Fundamentals of High Frequency Wave Propagation

Physics laboratory practice

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Chapter 1

Introduction

The goal of the laboratory practice is to give an insight into high frequency ($f > 1 - 10$ MHz) measurement techniques and to introduce the reader into the fundamentals of signal transmission theory. First of all, the most important to mention is that our previous knowledge based on low frequency networks fails when one tries to apply them on high frequency systems, where several unusual phenomena occur, for example the reflection of the signals. From the physics point of view, here, one has to use the Maxwell-equations for high frequencies, including wave effects for transmission cables.

By the middle of the XIXth century it became a huge need to transmit signals for a long distance, even through and connecting continents (e.g. underwater transmission cables)[1]. It turned out soon that wave effects have to be taken into account in signal transmission through cables. This technological development and the need for a theoretical description was close in time to the born of the Maxwell's equations (1861). Nowdays we know that the description of wave phenomenon in cables is described by the so called Telegrapher's equations. This is one of the applications of the electromagnetic wave phenomenon predicted by the Maxwell's equations. This practical approach had an inspiring influence on the discovery of the electromagnetic radiation (Hertz, 1886).

During our previous studies on the field of direct current (DC) and low frequency alternating current (AC) networks we neglected that the propagation of the signal is finite. We assumed, that connecting voltage at a certain point in the network it appears immediately at every equipotent sites. All of this obviously become invalid, when the propagation time required for the signal, $t = d/c$ (here d is the length of the cable, c is the speed of light in the medium), is comparable with the time period of the signal: $t \approx 1/f$ (in practice the $10 \times t \approx 1/f$ requirement is generally used). For example in the case of the trans-atlantean cable the frequency gained this way is $f = 6$ Hz. This result means, that without taking the wave phenomenon into account the trans-atlantean communication would work only at a lower frequency. Nowdays we would say that the communication is possible with the *bandwidth* of 6 Hz (i.e. 6 bits/sec).

Taking wave phenomenon into account is more important in modern communication devices, because e.g. at a 9 GHz carrier frequency (which is a commonly used communication channel) the wavelength is only 3 cm. Another practical example are computers, which typically operate with signals of $2 - 3$ GHz ($\lambda \approx 10$ cm), which are transmitted to $10 - 20$ cm, thus the wave phenomenon is evidently required to be taken into account at circuit design. In our further studies we will experience similar phenomena in NMR (Nuclear Magnetic Resonance)

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[1]The first trans-atlantean cable was deployed in 1858.

and ESR (Electron Spin Resonance) laboratory practices.

Chapter 2

Theoretical background
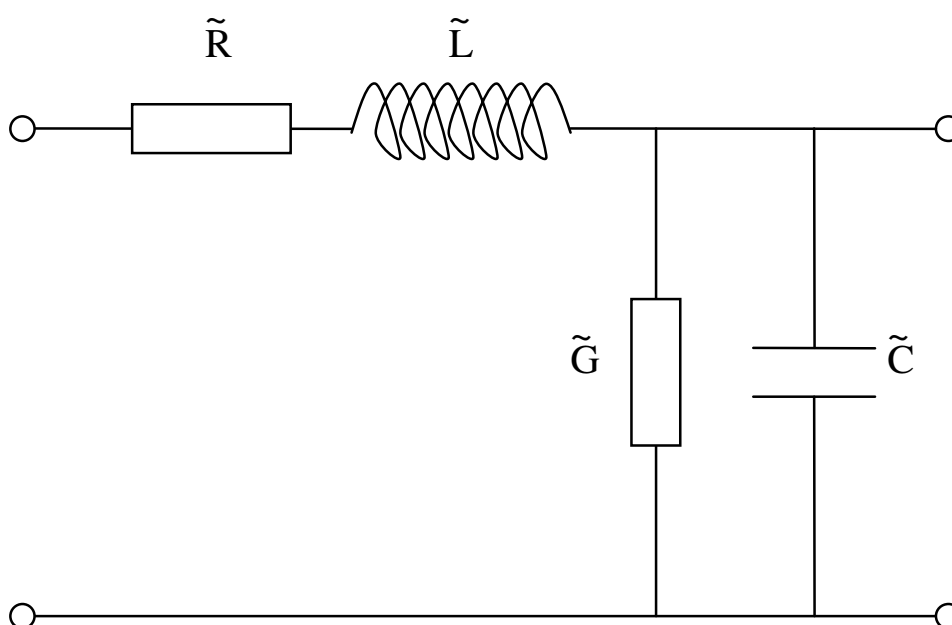
2.1 The Telegrapher's Equations

Figure 2.1: Circuit model of the piece of the investigated wire during wave propagation.

Let us consider an infinitesimally small piece of the signal transmitting cable, which is presented in Fig. 2.1. In general this system can be described with a series so-called *distributed resistance*, $\widetilde{R}$ (dimension: Ω/m), *distributed inductance*, $\widetilde{L}$ (dimension: H/m), *distributed capacity*, $\widetilde{C}$ (dimension: F/m) and a *distributed conductance*, $\widetilde{G}$, between the two cables (dimension: S/m). The reason of the series resistor is the loss in the wire pieces. The inductance is required due to the magnetic field surrounding the cable, therefore one wire thread has self inductance. The $\widetilde{G}$ parameter describes the electric conductivity between the two wires, which is also present when the conductors are separated with good dielectric medium. Because of the fact that the two wires are not equipotent, a $\widetilde{C}$ capacity appears between them.

