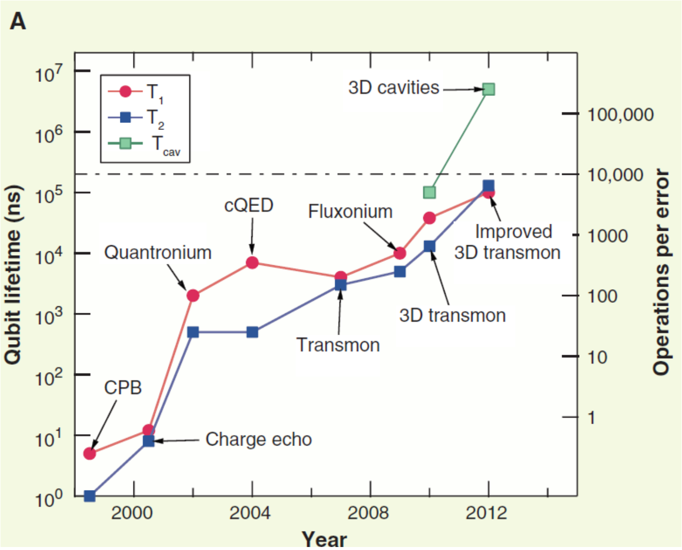
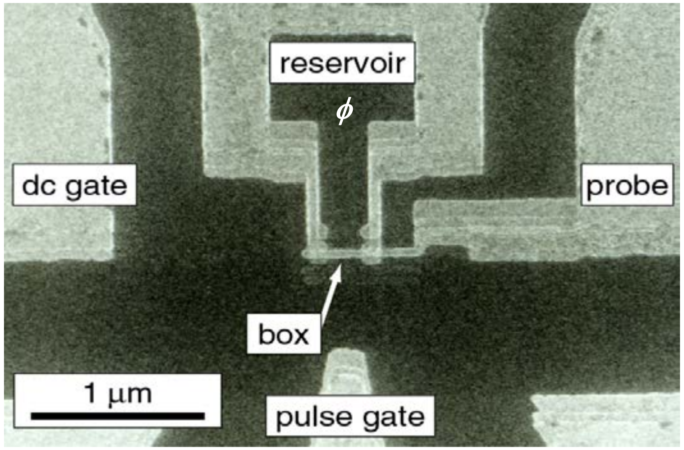
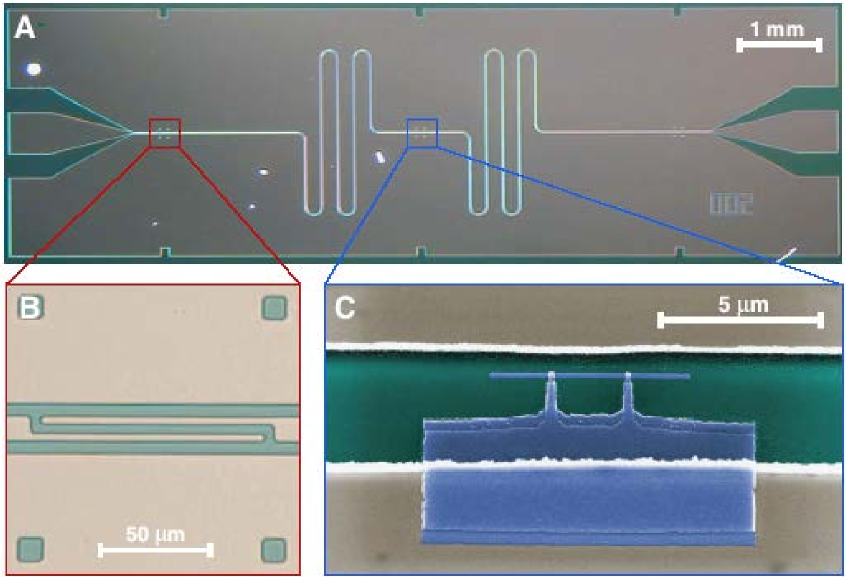
Quantum Computing Architectures

Budapest University of Technology and Economics 2018 Fall

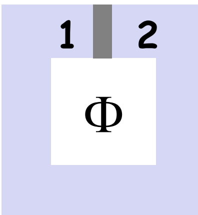




Lecture 7: Superconducting qubits
Basic architectures 2
Flux and charge qubits
cQED



RF SQUID



L loop inductance

Similarly to DC squid the phase difference equals the flux inside the loop

$$\frac{\Phi_0}{2\pi}(\phi_1 - \phi_2) = \int_1^2 \vec{A}\vec{dl}$$

$$\delta = \phi_1 - \phi_2 = \frac{2\pi\Phi}{\Phi_0}$$

The flux inside the loop will be partially screened by and induced circulating current

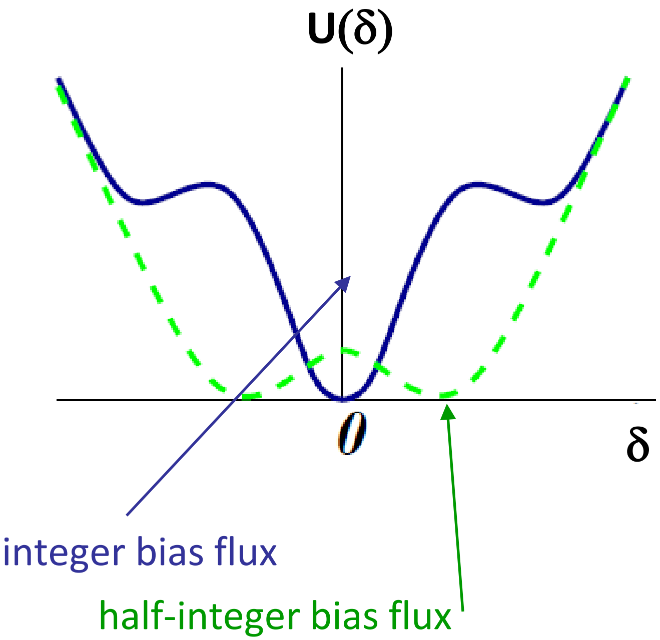
$$\Phi = \Phi_{ext} - LI_{circ}$$

Equation of motion, with the calculated current:

$$0 = C\frac{\Phi_0}{2\pi}\frac{d^2\delta}{dt^2} + \frac{\Phi_0}{2\pi}\frac{1}{R}\frac{d\delta}{dt} - I_c\sin(\delta) - \frac{1}{L}(\delta - \Phi_{ext})$$

$$U(\delta) = \frac{\Phi_0}{2\pi}I_c\left(1 - \cos(\delta)\right) + \frac{1}{2L}\left(\frac{\hbar}{2e}\delta - \Phi_{ext}\right)^2$$

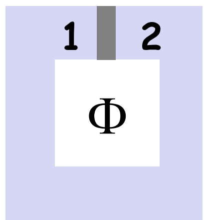
Potential – junction + magnetic energy



At half quanta the circulating current changes sign and a flux quanta jumps into the loop

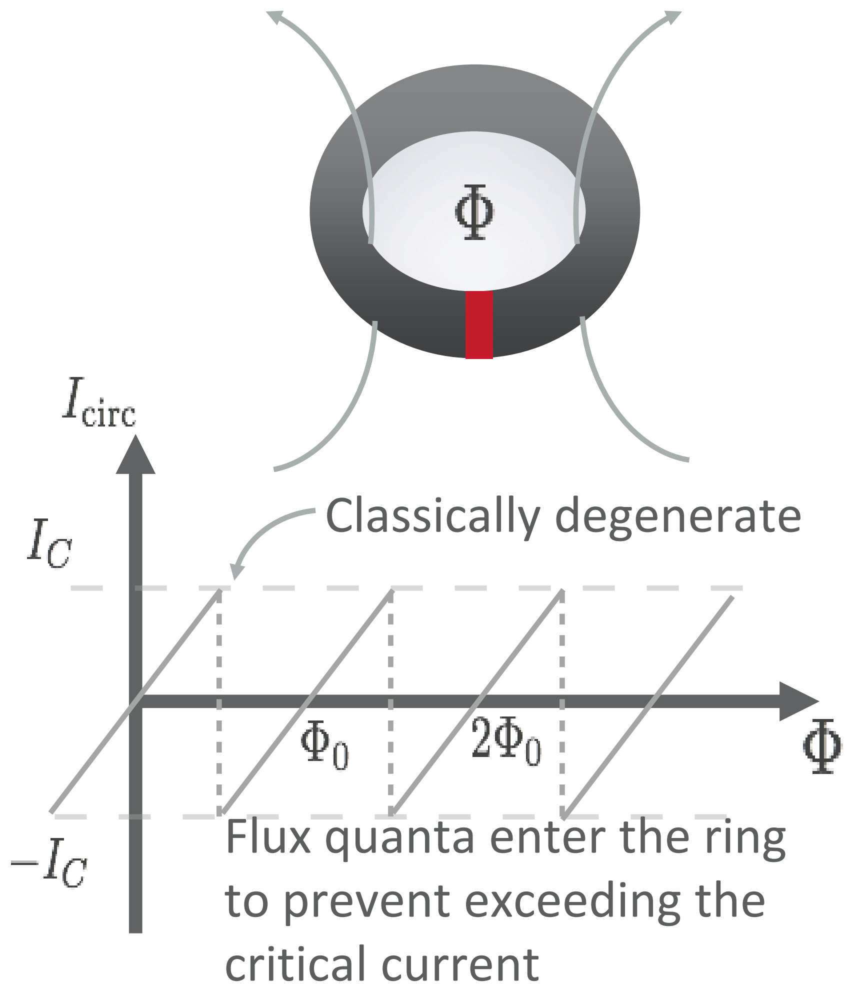
For half integer quantum, two minima:
two persistent current states, circulating in different direction

RF SQUID

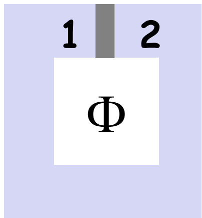


L loop inductance

Single-junction loop: rf-SQUID / Flux qubit

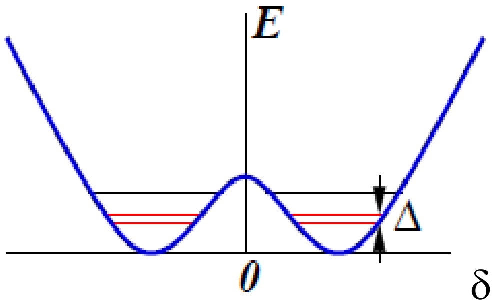


P. Forn-Diaz



Flux qubit

$$U(\delta) = \frac{\Phi_0}{2\pi} I_c \left(1 - \cos(\delta)\right) + \frac{1}{2L}\left(\frac{\hbar}{2e}\delta - \Phi_{ext}\right)^2$$

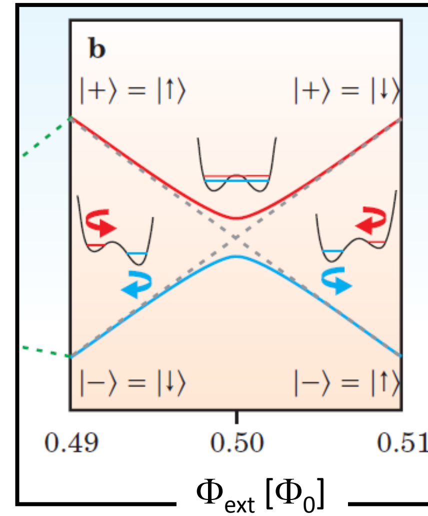


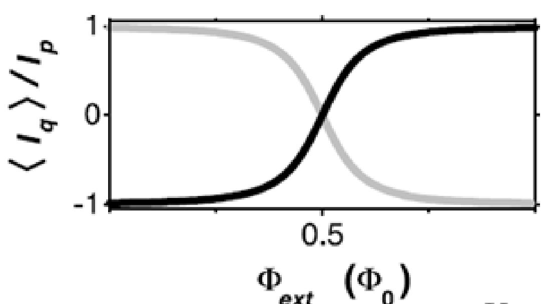
potential for half integer flux bias

Two wells → two levels – for symmetric potential degenerate flux states

The two states correspond to oppositely circulating persistent current

If tunneling is possible between the two wells (Δ), states hybridize and split up and the macroscopic tunneling determines the separation





The expectation value of the current as a function of the flux

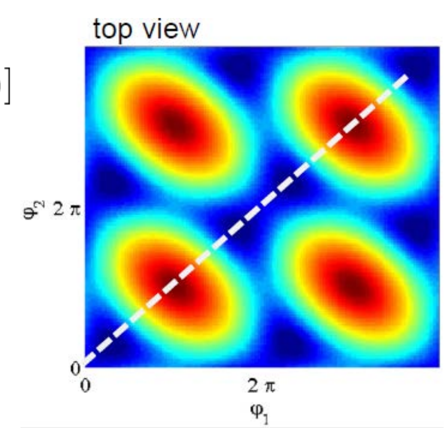
Away from half flux quanta, pure flux states

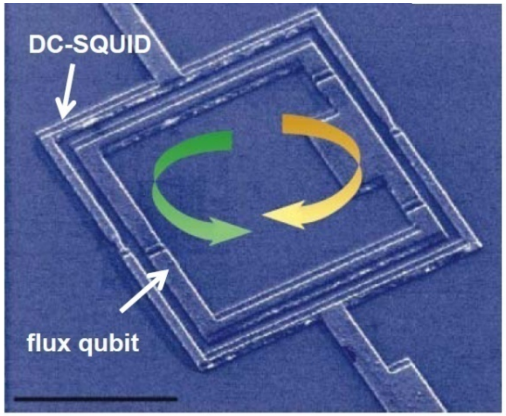
$$U = -E_j\left[\cos(\varphi_1) + \cos(\varphi_2) + \alpha\cos(\varphi_1 - \varphi_2 - 2\pi\Phi_{ext}/\Phi_0)\right]$$

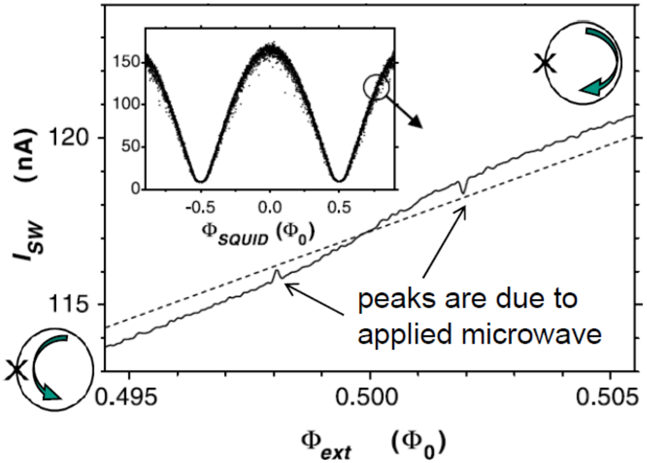
Hard to fabricate, big loop is needed for inductance matching (large noise pickup possible big decoherence)→ 3 JJ-s qubit (effectively the same).

$$\varphi_1 + \varphi_2 + \varphi_3 + 2\pi\Phi/\Phi_0 = 2\pi n$$

the potential is parabolic on the white intersection α tunes the macroscopic quantum tunneling.

