a, Munkatétel: $E_0 = \frac{1}{2} m v_0^2 = \frac{\ell}{d} U \cdot q \rightarrow \ell = \frac{E_0 \cdot d}{qU} = 7{,}5\,mm$

b, $v_0 = \sqrt{\frac{2E_0}{m}} = 1{,}8 \cdot 10^6 \,\frac{m}{s}$

c, A térerősség: $\frac{U}{d}$, tehát $a = \frac{\frac{U}{d} \cdot q}{m} = \frac{Uq}{md} = 2{,}1 \cdot 10^{14} \,\frac{m}{s^2}$

F5. $\varphi(x;y;z) = 3\,\frac{V}{m^2} \cdot x^2 + 7\,V = \alpha x^2 + \beta$

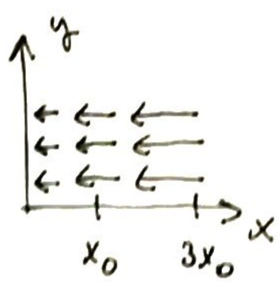
$x_0 = 1m$; $y_0 = 0$; $z_0 = 0$; $v_0 = 0$

a, Mivel a potenciál a +x tengely irányában növekszik, ezért a térerősség −x irányú. Tehát az elektron +x irányba indul. A potenciál megváltozása: $\Delta\varphi = \varphi(3x_0) - \varphi(x_0) = \alpha \cdot 9x_0^2 - \alpha x_0^2 = 8\alpha x_0^2$

munkatétel: $q \cdot \Delta\varphi = \frac{1}{2} m v^2 = E_{kin} = 8\alpha q x_0^2 = 24\,eV$

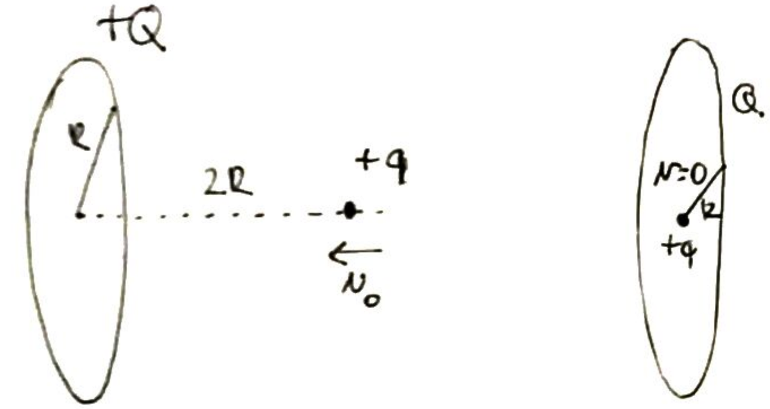
b, $v = \sqrt{\frac{2E_{kin}}{m}} = 2{,}9 \cdot 10^6 \,\frac{m}{s}$

c, $E(x) = -2\alpha x = -6\,\frac{V}{m^2}\,x$

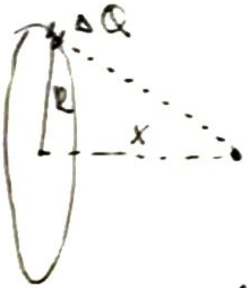


$a_0 = \frac{qE(x_0)}{m} = 1{,}1 \cdot 10^{12} \,\frac{m}{s^2}$

F6.



A potenciál x távolságra:



$$\varphi(x) = \sum \frac{k\Delta Q}{\sqrt{R^2+x^2}} = \frac{kQ}{\sqrt{R^2+x^2}}$$

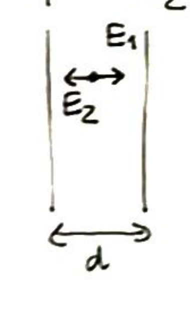
Energiamegmaradás: $$\frac{1}{2}mv_0^2 + U(x)\cdot q = 0 + U(R)\cdot q$$

$$\frac{1}{2}mv_0^2 + \frac{kqQ}{\sqrt{5}R} = \frac{kqQ}{R}$$

$$\frac{1}{2}mv_0^2 = \frac{kQq}{R}\,\frac{5-\sqrt{5}}{5}$$

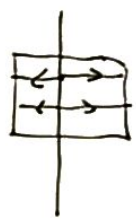
$$v_0 = \sqrt{\frac{2(5-\sqrt{5})}{5}\cdot\frac{kQq}{mR}}$$

F7.



$$C = \frac{\varepsilon_0 A}{d}$$

egy lemez:



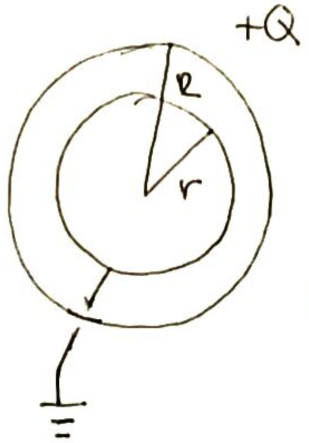
$$E\cdot 2A = \frac{1}{\varepsilon_0}\cdot Q$$

$$E = \frac{Q}{2\varepsilon_0 A}$$

Az eredő térerősség: $$E = \frac{Q_1-Q_2}{2\varepsilon_0 A}$$

$$U = E\cdot d = \frac{Q_1-Q_2}{2}\,\frac{d}{\varepsilon_0 A} = \frac{1}{2}\,\frac{Q_1-Q_2}{C}$$

(F8.)



A kisebb gömb potenciálja nulla (földelt). Ahhoz, hogy ez teljesüljön, negatív töltés kerül a kisebb gömbre

Szuperpozíciós elv:

$$0 = \frac{kQ}{R} + \frac{kq}{r}$$

külső gömb potenciálja az r helyen

q töltésű gömb potenciálja az r helyen

tehát:

$$q = -Q \cdot \frac{r}{R}$$