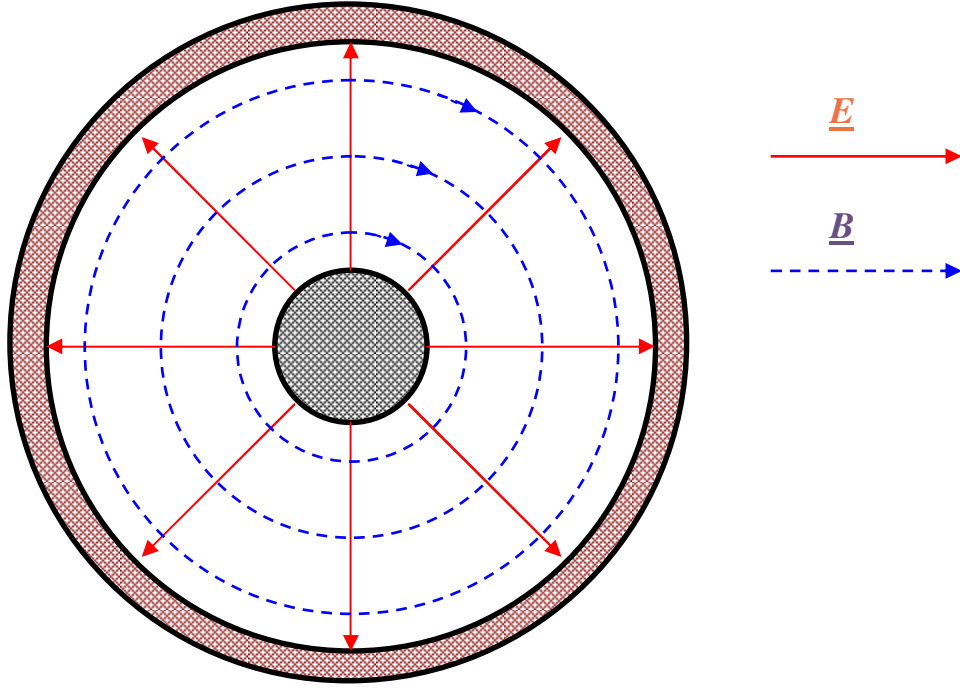It can be seen, that from the values above, $\widetilde{R}$ primarily depends from the quality of the

*Figure 2.2: The cross-section of the coaxial cable. Red and blue lines denote the electric **E** and magnetic **B** fields respectively for the so-called TEM00 mode. The inner wire is connected to an alternating voltage, while the outer is usually grounded.*

material[1], but on the other hand $\widetilde{L}$, $\widetilde{C}$ and $\widetilde{G}$ strongly depend from the relative position of the wires (e.g. for a stranded wire pair $\widetilde{L} = 0$, but $\widetilde{C}$ is large). In the case of a pair of wires at a given distance from each other the value of $\widetilde{L}$ is constant, meanwhile $\widetilde{C}$ strongly depends from the neighboring dielectric medium (the latter caused problems in the case of the trans-atlantean cable because of the salt water). For all of these problems the coaxial cable (Heaviside, 1880) gives a solution, in which the inner wire is located in the middle of the grounded metallic coating. The benefit of this geometry is that all of its parameters are well defined, both electric and magnetic field lines are inside the space between the two wires, as it can be seen in Fig. 2.2. In the past, coaxial cables had rigid coating and were filled with air. Nowadays they use elastic dielectric filling (typically $\varepsilon_r = 2 - 3$ and $\mu_r = 1.0$).

The capacity and the self inductance for a unity length coaxial cable can be written from the definitions of these parameters:

$$\widetilde{C} = \frac{2\pi\varepsilon_0\varepsilon_r}{\log(D/d)}, \tag{2.1}$$

$$\widetilde{L} = \frac{\mu_0\mu_r \log(D/d)}{2\pi}, \tag{2.2}$$

where D is the internal diameter of the coating and d is the outer diameter of the internal wire. ε_0 and μ_0 are well known physical constants, ε_r and μ_r are characteristic parameters of the material.

To introduce to Telegrapher's equations first we take the simplest case, when both wires are perfect conductors ($\widetilde{R} = 0$) and perfectly insulated from each other ($\widetilde{G}=0$), thus the phenomenon

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[1]At high frequencies its value increases due to the *skin effect*.

depends only from $\widetilde{L}$ and $\widetilde{C}$.[2] In this case both the voltage ($U(x,t)$), both the current ($I(x,t)$) become space and time dependent. To describe these quantities one can gain two coupled, first order, partial differential equations (Heaviside, 1880)[3], which can be derived from Maxwell's equations with finite differences:

$$\frac{\partial U(x,t)}{\partial x} = -\widetilde{L}\frac{\partial I(x,t)}{\partial t}$$
$$\frac{\partial I(x,t)}{\partial x} = -\widetilde{C}\frac{\partial U(x,t)}{\partial t}. \tag{2.3}$$

The (2.3) Telegrapher's equations can be combined into two equivalent wave equations, one for the voltage and one for the current.

$$\frac{\partial^2 U(x,t)}{\partial t^2} = \frac{1}{\widetilde{L}\widetilde{C}}\frac{\partial^2 U(x,t)}{\partial x^2}$$
$$\frac{\partial^2 I(x,t)}{\partial t^2} = \frac{1}{\widetilde{L}\widetilde{C}}\frac{\partial^2 I(x,t)}{\partial x^2}. \tag{2.4}$$

From the well known wave equations it can be read, that propagation speed is $v = \frac{1}{\sqrt{\widetilde{L}\widetilde{C}}}$, and the most general solutions for the voltage and the current are:

$$U(x,t) = U^+ f(\omega t - kx) + U^- f(\omega t + kx)$$
$$I(x,t) = I^+ f(\omega t - kx) + I^- f(\omega t + kx), \tag{2.5}$$

where ω is the angular frequency and $k = \omega/v$ is the wavenumber of the propagating wave. U^+ and U^- are the amplitudes of the positive and negative direction of propagation among the x axis, respectively, and f is an arbitrary function. Let us notice, that the dimension of v is truly m/s.

A special case is, when only propagation in a certain direction is present. This solution is:

$$U(x,t) = U_0\, e^{i(\omega t - kx)}$$
$$I(x,t) = I_0\, e^{i(\omega t - kx)}. \tag{2.6}$$

Substituting the solution given by (2.6) into the Telegrapher's equations, one can obtain that for the propagating wave the ratio of voltage and the current is:

$$\frac{U(x,t)}{I(x,t)} = \sqrt{\frac{\widetilde{L}}{\widetilde{C}}} = Z_0, \tag{2.7}$$

where Z_0 has the unit of resistance and called *wave impedance*.