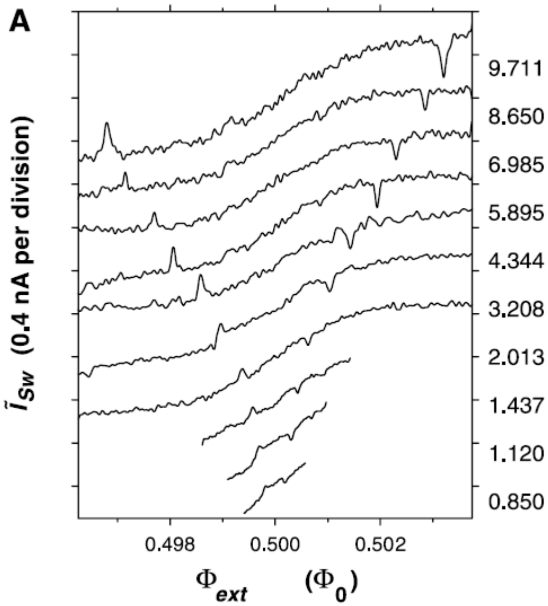


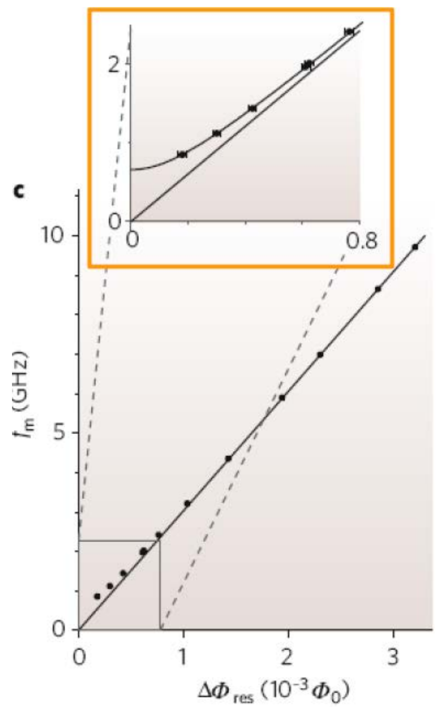


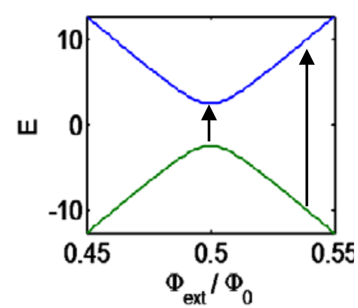


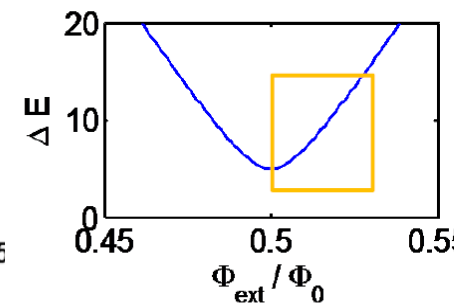
Readout – by DC squid measuring the opposite supercurrents in the qubit. Measurement with squid – measure the switching currents

During the sweeping of the magnetic field, microwave applied. transition causes supercurrent flowing opposite direction → change in field measured by squid (change in switching current)
- the resonance seen for different frequencies at different flux points.
- peaks indicate switching between flux states
-the excitation spectra is nicely reproduced
- at zero detuning the avoided crossing of the two levels is extrapolated

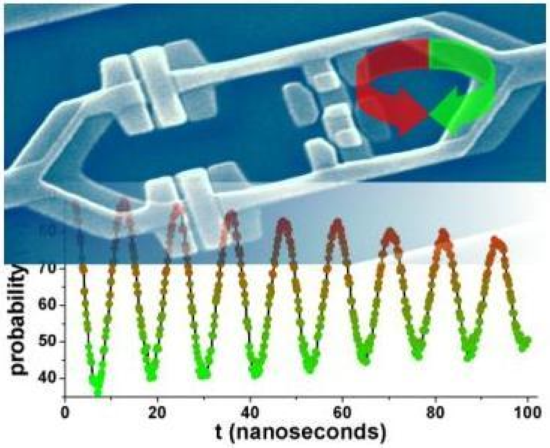


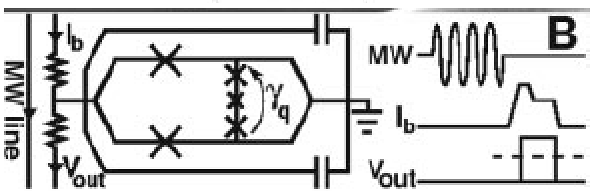


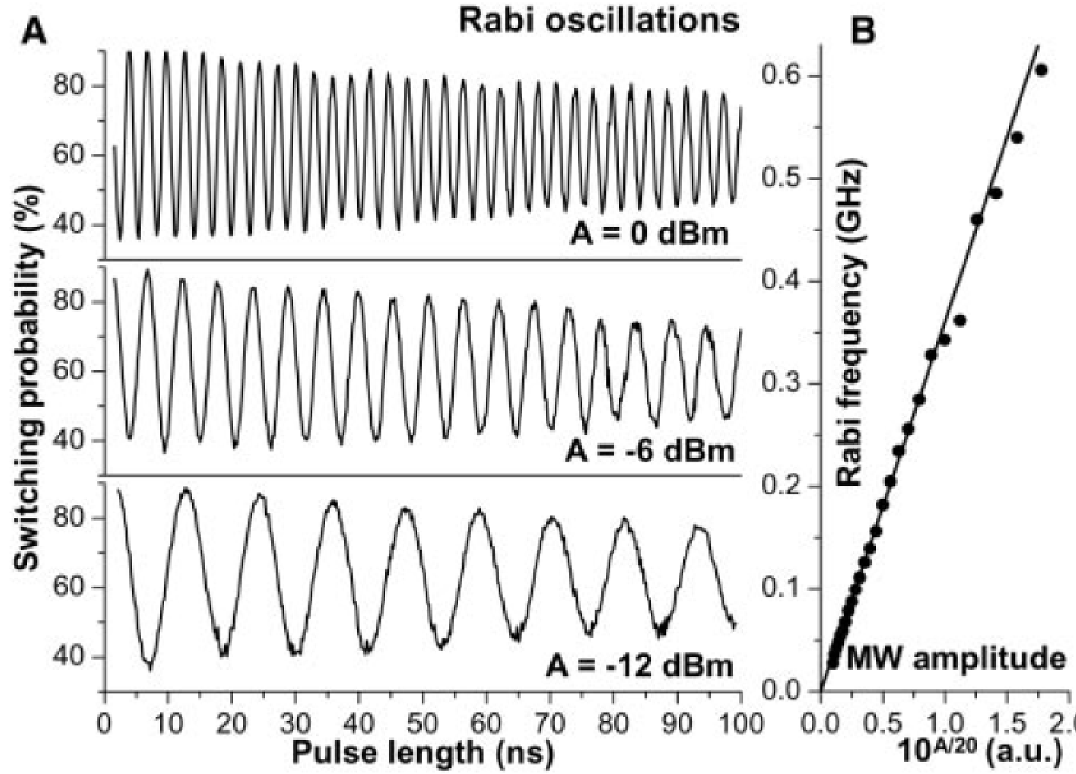




Caspar H. van der Wal et al., Science 290, 773 (2000)







Other design: squid is directly coupled to achieve higher sensitivity

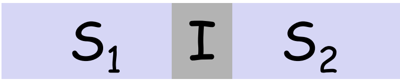
T_1~900ns, T_2~20-30 ns

Dephasing: likely flux noise → changes the qubit frequency randomly

Ideal opeation would be at $\Phi=\pi$, however this did not work for this devices.

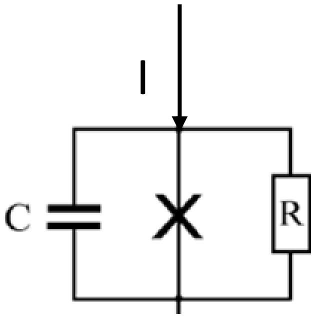
There $\delta E \sim \Phi^2$, less sensitive to flux noise →sweet spot

I. Chiorescu et al., Science 299, 1869 (2003)



$$\delta = \phi_2 - \phi_1$$

RCSJ model – energy terms



Neglect damping, SC state, R=0

E=K+U

$$K = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}C\left(\frac{\hbar}{2e}\right)^2\left(\frac{d\delta}{dt}\right)^2$$

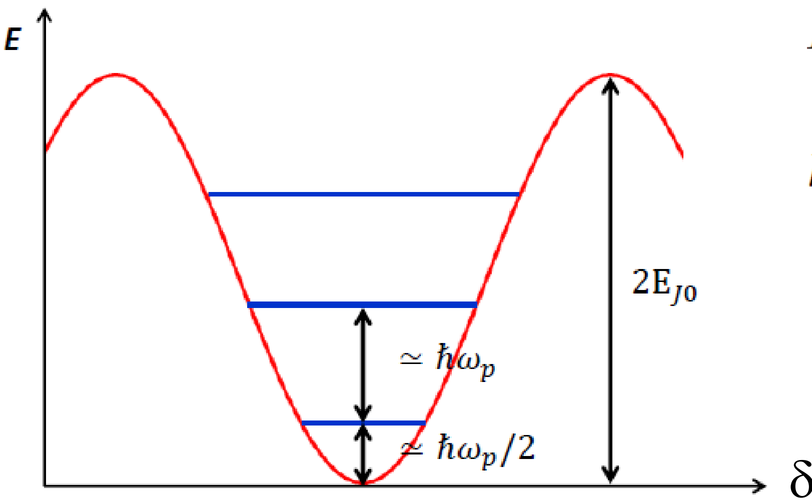
or using

$$M = \frac{\hbar C}{2e}$$

$$U = E_{J0}(1-\cos(\delta))$$

$$E_{J0} = I_C\frac{\hbar}{2e}$$

Josephson energy



$$E_C = \frac{e^2}{2C}$$

Charging energy

$$\hbar\omega_{pl} = \sqrt{8E_C E_{J0}}$$

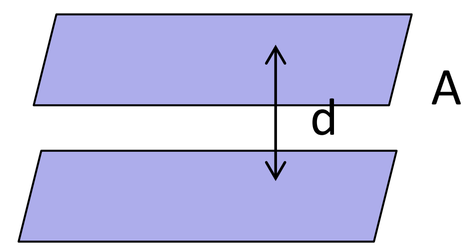
$$\hbar\omega_{pl} \ll E_{J0}$$

Classical treatment valid:
Oscillation only in the bottom of the potential well

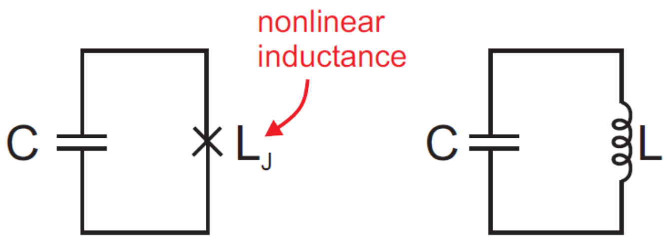
$$\downarrow$$

$$E_C \ll E_{J0}$$

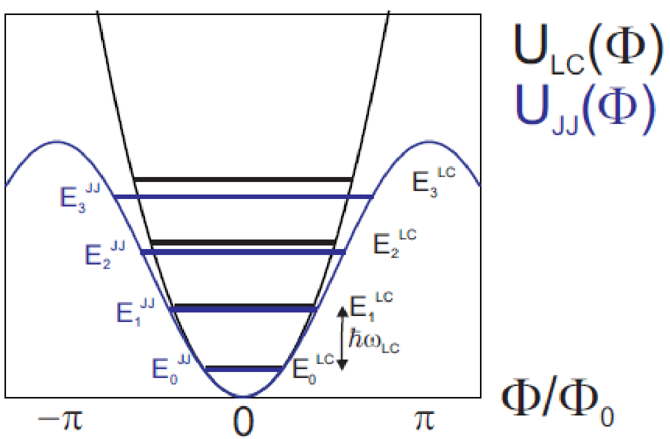
Homework: How to enter the quantum regime? Investigate scaling with the junction area. Suppose d=1nm, ε=10, I_c= 100 A/cm^2. What is the temperature range where the measurement should be done?



Energy terms
Why JJ, not a simple inductor?



