Measurements in Optics

I. FOCAL LENGTH OF LENSES

The behavior of simplest optical devices can be described by the method of geometrical optics. For convex or converging and concave or diverginig lenses the ray diagrams can be seen in the following figure.

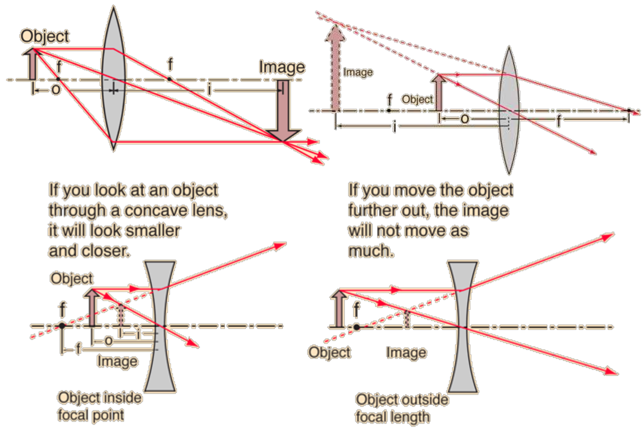

FIG. 1. Ray diagrams for convex (upper line) and concave (lower line) lenses[a]

[a] HyperPhysics

If the position of an object is at a distance of o from the lens of focal length f, and that of the image is i, then the lens equation is

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}.$$

In order to determine the focal length of a lens, Bessel-method is used. In this method the distance s between the object and the image (screen) is fixed. By changinig the position of the lens between the object and the screen, there are two positions with which perfect image can be obtained. The distance of this two lens positions is d. From the lens equation it can be shown that the focal length of the lens is

$$f = \frac{(s-d)(s+d)}{4s}. \quad (1)$$

II. REFRACTIVE INDEX OF A PRISM

If light propagates in a medium, its speed is lower than that in vacuum. The ratio of speeds gives the refractive index:

$$n = \frac{c_{\text{vacuum}}}{c_{\text{medium}}},$$

which is larger than 1. If a light is passing through a boundary of two different media, there is a relationship between the angle of incidence (ϑ_1) and refraction (ϑ_2), called Snell's law (FIG. 2):

$$\frac{\sin\vartheta_1}{\sin\vartheta_2} = \frac{c_1}{c_2} = \frac{n_2}{n_1}, \quad (2)$$

where the speed of propagation and the refractive indices of the two media are c_1, n_1 and c_2, n_2, respecitvely.

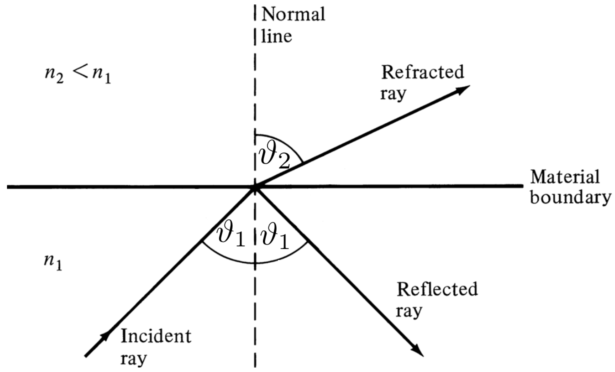

FIG. 2. Snell's law[a]

[a] https://physi.wordpress.com/2010/01/13/snells-law/

In our case $n_2 = 1$, which is the refractive index of air, n_1 is for glass. From eq. (2) it can be seen that for a given n_1 the angle of refraction can be $\vartheta_2 = 90°$:

$$\sin\vartheta_c = \frac{1}{n_1}.$$

This ϑ_c angle is the critical angle of incidence for total internal reflection. If a light falls on the boundary with larger angle of incidence than this critical one, it cannot leave the medium, the light is reflected.

The total internal reflection can be used to measure the refractive index of a prism. If a light enters a prism of apex angle φ perpendicularly, depending on this angle the light is reflected by the opposite side of the prism (FIG. 3). If the prism is rotated about the axis perpendicular to the plane where the light propagates, a position can be obtained when the light just leaves the prism, that is it propagates parallel to the opposite side of the prism.

The refractive index generally depends on the wavelength of the light propagating through the prism (this is called dispersion), the incident white light decomposes on different colored beams leaving the prism like a rainbow. The colored beams leave the prism at different angle of rotation. This way it is possible to measure the refractive index for different wavelengths in the visible light range.