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[2]The general case can be solved as well, but it leads to complex solutions.
[3]equations (2.3)

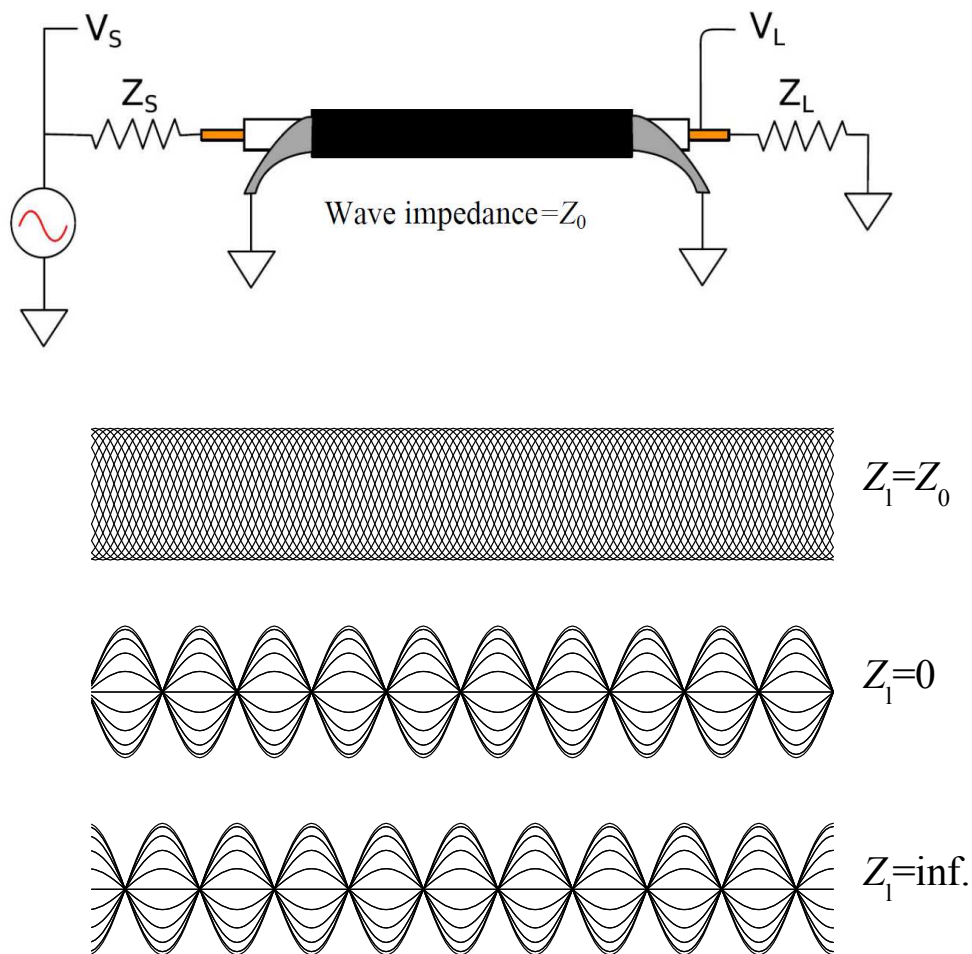## 2.2 Reflections from the end of the cable

Figure 2.3: Schematic circuit with sine driving generator, which has Z_f output impedance, coaxial cable which ends with a Z_l terminating impedance. The generator operates at the maximum power, when $Z_f = Z_0$. The voltage distribution in the cable is also shown for 20 periods, when $d = 5\lambda$ for 3 terminating impedance values. Notice, that in case of $Z_f = Z_0$ the maximum of the voltage is homogeneous in the cable, in case of $Z_f = 0$ both ends have a node (in this case we measure 0 voltage on the source), and in case of $Z_f = \infty$ there are antinodes at both terminations.

The exact solutions of the wave equations (2.4) can be obtained through fitting the initial and boundary conditions (for instance the amplitude at the end of the wire). The following two cases are analogous with high school acoustic phenomenon: when the termination of the cable is shorted ($Z_l = 0$), and when the coaxial cable has open end ($Z_l = \infty$). These two cases are presented in Fig. 2.3. If a well defined relation exists between the length of the cable and the exciting wave frequency ($d = n \times \lambda/2$ for terminated ending and $d = (2n+1) \times \lambda/4$ for opened end, where n is integer), voltage standing waves appear along the cable. The nodes and antinodes are analogous with the well known closed and open ended whistles. These two instances, according to the analogous behavior with acoustic waves, can be understood in that manner that from both closed and opened ends the wave is being reflected. Thus the standing wave behavior of the voltage among the cable is a result of the interference of propagating and

reflected waves. With our knowledge of DC networks we would say that the voltage in the shorted pair is always 0, while in the open circuit it takes the voltage value of the generator.

High frequency data- and energy transfer obviously requires, that there should not be any reflection from the end of the cable. In Appendix A. we show that this case applies, when the terminating impedance is:

$$Z_l = Z_0, \tag{2.8}$$

where $Z_0 = \sqrt{\widetilde{L}/\widetilde{C}}$ is the wave impedance of the cable. If (2.8) condition is not met, then for the amplitudes of the reflected and arriving waves the following ratio can be obtained:

$$\Gamma = \frac{Z_l - Z_0}{Z_l + Z_0}, \tag{2.9}$$

where Γ is the so called reflection coefficient; because of the fact that Z_l is a complex quantity Γ is also complex, this means that the phase of the reflected wave is not a multiple of π. Notice that in the two cases discussed above, $Z_l = 0$ and $Z_l = \infty$, when the reflection is maximal, the phase shift of the reflected wave is π or 0 degrees, respectively.

The wave impedance of the most commonly used coaxial cable is 50 Ω. This value is coming from an agreement, and has become a widely used industrial standard since the '60s. In some applications we can also run into coaxial cables with 75 Ω impedance. About the historical background of this and 50 Ω values can be found in Appendix B.

Using DC and low frequency (few KHz AC) devices we got used to it, that an ideal source has 0 internal resistance, while an ideal voltmeter has infinity input resistance. In high frequency networks all the measurement aperture has 50 Ω input and output resistance, because we connect cables with this impedance to them. If we close a cable with a given impedance with an appropriate resistance, then there will be no standing waves present (due to no reflection from the termination), and one can measure the same voltage in the full length of the cable.