Josephson junctions is a *non-linear inductance:* the energy spectra is anharmonic. The qubit can be separated from excited states



LC - oscillator

$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 \qquad V = L\frac{dI}{dt} \qquad \Phi = LI$$

$$I = C\frac{dV}{dt} \qquad Q = CV$$

Josephson junction

$$I = I_c \sin(\delta) = I_c \sin(2\pi\Phi/\Phi_0)$$

$$\frac{dI}{dt} = L_J^{-1}V \qquad L_J^{-1} = \frac{2\pi I_c}{\Phi_0}\cos(2\pi\Phi/\Phi_0)$$

for small Φ $\qquad L_J = \frac{\Phi_0}{2\pi I_c} \qquad I \simeq \frac{\Phi}{L_J}$

$$H = \frac{1}{2}CV^2 + \frac{1}{2}L_J I^2 = \frac{Q^2}{2C} + \frac{1}{2L_J}\Phi^2$$

Why else superconductors?

-Single non-degenerate macroscopic ground state
- no low energy excitations

Quantization of EM circuits

$$H = E + K = \frac{p^2}{2m} + \frac{1}{2}m\omega_{pl}^2 = \frac{Q^2}{2C} + \frac{1}{2L_J}\Phi^2$$

Energy of a harmonic oscillator

$$H = E + K = \frac{1}{2}C\left(\frac{\hbar}{2e}\right)^2\left(\frac{d\delta}{dt}\right)^2 + E_{J0}(1-\cos(\delta))$$

JJ: nonlinear Harmonic oscillator

$$p = mv = C\left(\frac{\hbar}{2e}\right)^2\frac{d\delta}{dt}$$

Knowing the mass, identify momentum

$$M = \left(\frac{\hbar}{2e}\right)^2 C$$

Quantization – using the momentum and position operators

$$\hat{p}_\delta = \frac{\hbar}{i}\frac{d}{d\delta} \qquad \hat{x} = \hat{\delta} \longrightarrow \left[\hat{\delta}, \hat{p}_\delta\right] = i\hbar$$

$$\boxed{\hat{H} = -4E_c\frac{d^2}{d\delta^2} + E_{J0}(1-\cos(\delta))}$$

Quantized JJ Hamiltonian

Phase representation (analogous to coordinate repr.)

<u>Charge, Cooper pair number, flux basis</u>

Homework:

$$Q = C\frac{\hbar}{2e}\frac{d\delta}{dt} \downarrow \hat{Q} = -2ei\frac{d}{d\delta}$$

charge

$$\hat{N} = -i\frac{d}{d\delta}$$

CP number

$$\hat{\Phi} = \frac{\hbar}{2e}\hat{\delta}$$

Flux

$$\longrightarrow \left[\hat{\delta}, \hat{N}\right] = i$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$$\boxed{\Delta N \Delta\delta \geq 1}$$

Either phase (flux) or number of Cooper pairs (charge) is well defined

→ Phase or charge regime

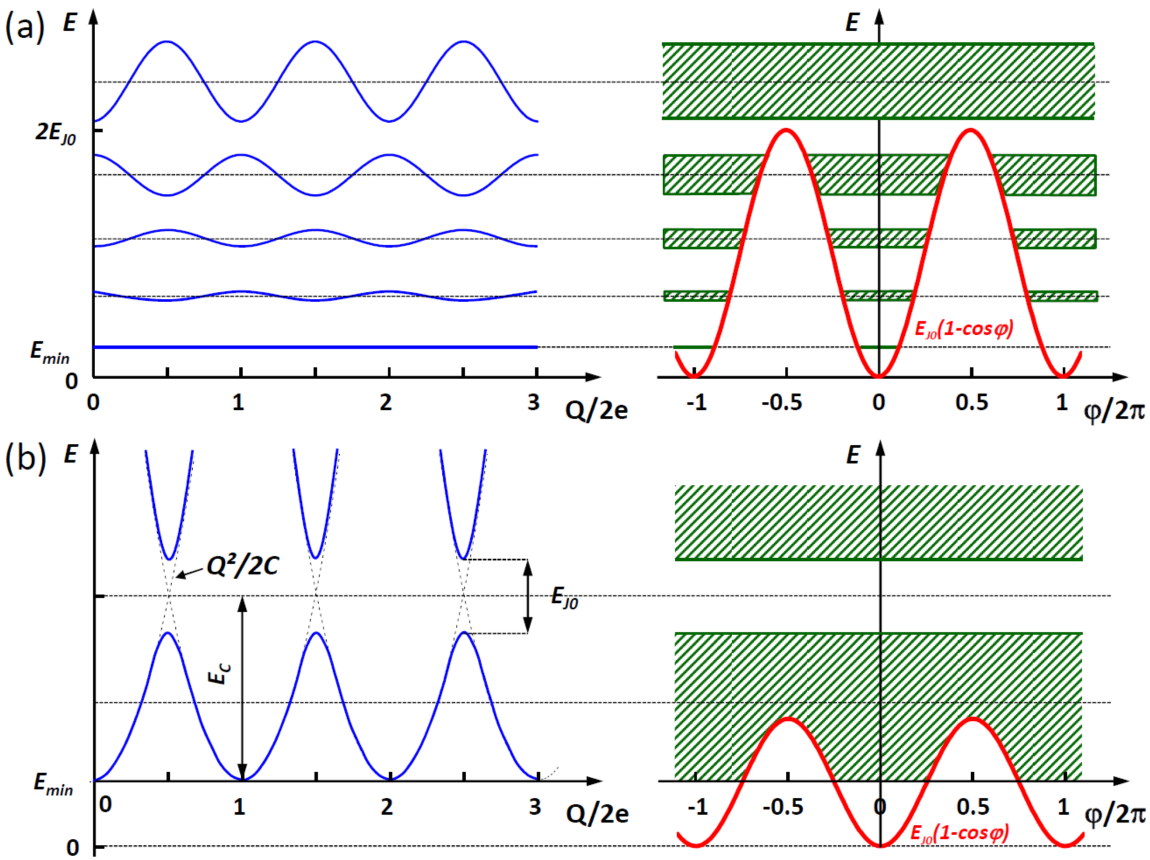
$$\Delta N \Delta \delta \geq 1$$

1) Phase regime $\hbar\omega_{pl} \ll E_{J0}$ and $E_C \ll E_{J0}$

phase is well localized in one of the minima, large charge fluctuations are possible (small E_c)

2) Charge regime $\hbar\omega_{pl} \gg E_{J0}$ and $E_C \gg E_{J0}$

e.g. a small island tunnel coupled, number of states well localized (Coulomb blockade), phase fluctuations are large



Analogous to the problem of electrons in a periodic potential

Strong phase potential → localized states (in phase)

Weak phase potential → extended states in phase space

R. Gross, A. Marx, Applied Superconductivity, Lecture notes (Walter-Meissner Institute)

Charge qubits

$$\hat{H} = \frac{\hat{Q}^2}{2C} + E_{J0}\left(1-\cos\left(\frac{2\pi\hat{\Phi}}{\Phi_0}\right)\right) = E_c\hat{N}^2 + E_{J0}\left(1-\cos\left(\hat{\delta}\right)\right)$$

Hamiltonian: nonlinear ocillator

$$\hat{H} = -4E_c\frac{d^2}{d\delta^2} + E_{J0}(1-\cos(\delta))$$

Phase representation (~x repr.)

$$|\delta\rangle = \sum_{N=-\infty}^{\infty} e^{iN\delta}|N\rangle \longleftrightarrow |N\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{-iN\delta}|\delta\rangle$$

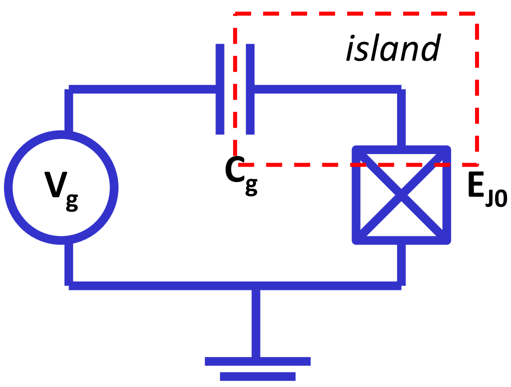
Number representation (~ k repr.). Transformation: Fourier tansform

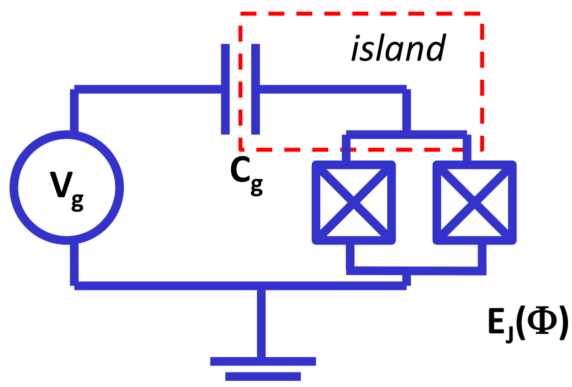
Homework to show: $e^{i\hat{\delta}}|N\rangle = |N-1\rangle$

$$\hat{H}_J \approx E_J\cos(\delta) = -\frac{E_J}{2}\sum_N |N\rangle\langle N+1| + |N+1\rangle\langle N|$$

Josephson term in number basis (neglecting constant offset)

$$\hat{H} = E_c\sum_N (N-N_g)^2|N\rangle\langle N| - \frac{E_J}{2}\sum_N |N\rangle\langle N+1| + |N+1\rangle\langle N|$$



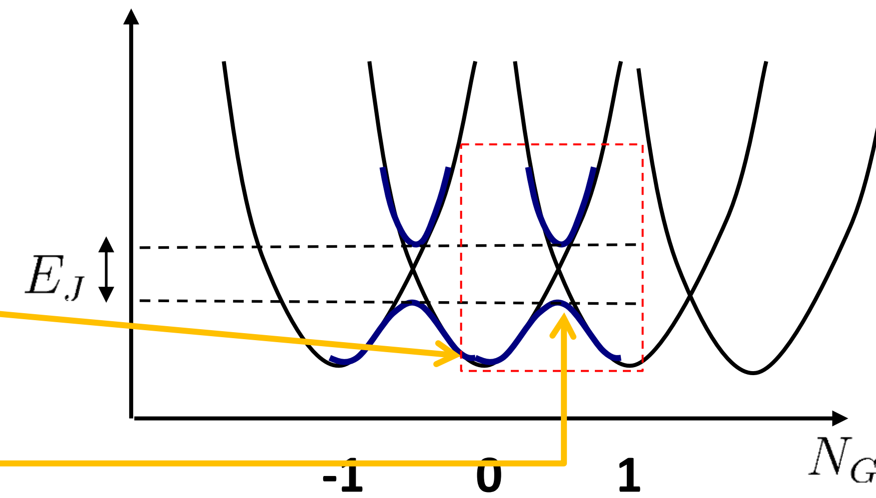


N_g: offset charge from gate electrode
Enchance Ec: make a small SC insland

Charge qubit/Cooper pair box – small SC island connected with a single lead to an large SC, and to a gate electrode. The island has large charging energy. Using a SQUID loop E_J is flux tunable

$$\hat{H} = E_c \sum_N (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \sum_N |N\rangle\langle N+1| + |N+1\rangle\langle N|$$

If E_c>>E_J: well defined charge states. The Josephson term connect neigbouring charge occupations (measured in 2e – Cooper pair tunneling!)



Good ground state: good for inicialization, charge states are far

Good qubit: degeneracy points: 2 levels close by next level far away

$$N_g = \frac{1}{2} + \Delta_g$$

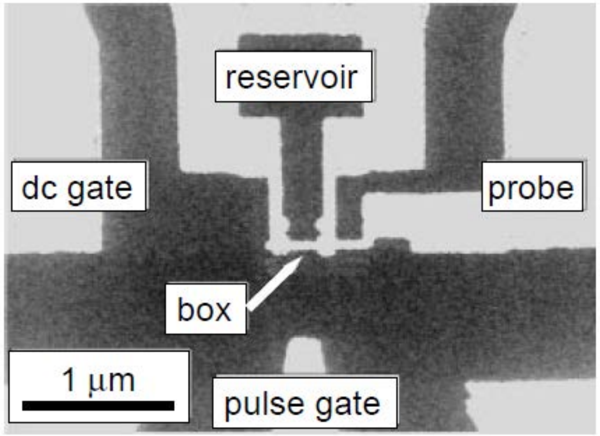
... To check. Up to contant terms:

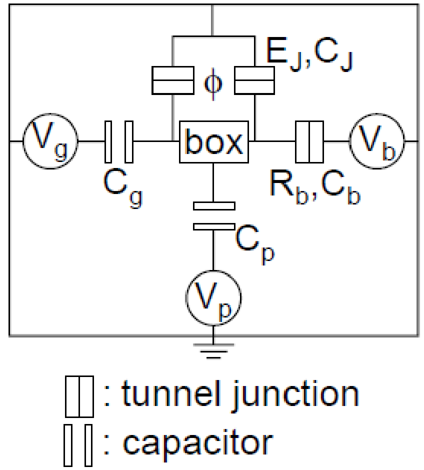
$$H = E_C \Delta_g \sigma_z - \frac{E_J}{2}\sigma_x = \begin{pmatrix} E_C\Delta_g & -E_J/2 \\ -E_J/2 & E_C\Delta_g \end{pmatrix} \longrightarrow E = \pm\frac{E_J}{2}\sqrt{1 + \frac{4E_C^2\Delta_g^2}{E_J^2}}$$