From the Snell's law it can be shown that the refractive

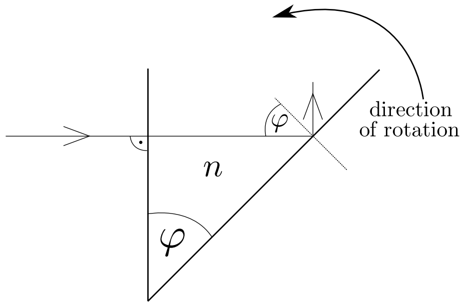

FIG. 3. Perpendicular incidence of light on a prism of apex angle φ

index of the prism for this total internal reflection case is

$$n = \sqrt{\frac{1 + \sin^2\alpha + 2\sin\alpha\cos\varphi}{\sin^2\varphi}}, \qquad (3)$$

where α is the rotation angle measured from the perpendicular incidence position.

III. POLARIZATION OF LIGHT

Light is transversal wave, which means that the electric field (and magnetic field) vector is perpendicular to the direction of propagation. This type of wave was studied in Standing Waves measurement in which the displacement of the string was perpendicular to the direction of propagation. The other type of waves is longitudinal wave for which the displacement of the particles is parallel to the direction of propagation (for example sound waves). For transversal waves there are a lot of planes perpendicular to the direction of propagation; this is the case for the sunlight, the light of a bulb, LED. These light sources provide *non-polarized* light. For longitudinal waves we cannot talk about polarization.

However, it is possible to reach that the electric field vector oscillates in a given plane. In this case the light is *polarized*. The light of a laser is polarized. Non-polarized light can be turned into polarized one with polarizing filters.

If we put a vertical polarizing filter (called polarizer) next to a light source providing non-polarized light, the light becomes polarized vertically. Then, if we put another but parallel polarizing filter (called analyzer), nothing will change, the detector reads maximum intensity. Now, when we start to rotate the analyzer, the intnsity read by the detector decreases, and when the direction of the analyzer is perpendicular to that of the polarizer, no light can get through the system to the detector. If we keep rotating the analyzer, the intensity increases, and again it reaches its maximum when the analyzer is parallel to the polarizer.

The quantity which measures the portion of polarized wave in a light source is the degree of polarization. By rotating the polarizer between the light source and the detector the maximum and the minimum intensity are $I_{\max}$ and $I_{\min}$, respectively. Then the degree of polarization is:

$$\beta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}. \qquad (4)$$

IV. INTERFERENCE OF LIGHT BEAMS

Since a light beam can be modelled as oscillating electric and magnetic field, when two or more beams meet the electric and magnetic fields are added according to the superposition principle. That is, in every point of the space the electric and magnetic field is the vectorial sum of fields of each beams (called interference).

If the beams originate from different sources, generally there is no connection between the electromagnetic oscillations. At every moment there are points in space where adding the fields results in maximum and points where results in cancellation. Since there is no definite connection, where a maximum was at a given moment, in the next moment there can be a minimum. The frequency of the visible light is too high for the human eye, it will average out the fast oscillation of the intensity.

If the beams originate from the same source, there is a correlation between the the phase and the frequency of the oscillations, so points can exist, where the beams are in phase continously (called coherent beams). In this case the total fields are always maximal in these points, and bright spot is visible there. In the points where the beams are always in opposite phase, the total intensity is minimal, a dark spot is visible.

First Thomas Young made interference pattern with two closely placed slits. The resulting image on screen far from the source consisted of bright and dark lines. This experiment was an important proof of the wave nature of light. The slits can be used as simple interferometer. If the distance between the slits is known, the wavelength of the light can be determined by measuring the distance of the maxima and minima on the screen. If the wavelength is known, the distance of the slits can be determined.

A. Michelson interferometer

In 1881, 78 years after Young, A. A. Michelson built a similarily working interferometer. Michelsons original intention was to prove the existance of Aether the hypothetical medium which was hold for responsible for the propagation of electromagnetic radiation (such as light). Partially his work proves that this hypothesis was wrong. Furthermore Michelson interferometer is widely used to measure the wavelenght of light, or if it is known, to measure very small displacement.