2.3 The terminating impedance

Figure 2.4. shows an interesting realization of the cable terminating impedance. The speciality of the circuit is that it act as a real 50 Ω termination, while it does not contain significant resistive element except the small ohmic part of the coil. Figure 2.4b shows the real and imaginary part of the impedance in a circuit, where C_T and L are connected parallel. The used values for the figure are $C_T = 217$ pF, $L = 1$ μH, and we have taken into account that the coil also has a small, approximately 1 Ω real resistance as well[4]. For this part of the circuit at 10 MHz the real part of the impedance is 50 Ω, while the imaginary part is a high positive number (440 Ω), which can be compensated with a series connected $C_M = 36$ pF capacitor. With this method one can achieve, that the circuit will act as a real 50 Ω impedance termination. This circuit is used in nuclear magnetic resonance spectroscopy to broadcast and receive radio frequency impulses.

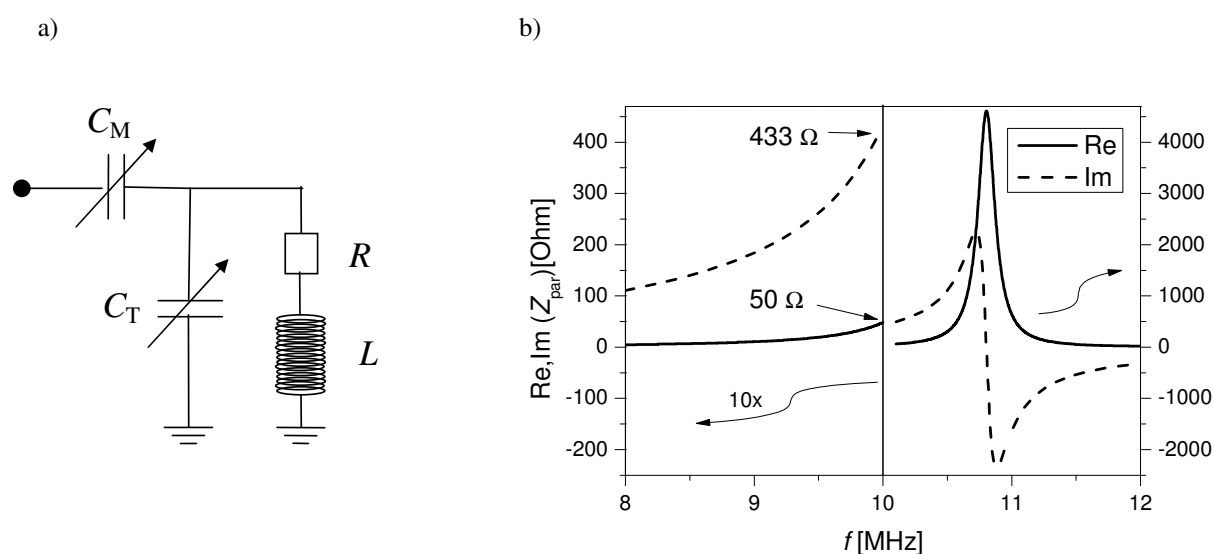a)   b)

Figure 2.4: a) The circuit containing no resistive part, which can be tuned to the cable's wave impedance. In our case we will change only the value of C_M. b) The real and imaginary part of impedance, Z_{par} for the parallel connected C_T and L part of the circuit against the frequency for fixed values (the region below 10 MHz is magnified by 10 times). Notice that, at 10 MHz the imaginary part is a large positive value, while the real part is 50 Ω.

In Fig. 2.5. we present the $|\Gamma|$ reflection of the 2.4a circuit against the frequency for three values of C_M matching capacitor. With the optimally adjusted C_M the reflection becomes zero.

2.4 Examination of reflections

The presence of reflections can be examined in two ways, by the examination of the output voltage of the source and by the help of a device called *duplexer*. In the first case we put a BNC T distributor to the oscillator source output. Then the divided signal is sent via a long (e.g. $10 - 25$ m) cable and a significantly shorter (e.g. 0.5 m) BNC cable to the oscilloscope.

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[4]The copper ($\rho = 1.7 \times 10^{-8}$ Ωm) coil used during the practice is 0.5 m long, has a diameter of 1 mm and its DC resistance is 11 mΩ. At 10 MHz the penetration depth is 20 μm, therefore the resistance goes up to 130 mΩ.
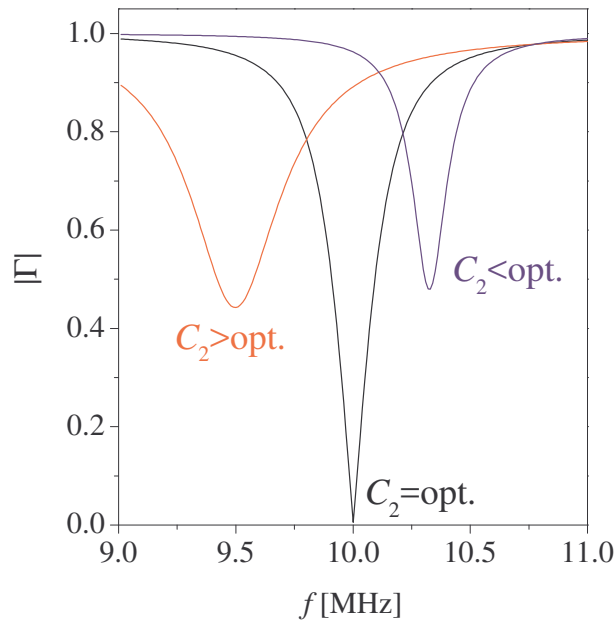
Figure 2.5: The reflection coefficient of the presented circuit, $|\Gamma|$ against the frequency with the given parameters in the text for three values of C_M: optimal (36 pF), below optimal (20 pF), and above optimal (65 pF). The latter two are also called under and over coupled cases. Please notice, that at the optimal value of C_M the reflection becomes 0 at a certain frequency, while in the other cases it is finite for all frequencies.