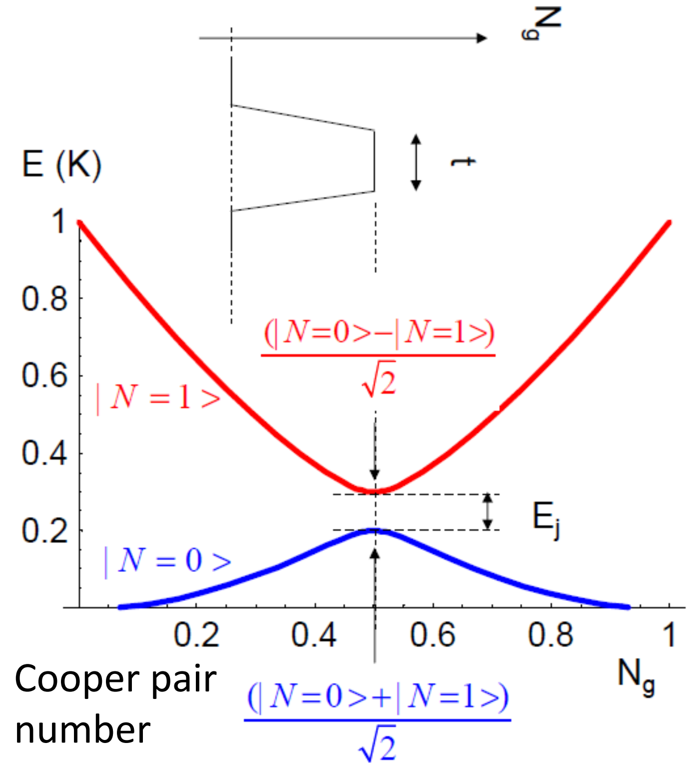
Around the splitting the spectrum is quadratic:

$$\frac{4E_C^2\Delta_g^2}{E_J^2} \ll 1 \qquad \Delta E = E_J + O(\Delta_g^2)$$

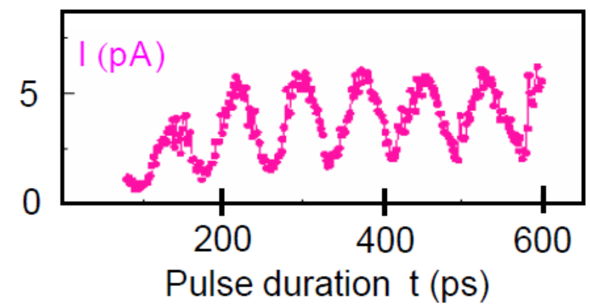
Charge qubits
Experiments



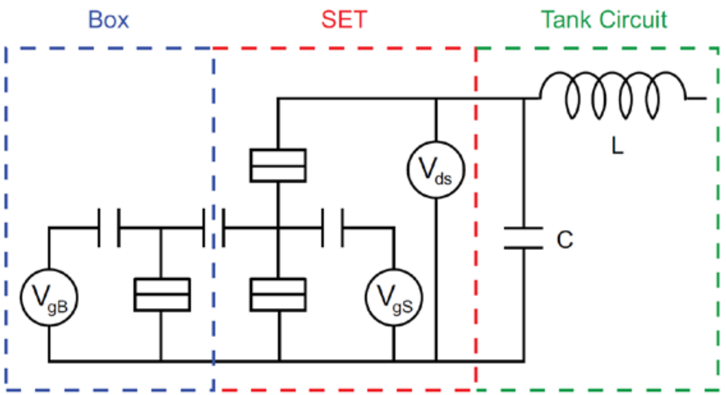


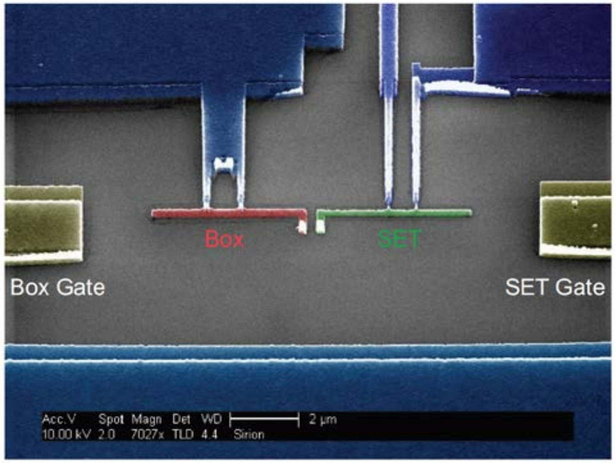


- First the qubit is prepared in state |0> by relaxation at N_g=0 (σ_Z eigenstate)
- Fast DC pulse to the gate to N_g=0.5 → not adiabatic, it remains in |0>. This is not an eigenstate (eigenstates are of σ_X)
- It starts Rabi-oscillating between |+> and |->, and evolves during the pulse length (*t*). After time t, bring it back to N_g=0
- In Larmor language: N_g=0, B_z field, and a ↓ is prepared. Than B rotated fast to B_Y. Larmor precession in the x-z plane. Then measurement again at B_Z basis.
- Detection: If after the pulse, the qubit is in |1> decays to probe electrode (properly biased) through 2 quasi particle tunneling events - **single shot readout**
- By adjusting *t*, the length of the pulse Rabi oscillation is seen
- Relaxation <5 ns, probably due to charge fluctuations



Y. Nakamura et al., Nature, ***398****, 786 (1999)*



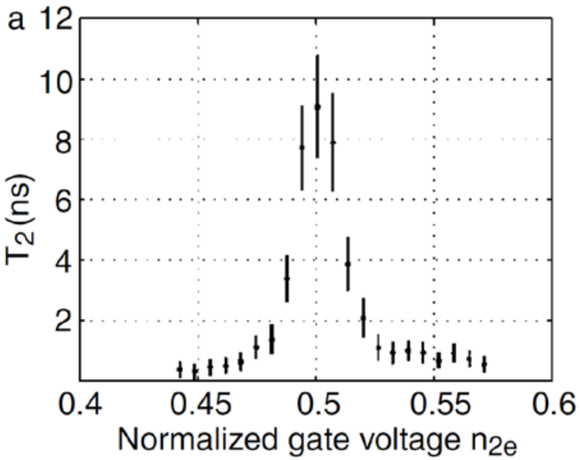


Other readout for charge qubit: with SET

Julia Love, PhD Thesis

SETs are capacitively coupled to the CPB. The change of the number of electrons on the CPB shifts the levels of the dot. The transport through the dot is measured. Or, SET coupled to RF circuit, and frequency shift of the resonator is measured.

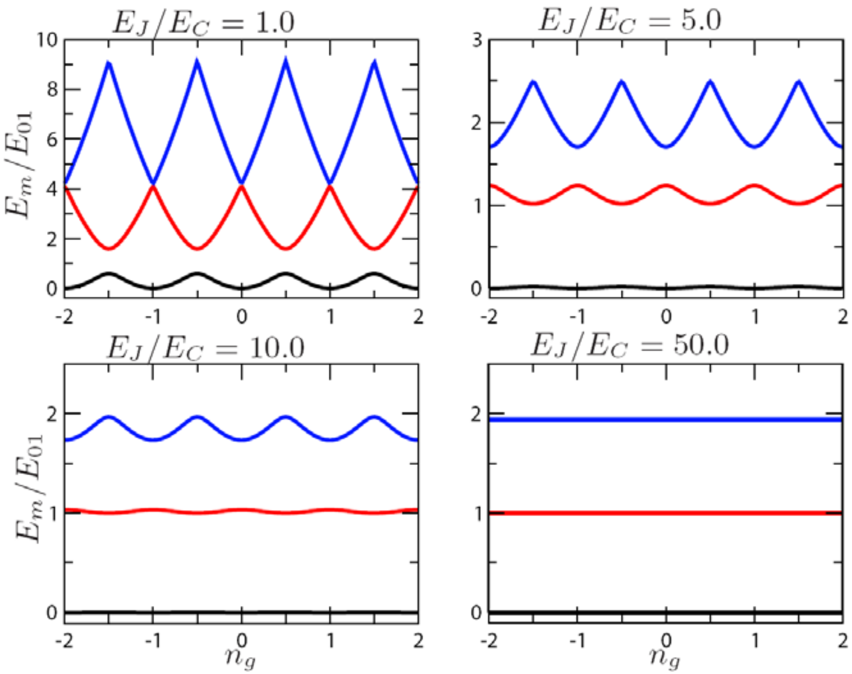
Decoherence: limited by charge noise – 1/f noise. This gives fluctuation in gate voltage (not stable instruments, fluctuations in tunnel barriers, nearby trap charges), which changes the qubit energy splitting a lot → leads to small T_2 (high frequency noise enters T1). The least sensitive to noise at degeneracy point. Here $\delta E \sim n_g^2$, only quadratically sensitive to noise: **sweet spot**



K. Bladh et al, New J. Phys. 7 180 (2005)

Best numbers:
T_1~ 7 µs
T_2~ 500 ns @ sweet spot
A. Wallraff et al., Phys. Rev. Lett. 95, 060501 (2005)

Transmon regime



Idea: flatten the dispersion relation such, that the it becomes a **sweet spot everywhere**

Increase E_J/E_C ratio – technically done by make a large parallel capacitance (than E_J is not tuned) – increase C, decrease E_C

How does this change the

- Charge dispersion? – decreases, becomes flat
- Anharmonicity? - decreases

Anharmonicity –decreases linearly

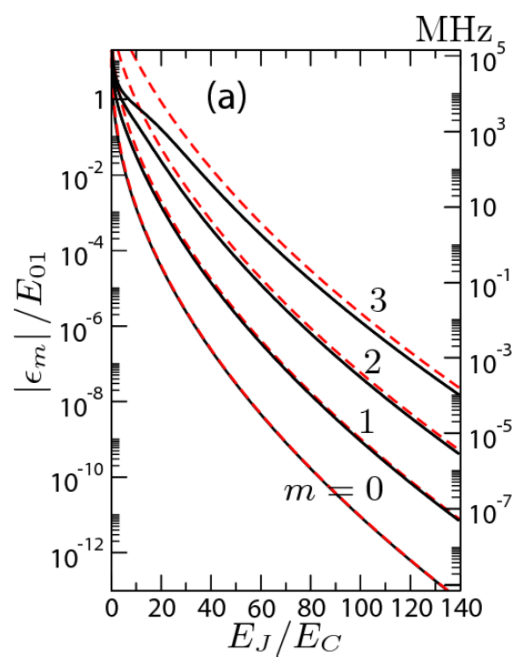
$$\alpha_r = \frac{E_{12} - E_{10}}{E_{10}} = \sqrt{\frac{E_c}{8E_J}}$$

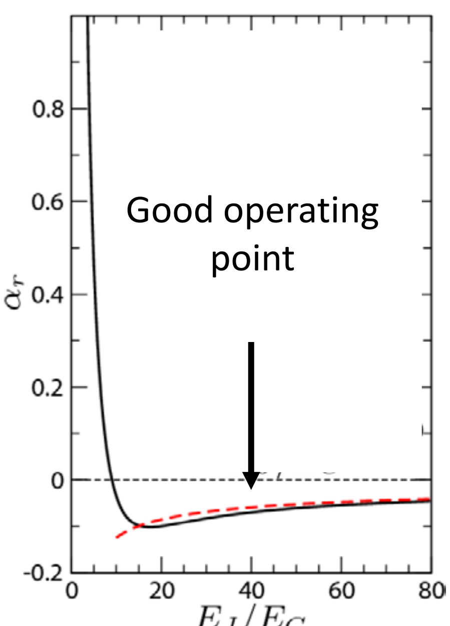
Charge dispersion – decreases exponentially (m: band index)

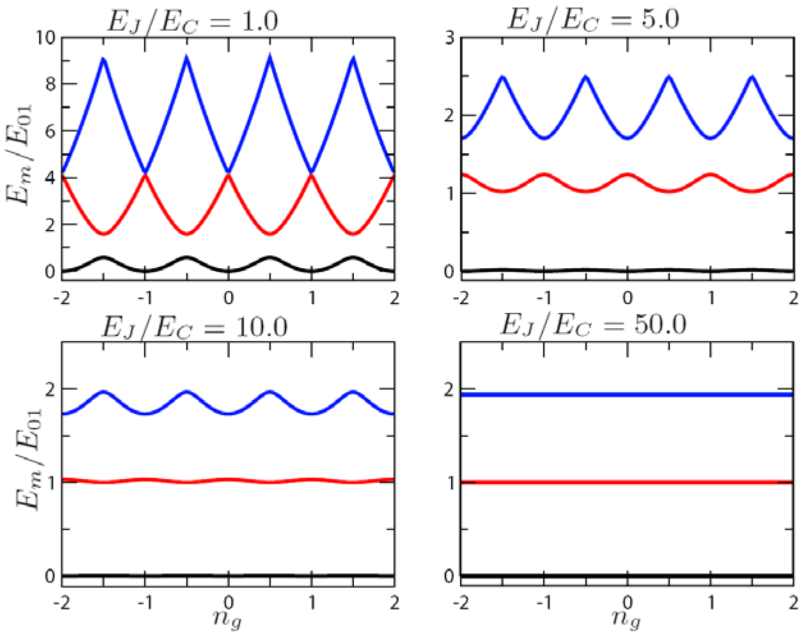
$$\epsilon_m = E_m(n_g = 1/2) - E_m(n_g = 0) \sim e^{-\sqrt{8E_J/E_c}}$$