FIG. 4 shows the sketch of Michelson interferometer. The beam of laser is split by a beam splitter, which reflects the 50% of the intensity and transmits the other

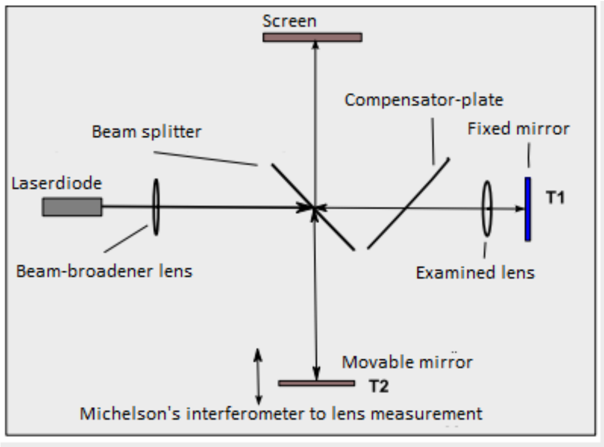

FIG. 4. Sketch of Michelson interferometer

50%. One of the split beams is reflected on a fixed (T_1), the other is relfected on a movable mirror (T_2). Both of the beams return to the beam splitter. Now the half of light reflected from the movable mirror goes to the screen, also the half of the other beam goes to the screen.

This way the original beam is split to two different paths and some parts of them merge again. Since the beams originate from the same source, their phase are strongly correlated. If a lens is used between the laser and the beam splitter, the beams are broadend and an interference pattern consisting of bright and dark rings is visible on the screen (FIG. 5).

FIG. 5. Fringe pattern of Michelson interferometer

Since the two interfering beams of light were split from the same initial beam, they were initially in phase. Their relative phase, when they meet at any point on the screen, depends on the difference in the length of their optical paths before reaching that point.

By moving mirror T_2, the path length of one of the beams can be varied. Since the beam traverses the path between T_2 and the beam-splitter twice, moving mirror T_2 by 1/4 wavelength nearer to the beam-splitter will reduce the optical path of that beam by 1/2 wavelength. The interference pattern will change; the radii of the maxima will be reduced so they now occupy the position of the former minima. If mirror T_2 is moved an additional 1/4 wavelength closer to the beam-splitter, the radii of the maxima will again be reduced so maxima and minima trade positions. However, this new arrangement will be indistinguishable from the original pattern.

By slowly moving mirror T_2 by a distance of d_N, and counting N which is the number of times the fringe pattern is restored to its original state, the wavelength λ of the light can be calculated as follows:

$$\lambda = \frac{2d_N}{N}. \tag{5}$$

If the wavelength of the light is known, the same procedure can be used to measure d_N.

One of the beams crosses the beam splitter only once, while the other three-times. If the light source is strongly coherent and monochromatic (sonsists of a single wavelength), such as a laser, it is not a problem. Although in case of other light sources it can result in error. As the length of optical path increases, the coherence of split beams is decreasing, which can distort the interference pattern. Using a compensator plate (see FIG. 4) both beams cross glass object three-times, which solves this problem. The compensator plate is similar to the beam-splitter but without reflecting coating.

With Michelson interferometer one can examine optical devices. E.g. placing a lens, as only one of the split beams crosses it, every small deviation of the lens will result in strong distortion of the interference pattern.

B. Piezoelectric effect

In several materials mechanical stress can cause change in the polarization. This is called piezoelectric effect. These materials are piezoelectric materials (for example quartz, certain ceramics). In practice the polarization change due to mechanical impact can be converted into electric signal. In general the electric signal is proportional to the mechanical strain or stress, which makes it easy to measure force or deformation.

The effect can be used for detection of sound since the pressure oscillation on a surface generated by the sound can be converted into electric signal. The operation of sensors in ultrasound devices are based on this effect. Moreover, this is the basic of the operation of piezo igniters, as well. The electrodes of the piezolelectric material placed in the igniter are not connected, the wire coming from the electrodes is broken (see FIG. 6). Due to the sudden mechanical strain polarization charges occur, which causes large voltage between the ends of the broken wire, and electric spark is created in the air.

The piezolelectric effect can be reversed (called inverse piezolelectric effect): If a piezoelectric material is placed into an electric field, mechanical strain occurs. The strain is increasing by increasing the electric field, which makes it possible to create small displacement with electric field.