We also connect a 50 Ω termination to the same input with a T distributor. The magnitude of the signal as a function of the frequency measured on the oscilloscope, depends on the value of the termination. From this one can evaluate the speed of light in the cable, or knowing this, the length of the cable.
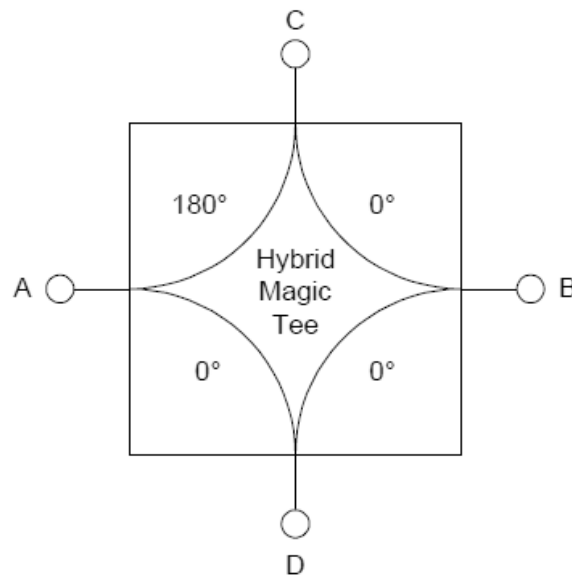

Figure 2.6: Schematic figure of the duplexer or hybrid magic tee *used in our experiment (type: Anzac HH107, $2-200$ MHz). In reality the connectors placed differently then on this schematic figure; pay attention to the marks.*

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The duplexer is a general notion, essentially it gives the possibility to determine the reflected voltage from the cable for a sent signal. In our case the radio frequency duplexer is a so called *hybrid magic tee*, which is shown in Fig. 2.6 and its operation is described in Appendix C. The essence of the duplexer is that, it divides the voltage from input port A, between ports C and D, while the the phase is shifted on port C.[5] Reverting the propagation of the signal: at the port A the difference of C and D appears: $D - C$. In our case to examine reflections, when using the *hybrid magic tee* the ports are the following: A: the source, D: connected to the oscilloscope CH1 with a 50 Ω termination, C: cable investigated for reflections, B: the other input of the oscilloscope with 50 Ω termination. If we measure 0 voltage on port B, then there is no reflextion from the investigated cable.

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[5]The *hybrid magic tee* features a small (10%) crosstalk between the ports.

Chapter 3

Measurement exercises

3.1 Reflection from the termination of the cable

3.1.1 With oscilloscope

Here, we measure with oscilloscope the effect of reflection from the termination of the cable to the output of the source.

1. Record the output voltage of the high frequency signal generator with oscilloscope for the three cases of terminating impedances (0, 50 Ω and break) as a function of the frequency (0.001 MHz - 15 MHz) with the measuring program (*freqswe* command). (Help: BNC T is on the output of the source; one termination on the oscilloscope with the shortest cable available terminated with 50 Ω, on the other termination there is a cable with a known length. Manual of the measuring device is in Appendix D. Use the SYNC output of the source for oscilloscope triggering.)

2. Using the measured curves determine the propagation speed in the cable. From this determine the relative dielectric constant ε_r of the BNC cable ($\mu_r = 1$), and write down in a few words, how this method can be used to find the break in the cable.

3.1.2 With duplexer

Here we examine the reflection from the termination with a *hybrid magic tee* (Tee in the following). The examined range of frequency is $2 - 20$ MHz.

3. Wiring of the Tee: *A*: source, *D*: CH1 port of the oscilloscope (trigger source), with 50 Ω termination, *C*: to the other input of the oscilloscope, CH2 with 50 Ω termination. Now we have to see the signal with opposite phase on CH1 and CH2. Record the curves (*readcope* command). In the following case: *A*: source, *D*: oscilloscope CH1 with 50 Ω termination, *C*: 50 Ω termination, *B*: other input of the oscilloscope, CH2 with 50 Ω termination. In ideal case we would see 0 signal, but due to the imperfection of the duplexer we observe a small signal on CH2. Record it.

4. Record the magnitude of the reflected signal from the termination of the cable as a function of frequency ($2 - 15$ MHz) using the Tee, similarly to the previous exercise, but put

the previously examined cable to port C with shorted, unclosed and with 50 Ω termination. How does the signal measured on CH2 of the oscilloscope look like as a function of the frequency for these three cases and why? (Help: the duplexer shows the magnitude of the reflected signal, which does not interfere with the source's signal. Therefore here we does not expect the emergence of standing waves like in exercise 1.) At 10 MHz examine, how does the phase of the signal change on CH2 compared to CH1 when the cable is shorted or open. Record the signal of the oscilloscope for these two cases.

3.1.3 Examination of reflection from the cable termination using pulse signal

5. Set the function generator to a 1 MHz sine signal. After pressing SHIFT + BURST it ejects one period of the signal with a low frequency, 100 Hz repetition time. Use the configuration from exercise 4 to measure the reflection from the termination of the 25 m long cable. Record the signal of the oscilloscope. Let us consider the time shift of the pulse as the difference of the identical phase signals in time. What do we see in the case of short circuit, break and 50 Ω termination? From the reflection time of the impulse determine the speed of light in the cable and compare the result with the one from exercise 2. Try the same with squre wave instead of sine, and record the signal recieved this way.

3.2 Examination of terminating impedance

6. Connect the circuit seen in Fig. 2.4 with the Tee to the source using our previous knowledge, to examine the reflection from the circuit. Set the value of C_M trimming capacitor to minimize the reflection from the circuit at a given frequency. In this case, we must get a similar curve to the one shown in Fig. 2.5. Help: set the value of C_M near to 30 pF, with this we shall observe a minimum at about 9 MHz. Try to lock near to this frequency. By changing the frequency of the generator and the trimmer capacitor simultaneously achieve that the minimum of reflection has to be smaller than 10% of the minimum of the reflection at distant frequencies. Alternatively, but a bit slower method is that if we record the reflection continuously with the *freqswe* command, while we adjust the value of C_M. Then we can observe similar curves to the ones can be seen in Fig. 2.5.