E_J/E_c ~ 50 ideal

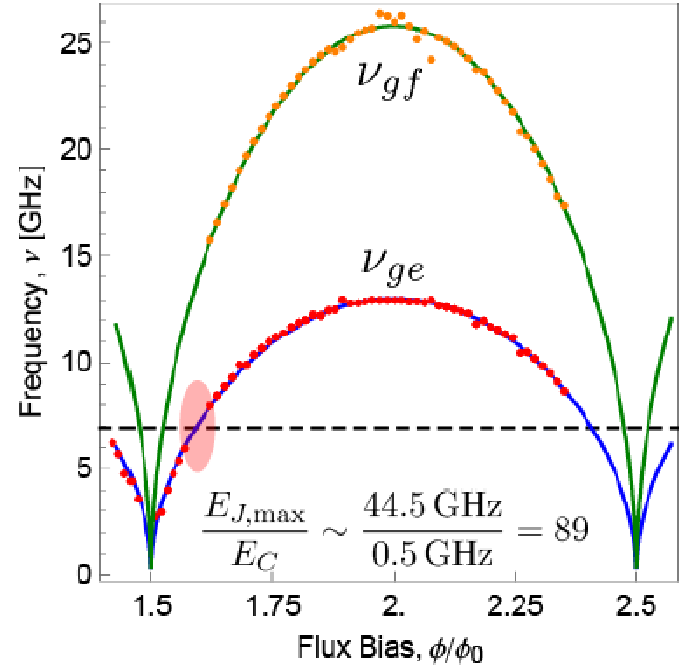
J. Koch et al., Phys. Rev. A., 76, 042319 (2007)

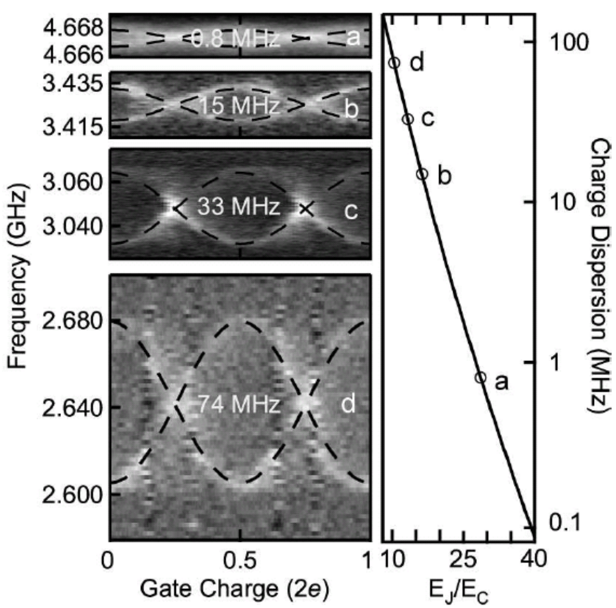


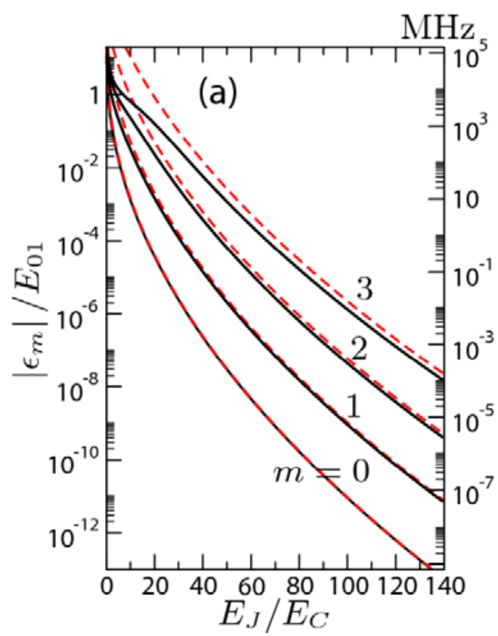




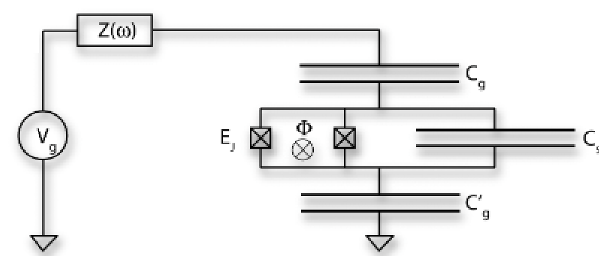
Charge becomes localized, phase compact in [0,2π]
Energies flux tunable





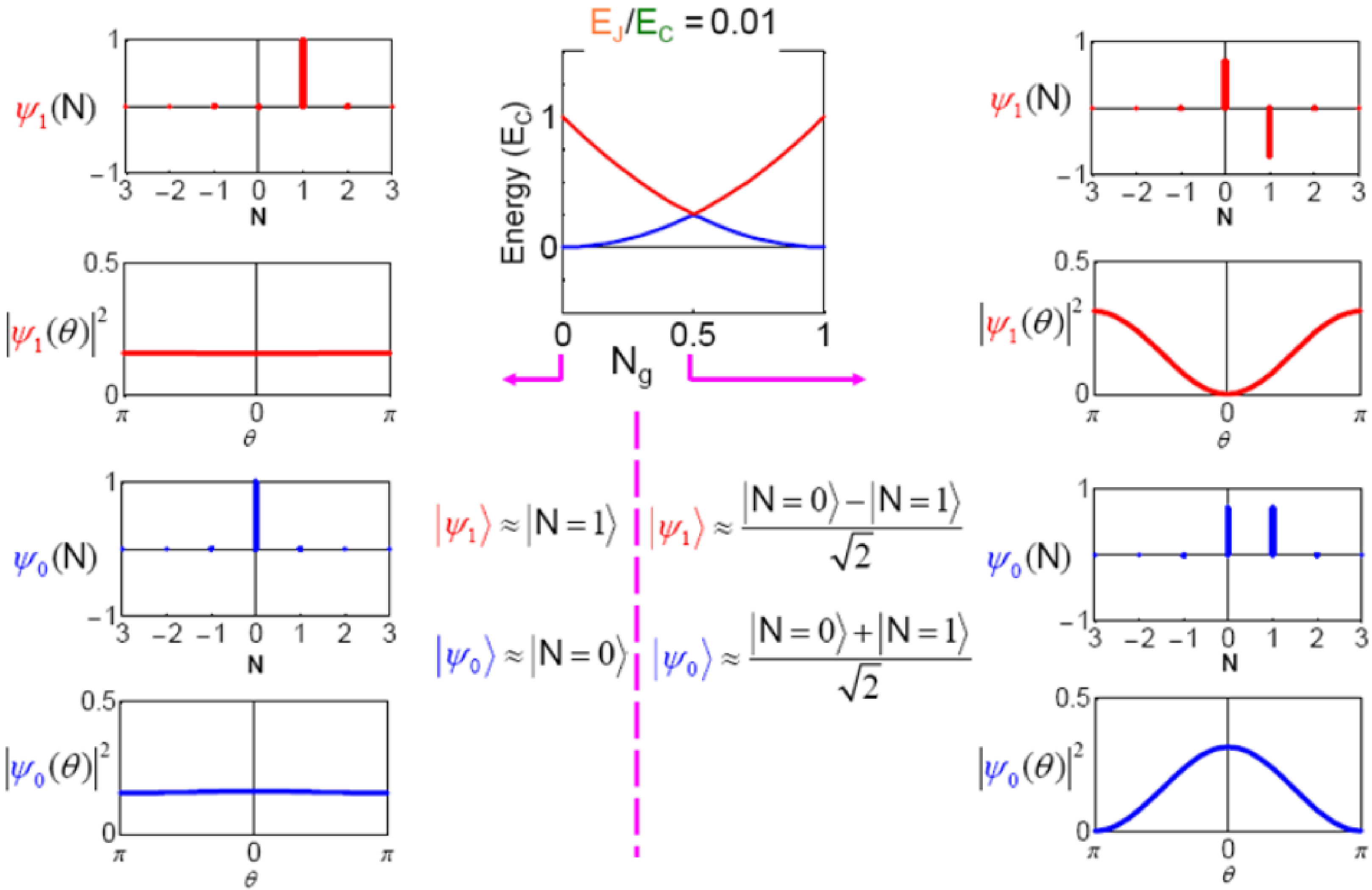


Measurement of charge dispersion on transmon qubits: follow well the expectation (doubling: Quasi-particle poisoning)



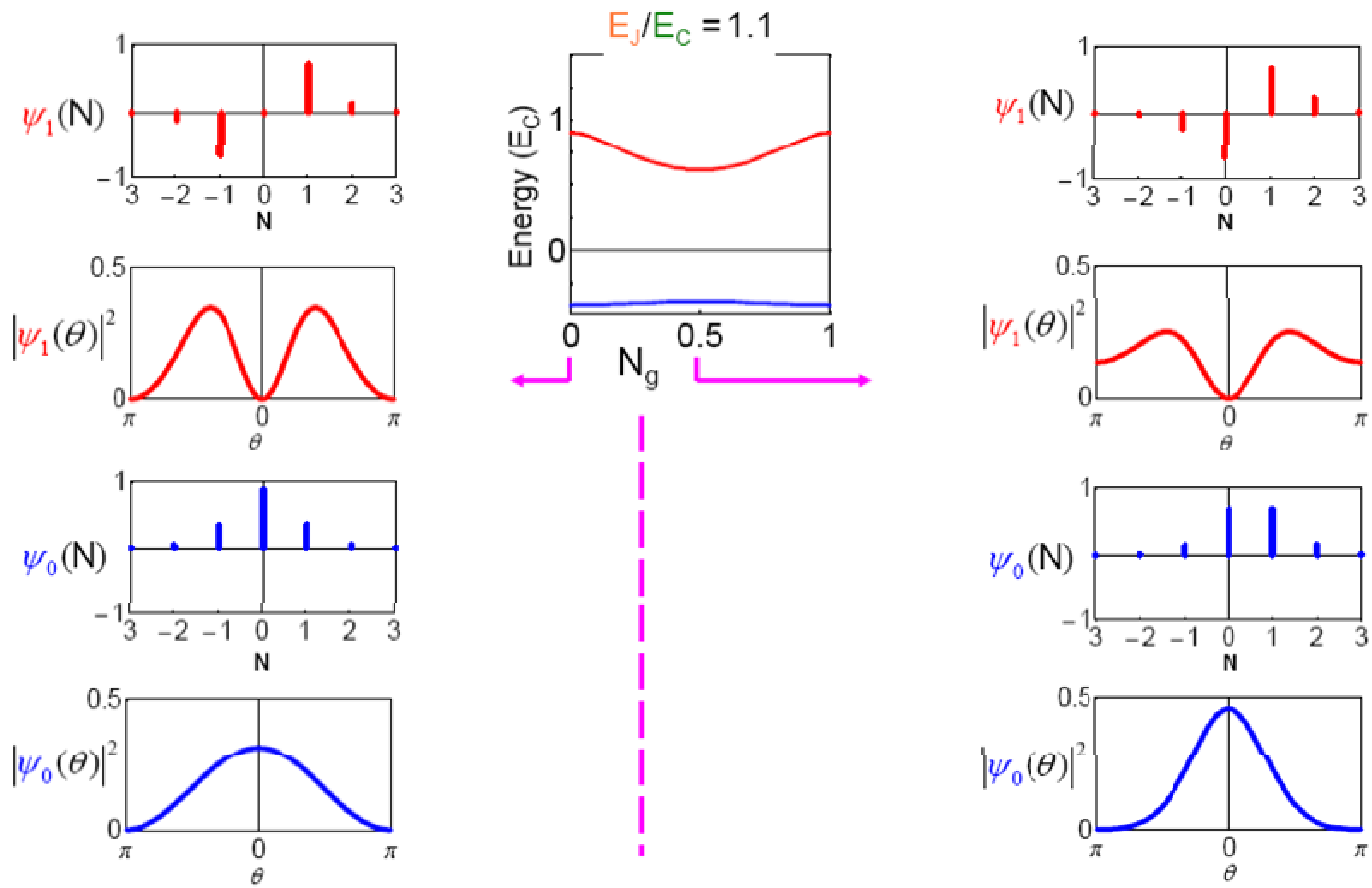
J. A. Schreier et al., Phys. Rev. B 77, 180502(R) (2008)