Applying alternating electric field by the effect, the piezolelectric materials can be deformed cyclically with the frequency of the exciting field, which is the basic of the piezolelectric sound generators used in ultrasound

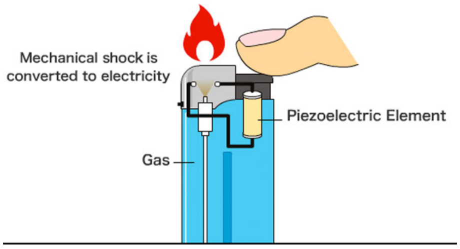

FIG. 6. Piezoelectric igniter[a]

[a] cheaphumidors.com

devices mainly. In quartz watches this phenomenon is used to make the piezolelectric chip vibrate which ensures the stable frequency of the watch (nowadays the material is mostly no longer quartz).

For generating reproducible small displacements piezolelectric materials are excellent tools. Displacements even at nanoscale can be realized; in scanning tunneling microscope (STM), which is one of today's modern devices in material science, the movement of the probe is carried out by this sensitive method.

V. MEASUREMENT TASKS

1. *Measuring the focal lengths of lenses with Bessel method*

   The instructor gives three lenses. Use the halogen lamp as light source, with diaphragm. Put the object in front of the diaphragm. Measure the distances with measuring tape. For a lens use three different s distances. Insert the measured data in a table and calculate the focal length for every s by using eq. (1). The focal length of a lens is the average of the three calulated values.

2. *Measuring the refractive index of a trapezoidal prism by total internal reflection*

   As a light source use the halogen lamp with diaphragm. Put the prism in the middle of the disk marked in degrees. Also put a slit right in front of the disk by using the slidable holder. Set up the perpendicular incidence according to FIG. 3 (the lines on the disk may help you). The apex angle of the given prism is $\varphi = 45°$. Read the rotation angle corresponding to the red and violet outgoing light and use eq. (3) for calculating the refractive indices for red and violet lights.

3. *Examining the polarization of tungsten light bulb and laser diode*

   Use the setup which can be seen in FIG. 7.

   The maximal intensity read by the lux meter (detector) should be at least $\sim 1000 - 1200$ lux. With the diaphragm the light beam can be made narrow.

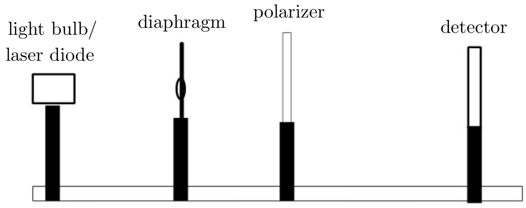

FIG. 7. Setup for measuring the degree of polarization

   Rotate the polarizer from its initial position by 10° and record the intensity value read by the lux meter. A complete revolution should be made, e.g. 360° rotation. Plot the intensity as a function of the rotation angle. Calculate the degree of polarization using eq. (4), and explain the results. The measurement must be done for both of the light sources.

4. *Measuring micrometer displacement with Michelson interferometer: calibration of piezolelectric actuator*

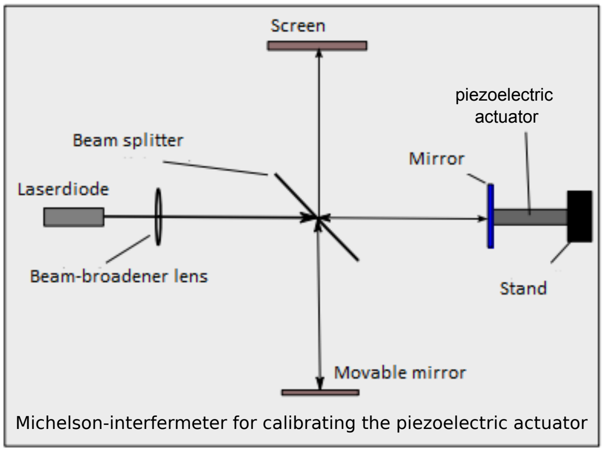

FIG. 8. Measurement setup

   Build up the interferometer seen in FIG.8, and find the pattern seen in FIG. 5. One of the mirrors of Michelson interferometer is fixed to one of the ends of a piezolelectric moving element. The other end of the element is fixed to the optical table containig the other parts of the interferometer. To the actuator a voltage supply is connected with which the voltage can be changed. Change the voltage slowly from 0 to 20 V and count N, the number of times the initial fringe pattern returns. From this result provide the displacement/voltage parameter (it measures the displacement for 1 V) of the piezoelectric actuator in unit of μm/V. The wavelength of the applied light is 650 nm.