7. Record the reflected signal from the circuit as a function of the frequency with a maximum range of $1 - 2$ MHz. Read the approximate value of C_M from the box. Evaluate at which value of C_M the impedance of the circuit will be $50 + i \times 0$ Ω ($f = 9.5$ MHz, $C_T = 220$ pF, $L = 1.15$ μH, $R = 0.5$ Ω.), and compare it with the two measured values.

Acknowledgments

Further reading

David M. Pozar: Microwave Engineering (4th Ed.)

Appendix A

Derivation of the reflection coefficient.

Previously we had seen, that the ratio of voltage and current for a propagating wave in the cable is the wave impedance Z_0 for all time and space. However, when the cable is closed with an arbitrary Z_l impedance, then on the termination the voltage-current ratio must be equal to this value. This is the reason why the reflected wave is formed, because this provides the required mathematical condition.

Let us now define the axis x, that at the termination $x = 0$. For this case the solutions of the Telegrapher's equations (2.4) are two propagating waves, in the opposite directions, both for the voltage and the current. The amplitudes for the different directions are not necessarily the same:

$$U(x) = U_0^+ \, \mathrm{e}^{\mathrm{i}(\omega t + kx)} + U_0^- \, \mathrm{e}^{\mathrm{i}(\omega t - kx)} \tag{A.1}$$

$$I(x) = \frac{U_0^+}{Z_0} \, \mathrm{e}^{\mathrm{i}(\omega t + kx)} - \frac{U_0^-}{Z_0} \, \mathrm{e}^{\mathrm{i}(\omega t - kx)}, \tag{A.2}$$

where U_0^+ and U_0^- are the waves propagation onward and backward to the termination, respectively. Notice the negative sign in the second term in the expression of the current, which comes from the Telegrapher's equations. This solution has to satisfy the condition $\frac{U(x=0)}{I(x=0)} = Z_l$, namely:

$$Z_l = \frac{U_0^+ + U_0^-}{U_0^+ - U_0^-} Z_0, \tag{A.3}$$

from which for the reflected amplitude we obtain:

$$U_0^- = \frac{Z_l - Z_0}{Z_l + Z_0} U_0^+. \tag{A.4}$$

From this, the previously introduced Γ coefficient can be directly expressed, which is the result of the formula (2.9).

Appendix B

Interesting facts from the topic

B.1   The role of impedance fitting for cables

Reflections can be used to examine cable breaks at the termination. In spy stories this method is used to check whether someone monitored (or bugged) the cable and the location of the penetration can be determined as well. The method is based on that it does not matter how small is the perturbation caused by the penetration, it will change the wave impedance of the cable at a certain point. This leads the change of reflection from the cable. The location of the modification can be determined from the frequency dependent reflection coefficient.

In physics nothing is accidental: if the combination of two physical constants results a third constant with a given dimension, it usually has some physical meaning. This is the case for the quantum of flux ($h/2e$), or for the Bohr magneton ($e\hbar/2m_e$), which are the quanta of the corresponding physical quantities. The $c = 1/\sqrt{\varepsilon_0\mu_0}$ relation is well-known, that describes the connection between the speed of light and electromagnetic quantities. Similarly, the $\sqrt{\mu_0/\varepsilon_0}$ also has physical content. This has the unit of Ω, its value is 377 Ω, and it is called the wave impedance of the vacuum.

The physical meaning of the vacuum's wave impedance is the following. From the magnitude of the electric and magnetic fields for the electromagnetic radiation propagating in vacuum: $Z_0 = |\mathbf{E}|/|\mathbf{H}| = \sqrt{\mu_0/\varepsilon_0}$, which means that the antennas receiving and emitting to vacuum ideally must have this impedance. On the other hand, this expression compared to coaxial cables means, that in the latter the magnitude of the electric field is smaller compared to the magnetic field, than in the freely propagating electromagnetic radiation. The main reason of this is that in coaxial cables $\varepsilon_r > 1$ (typically around 2.5).

We have seen in the introduction, that using coaxial cables for signal transmission have many benefits. However, in some fields, e.g. in audiotechnics, telecommunication we can encounter with different preferred impedances, e.g.: 8, 150 or 600 Ω. All of these have historical background, e.g. the reason of the 600 Ω is that the conventional, not coaxial telegraph wires (two copper wires with the distance of 1 feet from each other) had the impedance of this value. The first telephones used these cables, therefore, there was a time, when the 600 Ω was a telecommunication standard. Later, the twisted copper wire pair became widely used, which has a wave impedance of 150 Ω. Nowadays, the so called UTP[1] is widely used for personal computer network applications. It has a wave impedance of 100 Ω.

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[1]Unshielded Twisted Pair

With the spread of the coaxial cables the question arose, which wave impedance should be used as a standard. Interestingly, we cannot get a clear answer to this. From multiple sources[2] we find the following story.

The wave impedance appears in three, independent, technically relevant parameters: in the maximal transmission of power (optimal if $Z_0 = 30\,\Omega$), in the maximum of high voltage electric breakdown (optimal if $Z_0 = 60\,\Omega$), and in the minimum of the loss of the transmitted signal (optimal if $Z_0 = 77\,\Omega$). In the early radio frequency era the power of the sources were low, therefore the 77 Ω cables spread. Since no good and elastic dielectrics existed at that time, the cables were made up of two coaxial tubes superposed, filled with air.

50 Ω cable can be produced by putting a 3/4 inch copper wire into a 2 inch copper wire (both are widely used sizes in the USA), thus this solution has become used in high power radio frequency broadcasting since the 1930s. If we fill a 77 Ω wire with polyethylene ($\varepsilon_r = 2.3$) then its wave impedance become 51 Ω, which is close to 50 Ω. Another important viewpoint is that the two values mentioned above, namely 30 and 77 Ω have an arithmetic mean of 53.5 Ω and a geometrical mean of 48 Ω, so the 50 Ω is a good compromise between loss and power transmission: with the same cable we are able to broadcast and receive high energy signals. Another interesting point of view is that, if we take a 'nice' ratio between the diameter of the inner conductor and the outer shielding, this will result a cable with the wave impedance of approximately 50 Ω.