Charge and phase wave functions



D. Vion, Saclay

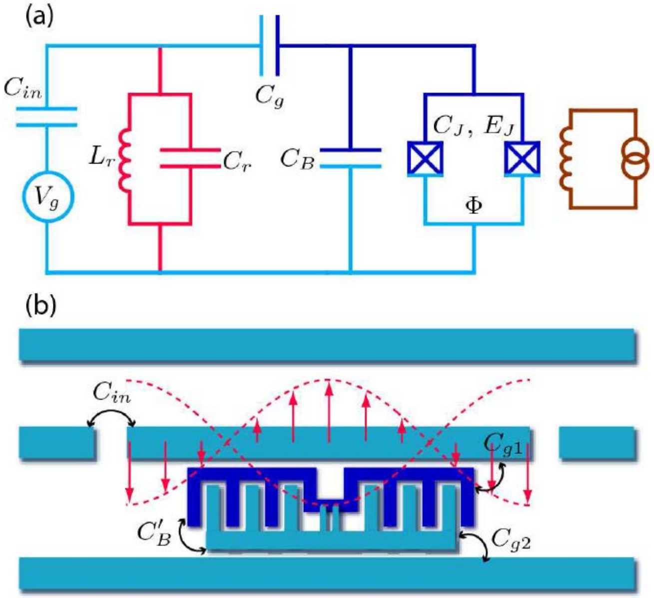
Charge and phase wave functions

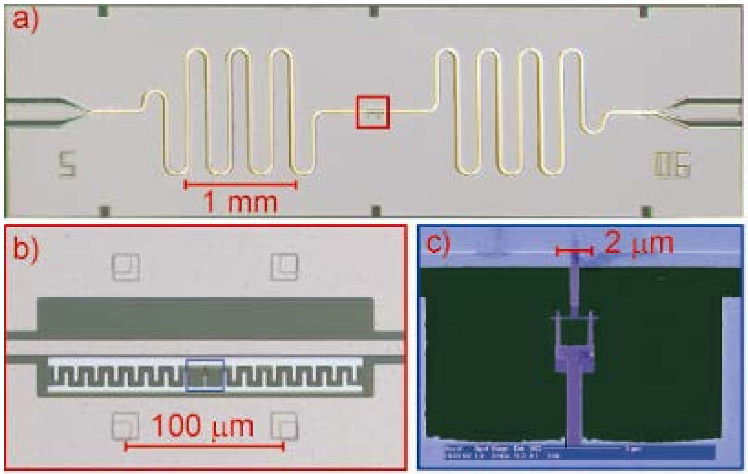


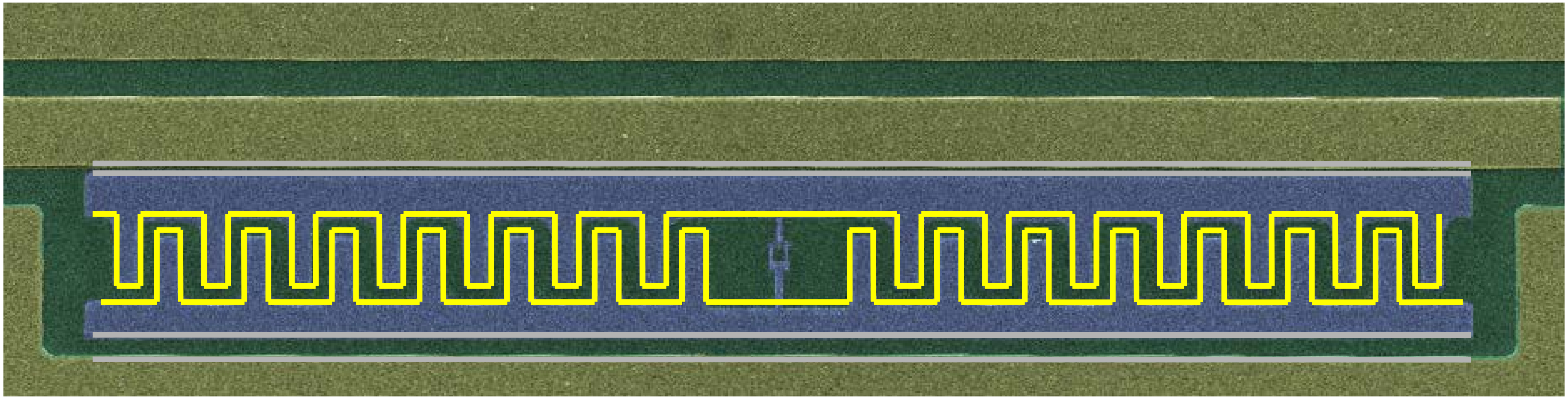
D. Vion, Saclay

Transmon

Cooper pair box+ large shunt capacitor to decrease E_c
Island volume ~1000 times bigger than conventional CPBs
E_J flux tunable
Readout – coupling to microwave resonator – RC circuit

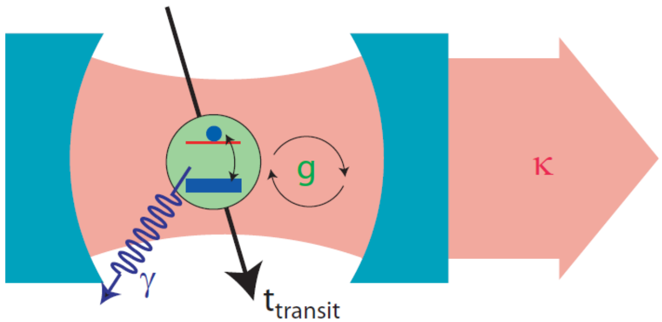




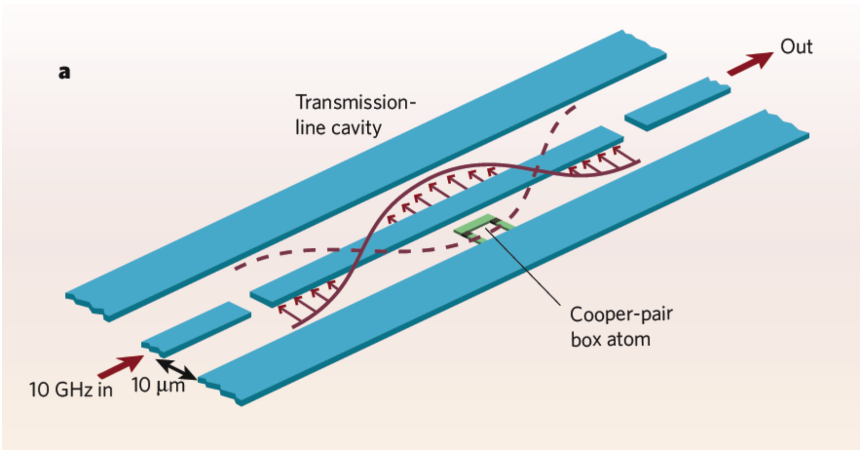


Source:

SC circuits

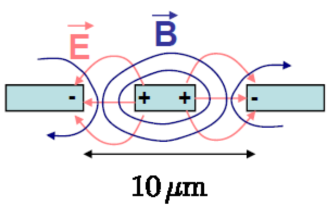


Fabry – Perot cavity for optics – using mirrors

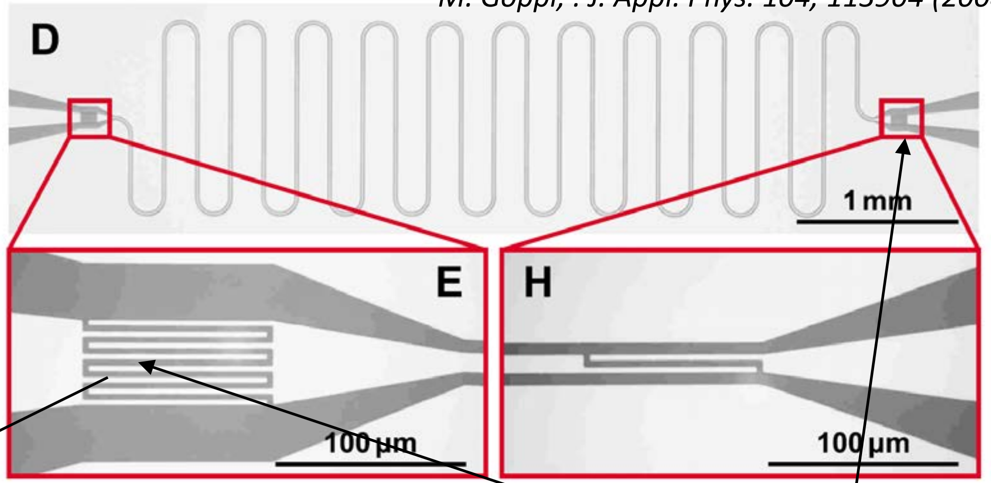


Fabry – Perot cavity for MW photons – capacitive mirrors

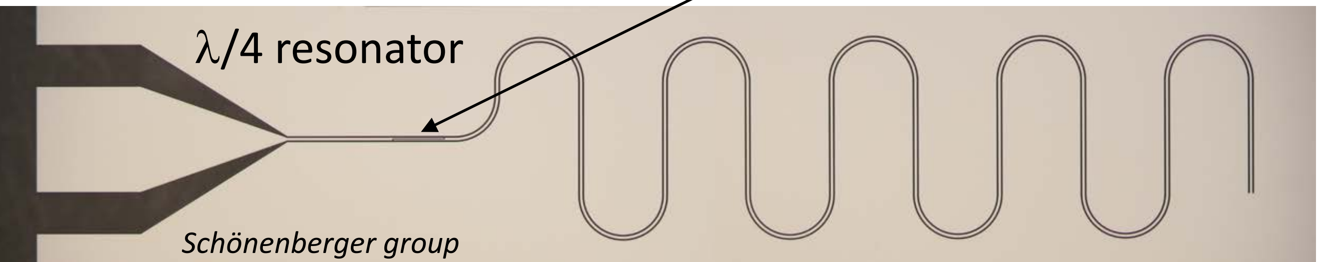
R.J. Schoelkopf et al., Nature 451, 664 (2009)
M. Göppl, : J. Appl. Phys. 104, 113904 (2008)

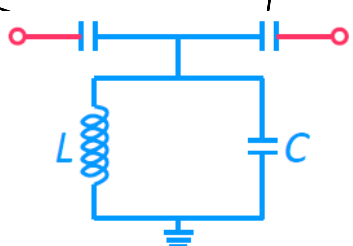


Central conductor and ground plane – essentially a coax



Superconducting circuit to minimize losses (white – SC material, black etched away)
Capacitors: voltage antinodes – zero current – good for electrical dipole coupling
Current antinode (voltage node) - maximal current – good for inductive coupling

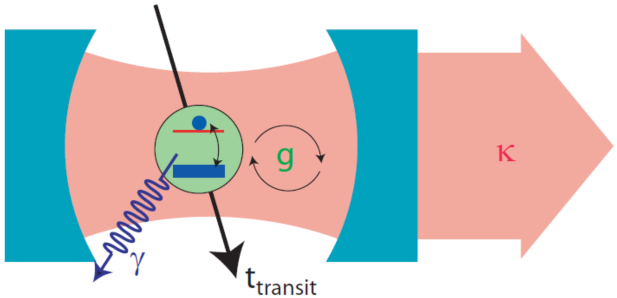




Readout: circuit QED

$$\hat{H} = 4E_c\left(N - N_g\right)^2 - E_J\cos\delta + \hbar\omega_r\hat{a}^\dagger\hat{a} + \underbrace{2\frac{C_g}{C_\Sigma}eV^0_{RMS}\hat{N}\left(\hat{a}^\dagger + \hat{a}\right)}$$

Coupling term – electrical coupling to charge (dipole)



$$\omega_r = \frac{1}{\sqrt{L_r C_r}} \qquad V^0_{rms} = \sqrt{\frac{\hbar\omega_r}{2C_r}}$$

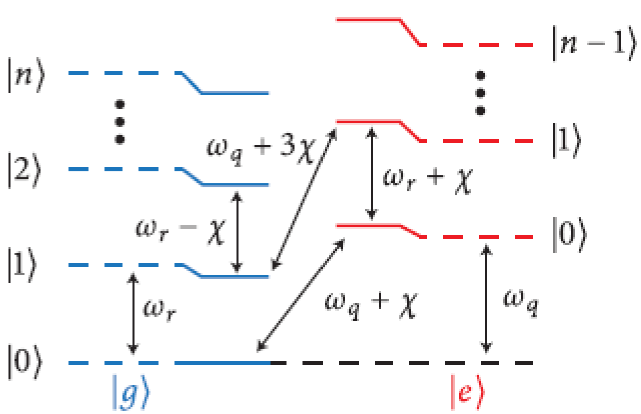
Can be mapped to J-C Hamiltonian

$$\hat{H} = \frac{\hbar\omega_q}{2}\sigma_Z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{a}^\dagger\sigma_- + \hat{a}\sigma_+\right) + H_\kappa + H_\gamma$$

Jaynes Cummings Hamiltonian

Qubit | Resonator | Coupling | Cavity decay | Qubit lifetime

b



$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta}\right)\sigma_Z + \left(\hbar\omega_r + \hbar\frac{g^2}{\Delta}\sigma_Z\right)\hat{a}^\dagger\hat{a}$$