Despite the incidence of 50 Ω, one can still find 75 Ω cables for weak video signal transmission, which has optimal loss, and 93 Ω cable to e.g. connect computers with displays. In case of the 93 Ω wave impedance cable the capacity for a unity length is the smallest, thus this has the highest cutoff frequency in the high frequency range. Therefore the square signal is transmitted better, without distortion.

B.2 Other examples to wave impedance fitting from the field of mechanics and optics

We can find impedance fitting not just in the case of electromagnetic wave propagation in cables, but in the case of light and sound waves as well. Sound waves reflect, instead of passing by, when they arrive to the boundary of two materials with very different refractive indices. In medical ultrasound examinations the used gel fits the impedance between the human body and the instrument. The funnel, which can be found on certain musical instruments and the geometrical shaping of speakers also fit the impedance between the source and the neighboring air.

In one of our most investigated mechanical example, when two bodies elastically collide, the energy transfer is maximal, when the masses of the bodies are equivalent. This can be interpreted as 'mechanical impedance fitting'.

In optical problems we find, that light reflects from boundaries of materials, which do not have the same refractive indices. The inverse of the refractive index, n, can be identified as optical or wave impedance: $Z = n^{-1}$. For a wave propagating from a medium with n_1 refractive

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[2]http://www.highfrequencyelectronics.com/Archives/Jun07/HFE0607_Editorial.pdf

http://www.rfcafe.com/references/electrical/history-of-50-ohms.htm

http://www.microwaves101.com/encyclopedia/why50ohms.cfm

http://en.wikipedia.org/wiki/Coaxial_cable

index to another one with n_2 we get $r = \frac{Z_2 - Z_1}{Z_1 + Z_2}$ to the amplitude of the reflected wave. This connection is known as the Fresnel equation in optics. In everyday life of this effect is the reflection from the lens of glasses (glimmering), which can be canceled with an anti-reflection layer. In this case the anti-reflection layer provides the impedance fitting.
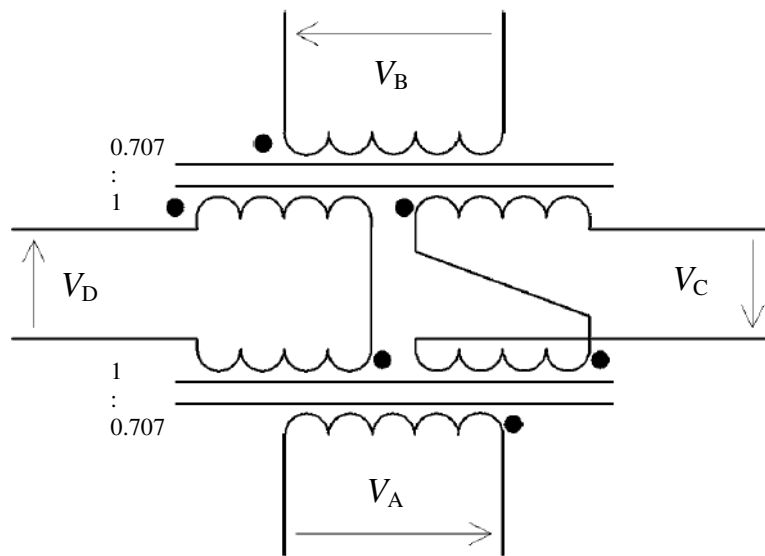
Appendix C

The radio frequency duplexer

Figure C.1: Realization of the radio frequency duplexer used in our case. Black points denote the points in phase of the transformer coils. Notice, that the ratio of number of turns is not 1 : 1, but close to $1 : 1/\sqrt{2}$ ratio.

In Fig. C.1. we show the schematic realization of the radio frequency duplexer. The essence is that we distribute or join the voltages among the ports with ferrite core transformers. Based on the properties of the transformers, the following relations are valid for the voltages measured on the ports:

$$V_A = (V_D - V_C)/\sqrt{2} \tag{C.1}$$
$$V_B = (V_C + V_D)/\sqrt{2} \tag{C.2}$$
$$V_C = (V_B - V_A)/\sqrt{2} \tag{C.3}$$
$$V_D = (V_A + V_B)/\sqrt{2}. \tag{C.4}$$

It can be seen, that the relation between these voltages have the same schematics that the one is shown in Fig. 2.6. So, for example the voltage on port C is the difference of the voltages on A and B. According to the conservation of the energy, it can be seen, that e.g. the signal enters at ports A and B, then it is distributed among ports C and D not with equal intensity, but

with the ratio according to the phase difference. If the phase difference is exactly $\pi/2$, then the intensities are distributed equally.

Appendix D

The use of the measuring instruments

Tektronix oscilloscope. The basic knowledge of using a digital oscilloscope is assumed to be known. After pressing the *trigger* button, the channel which we want to trigger to can be selected (the submenus can be reached using the buttons placed under the screen). In the *vertical menu* we can select the displayed channels using the buttons placed on the right hand side of the screen. Triggering to the reference signal coming from the source or when the duplexer is used, to the duplexer reference (usually port *D*) is expedient. The error of the voltage measurement is minimal, when the signal fits the screen, or a little bit (max. 20%) greater. The frequency setting is optimal when a few periods $(2-5)$ of the sine wave is visible.

The HP33120A signal generator. After pressing the appropriate function button (*frequency*, *ampltude*, etc.) the required value can be set using the knob. The desired decimal place can be set using the cursor buttons placed on the right hand side. Alternatively, the required value can be entered numerically after pressing the *enter number* button (the appropriate unit has to be selected with the buttons placed on the right as well). After using the measurement program, the device can be set back to local mode with pressing the *local* button.

The measurement program. After starting the measurement program using RF_Labor.exe placed on the desktop, under the menu bar a command line can be found, where the **freqswe** or **readscope** commands can be typed in. With the **freqswe** command the frequency sweep can be measured. Typical values:

<div align="center">

start freq: 1 [MHz]
end freq: 15 [MHz]
points: 100
dwell time/point: 0.1 [sec]
channel 1/2: 1.

</div>

Meanings: desired starting and final frequency (max. 15 MHz); points: number of measurement points, equidistantly between the starting and the final frequency; channel: the channel of the oscilloscope to be read. If we measure with e.g. 0.1 sec/point than the measurement time for 100 points is 25 seconds. Using more points and longer measurement time is only necessary, when we want to obtain a very pretty curve. Before starting the measurement we have to ensure the oscilloscope is well triggered, and the magnitude of the signals to be measured is optimal in the given range (the best is when the signal fits the screen of the oscilloscope).

The **readscope** command has no arguments, it reads and stores the channels, CH1 and CH2 of the oscilloscope. In *x*-axis the time read from the oscilloscope is stored.

Using any of the commands, the data has to be saved manually. For the files create a new folder on the *Desktop* under *\RF_Labor\201x_201x+1 \Surname /avoid special characters and spaces/*. The measured data can be saved under the *File* menu. It is useful to give long, informative names to the files. It is recommended to fill the *Sample ID* information under the *Header* menu of the *File* menu. The values can be read using a double click on the screen with the mouse.

The structure of the saved data: 5 lines of *header* '...', which contain the name of the command, the parameters, the date and the comments. After this there is a '2, 1000' from which the 2 denotes the number of data columns and 1000 the number of measured points. The *freqswe* command saves 2, while the *readscope* command saves 3 columns of data.

Appendix E

The Smith chart

The so-called Smith chart is an often used graphical tool to visualize and help problem solving in frequency dependent properties of circuits. It can be used to simultaneously display several parameters including impedances, admittances, reflection coefficients, S parameters, [1] and several other factors.

In Fig. E.1., we show the real and imaginary parts of the Γ reflection coefficient (introduced in Eq. (2.9)) for the resonance profile curves of the NMR circuit (which is shown in Fig. 2.4). The Smith chart shows the resonance profiles as circles of which only the optimally matched circuit crosses the origin. The Smith chart is often used to visualize the properties of a circuit when it is studied with a vector network analyzed (VNA).

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[1] The S_{ij}, or scattering, parameters describe the effect on the j-th port of a circuit if it is excited on the i-th port. E.g. for a two port circuit S_{11} is the reflection from a circuit, whereas S_{12} is the transmission.
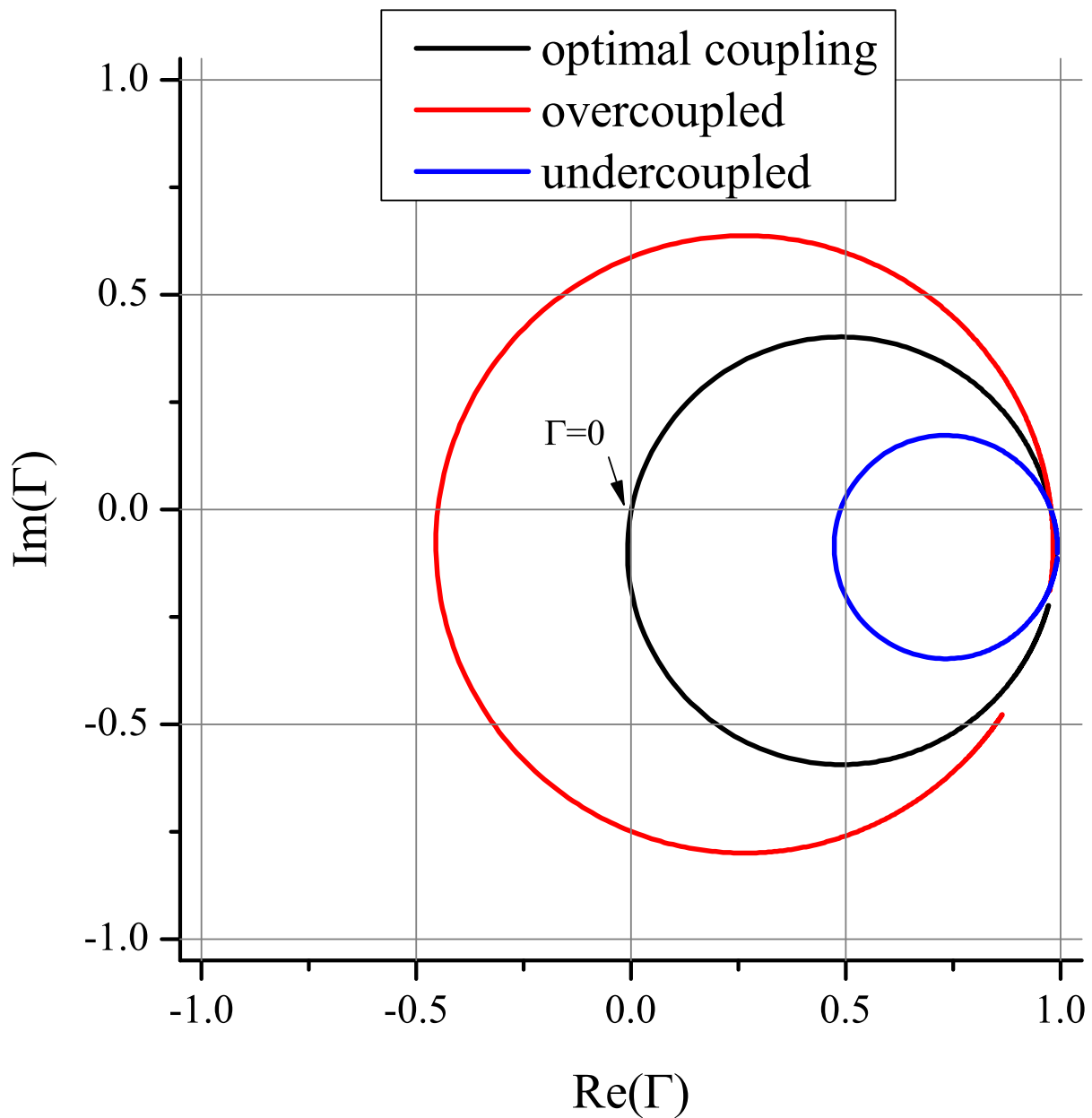
*Figure E.1: Smith chart for the reflection coefficients for the three cases discussed in Fig. **??** with the same color codes. Note that the reflection coefficient forms circles for all cases but only the optimallz matched case crosses the zero. Note that the center of the circles are shifted downwards.*