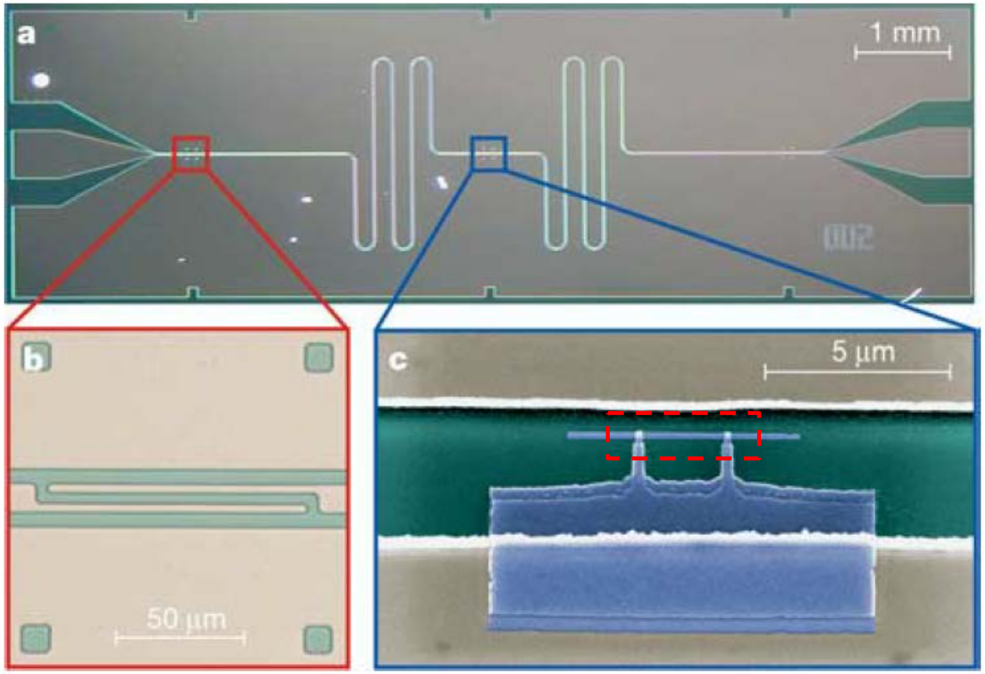
Lamb-shift

Qubit-state dependent resonance shift

$$\Delta = \omega_q - \omega_r$$

Readout: circuit QED

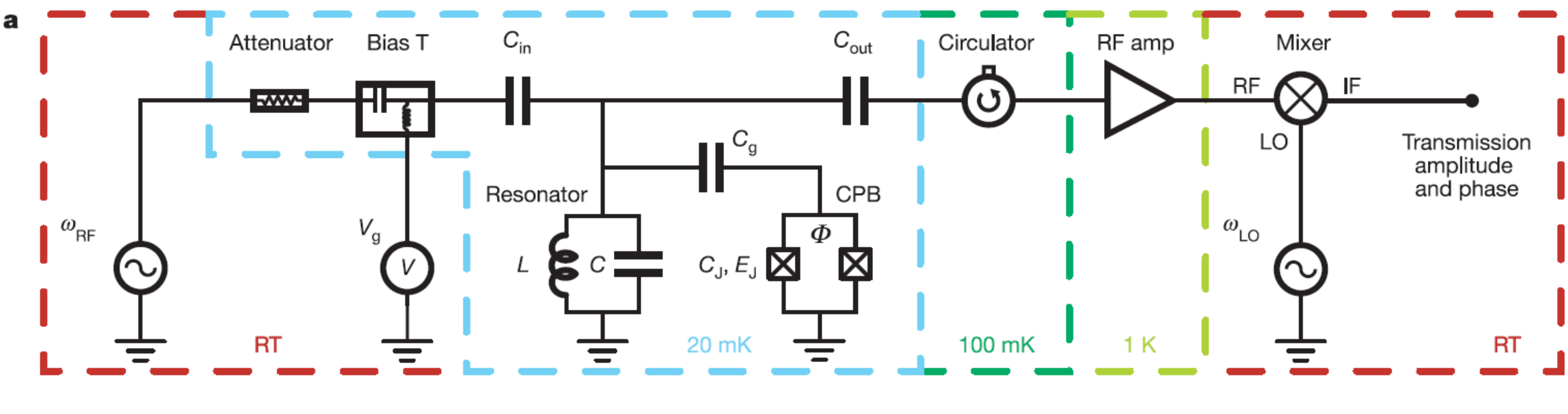
Spectroscopy on resonator



CP-box coupled (capacitively) to a MW cavity

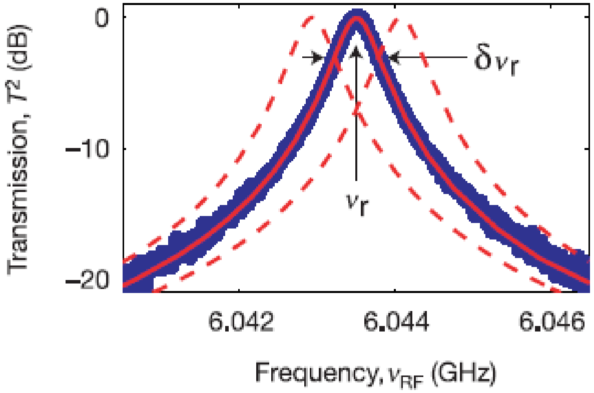
External B field tunes E_J

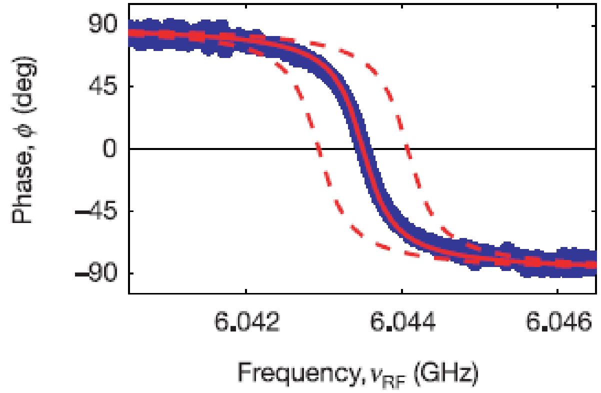
In the circuit model the qubit is a tunable capacitance which shifts the resonator

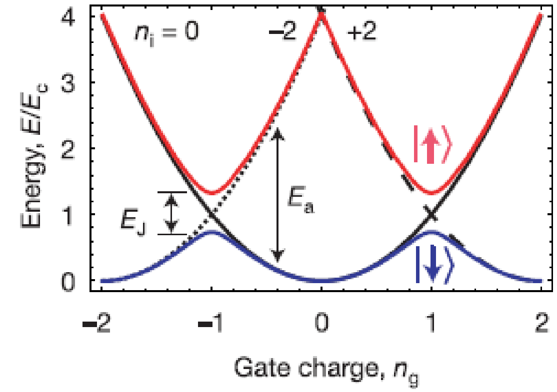


Many circuit elements are at low T (amplifier, circulator etc.)

A .Walraff et al., Nature 431, 162 (2004)

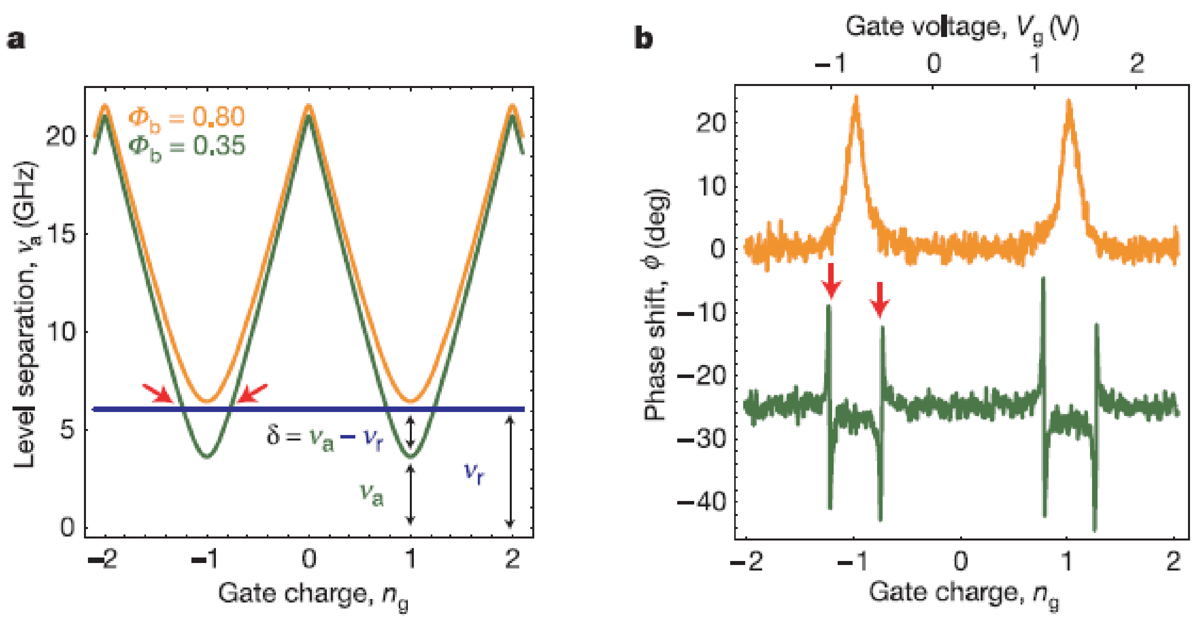






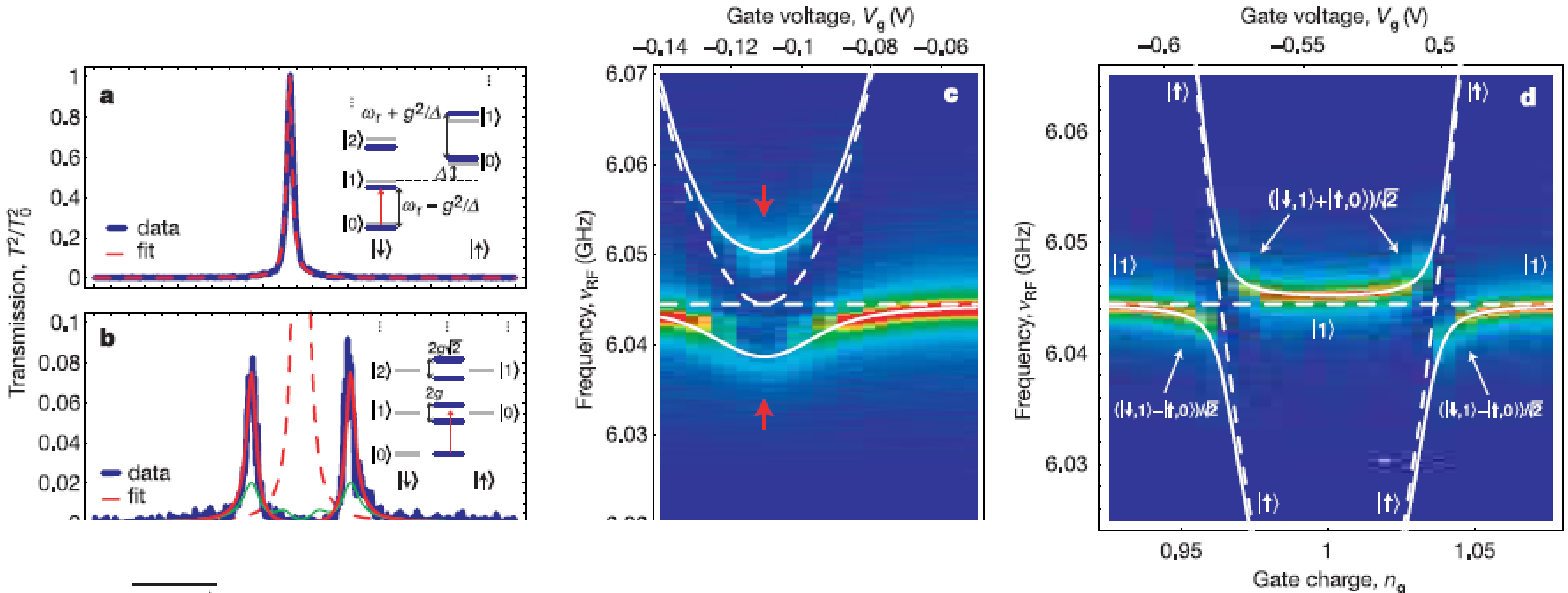
Resonator: Lorentz-like resonance curve with high Q. Phase response is more sensitive
Simulation: shifted curves for the two different qubit states. Idea: measurement at fixed frequency – measure phase response

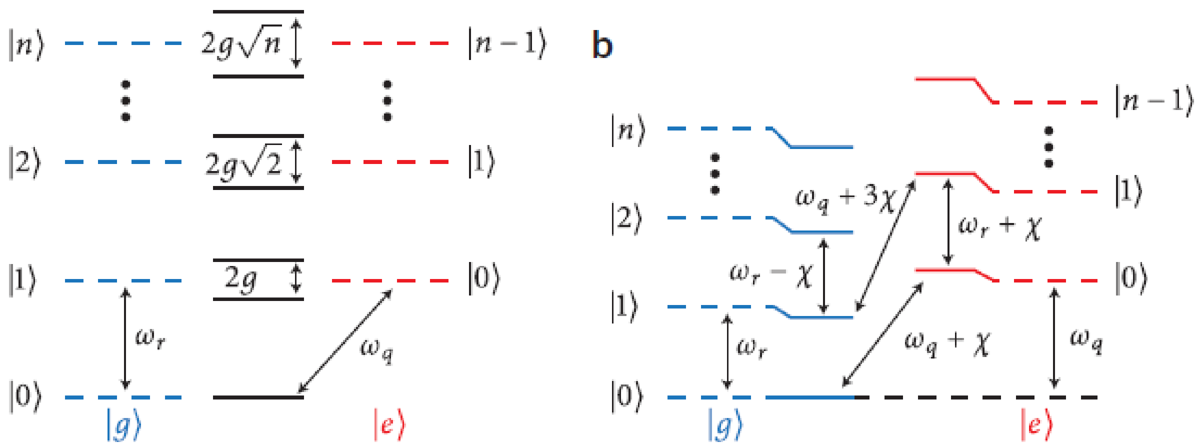
Reminder: $$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta}\right)\sigma_Z + \left(\hbar\omega_r + \hbar\frac{g^2}{\Delta}\sigma_Z\right)\hat{a}^\dagger\hat{a}$$



Here Qubit is in the ground state, and resonator is probed for different parameters
2 different flux biases: for one it goes through the resonance with the resonator (green), for the other not (orange).
Phase shift decreases by increasing detuning from resonance

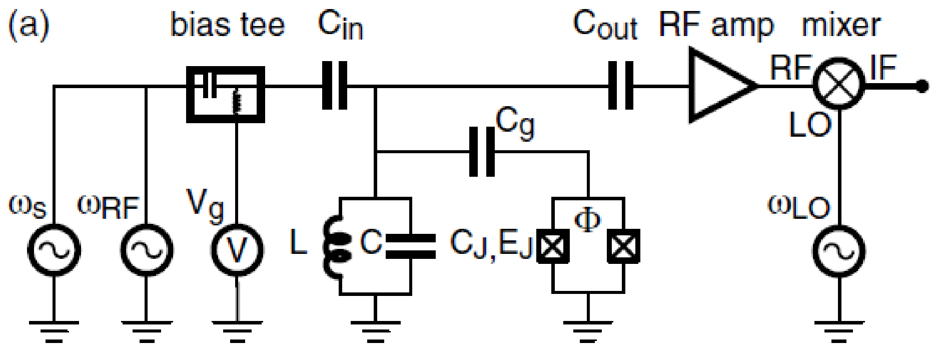
Strong coupling –
Spectroscopy measurement





Far away from crossing pure resonator states. Close to resonance an avoided crossing is seen. Bonding and anti-bonding states – entangled states with both photon and qubit character – „phobit" and „quton".

Here the photon number is small n<<1. Vacuum Rabi oscillation with frequency 2g. Continous photon emission and absorbtion.



Spectroscopy

2-tone measurements

2 tone:

ω_S: qubit frequency (here continuous)

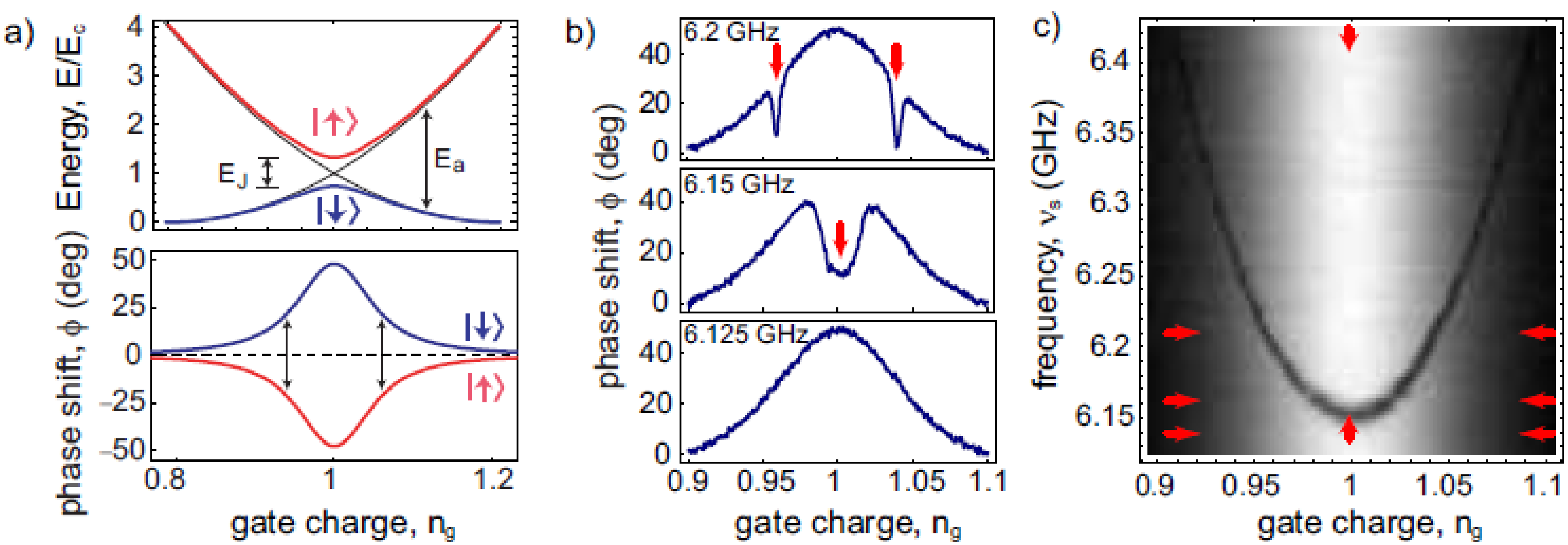
ω_{RF}: cavity frequency (here continuous)

Phase shift: opposite for the two states. If ω_S excites cavity than reduction in phase shift (red arrows). For high power, both states are equally populated and the shift averages to zero.

6.125 GHz- no resonance with qubit, just phase shift observed

6.15 GHz - at N_g=1 the qubit is driven. Reduction in the phase shift is seen. Similarly at 6.2 GHz.

For Rabi etc. pulsing at ω_S is needed (see later).



D. I. Schuster et al., PRL 94, 123602 (2005)

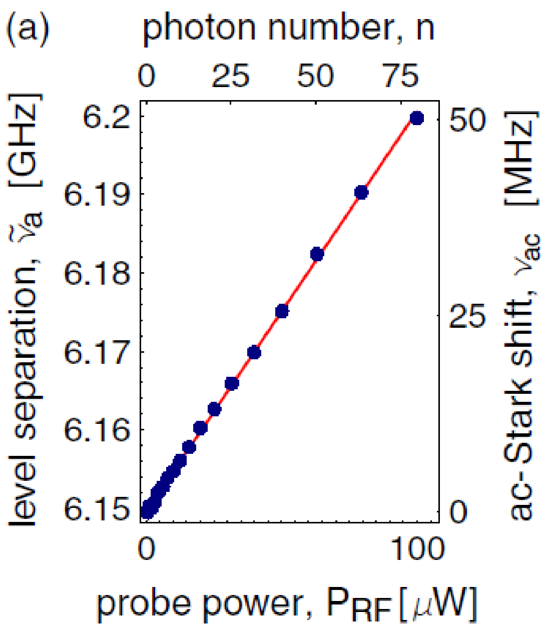
Back-action
Stark-shift

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta}\right)\sigma_Z + \left(\hbar\omega_r + \hbar\frac{g^2}{\Delta}\right)\hat{a}^\dagger\hat{a}$$

$$\downarrow$$

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta} + \boxed{\hbar\frac{g^2}{\Delta}\hat{a}^\dagger\hat{a}}\right)\sigma_Z + \hbar\omega_r\hat{a}^\dagger\hat{a}$$

Stark shift

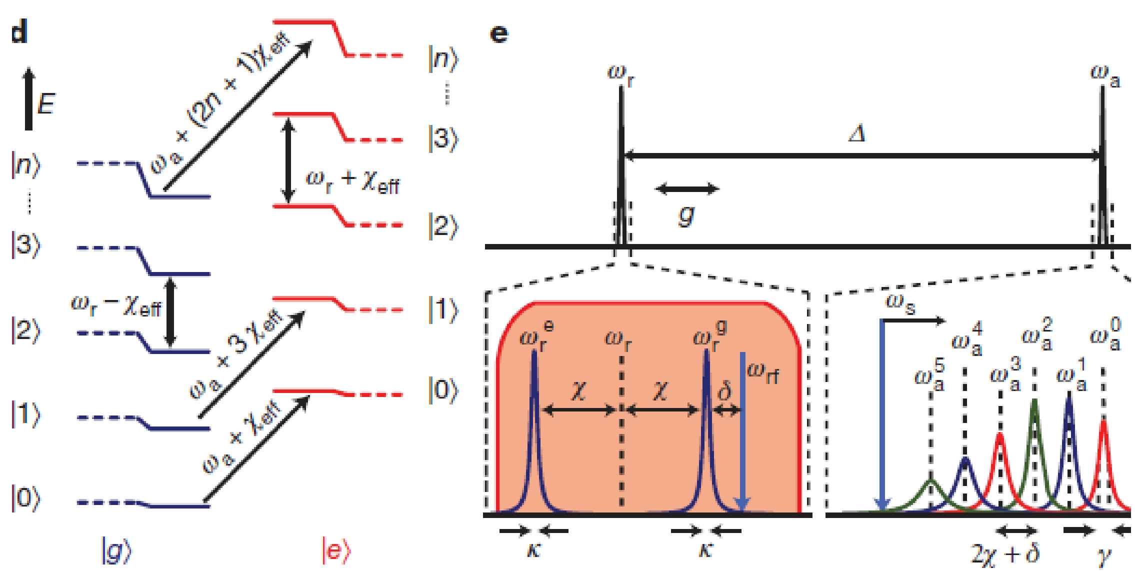


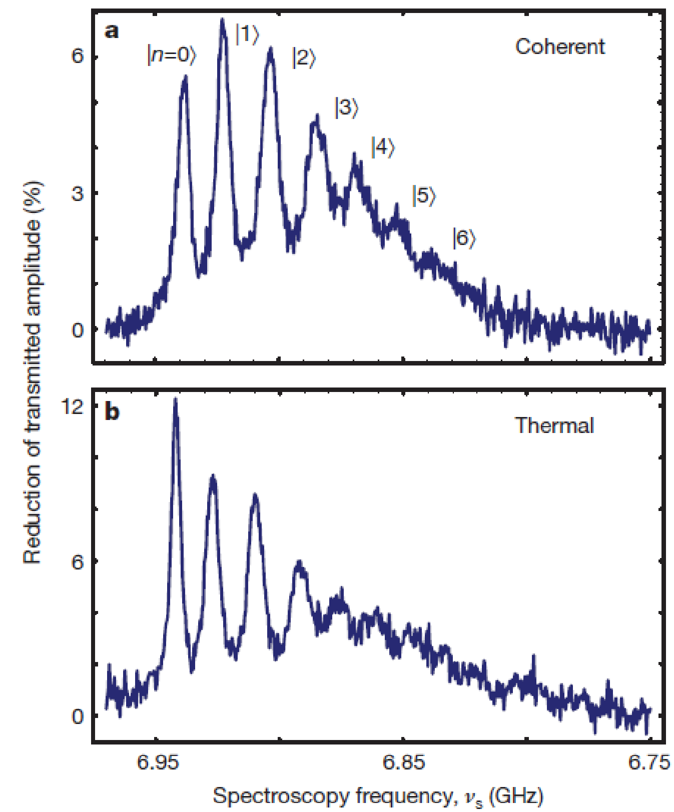
By increasing the resonator power, hence the average photon number, the qubit frequency shifts.

D. I. Schuster et al., PRL 94, 123602 (2005)

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta} + \boxed{\hbar\frac{g^2}{\Delta}\hat{a}^\dagger\hat{a}}\right)\sigma_Z + \hbar\omega_r\hat{a}^\dagger\hat{a} \qquad \chi = \frac{g^2}{\Delta}$$

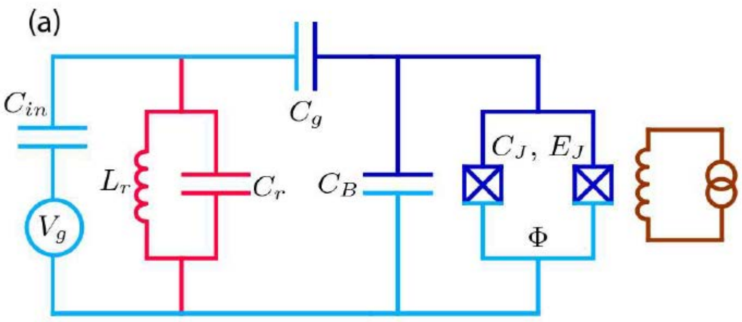
In the strong dispersive regime ($\chi >> \gamma, \kappa$) individual photon states resolved:
Populate resonator at wrf. Than sweep ws (qubit frequncy). If there were n photons in the cavity the resonance will be at $2n\chi$. If the qubit gets excited can be seen from the resonator frequency shift.
Individual photon states resolved.
Under usual drive close to coherent states observed.
Addig large thermal noise – thermal distribution.



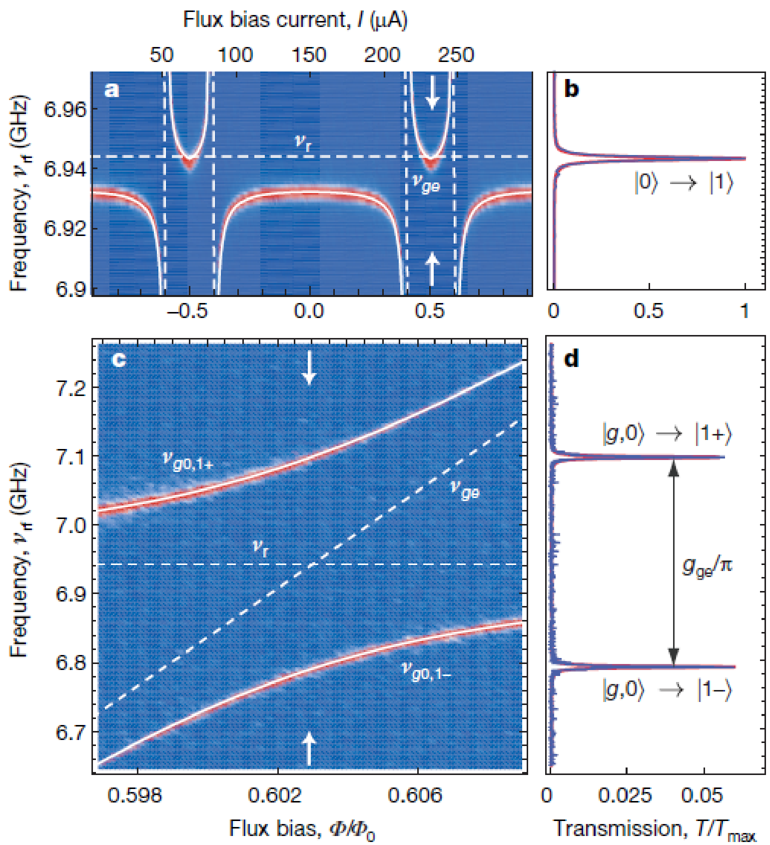


D. I. Schuster et al., Nature, 445, 515 (2007)

Transmon cQED



Strong coupling



Mostly the same, gate voltage not a useful parameter

Using the transmon wave function, RWA only the following relevant terms remain:

$$\hat{H} = \hbar \sum_j \omega_j |j\rangle \langle j| + \hbar\omega_r \hat{a}^\dagger \hat{a} + \left[\hbar \sum_i g_{i,i+1} |i\rangle \langle i+1| \hat{a}^\dagger + \text{H.C.}\right]$$

Multi level Jaynes Cummings Hamiltonian, where

$$\hbar g_{i,i+1} = 2e\frac{C_g}{C_\Sigma} eV^0_{rms} \langle i|\hat{N}|i+1\rangle \qquad \langle i|\hat{N}|i+1\rangle \sim \left(\frac{E_j}{8E_C}\right)^{1/4}$$

g – coupling term is large, even increases with increasing E_J

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_{01} + \hbar\chi_{01}\right)\sigma_Z + \left(\hbar\omega_r - \hbar\chi_{12} + \hbar\chi\sigma_Z\right)\hat{a}^\dagger\hat{a}$$

$$\chi = \chi_{01} - \chi_{12}/2 \qquad \chi_{ij} = \frac{g_{ij}}{\omega_{ij} - \omega_r}$$

Higher levels matter a bit, otherwise the same

Strong coupling achieved
For 0-1 state 2g Rabi frequency
For 1-2 state √2*2g as J-C says

J. M. Fink et al., Nature 454, 315 (2008)