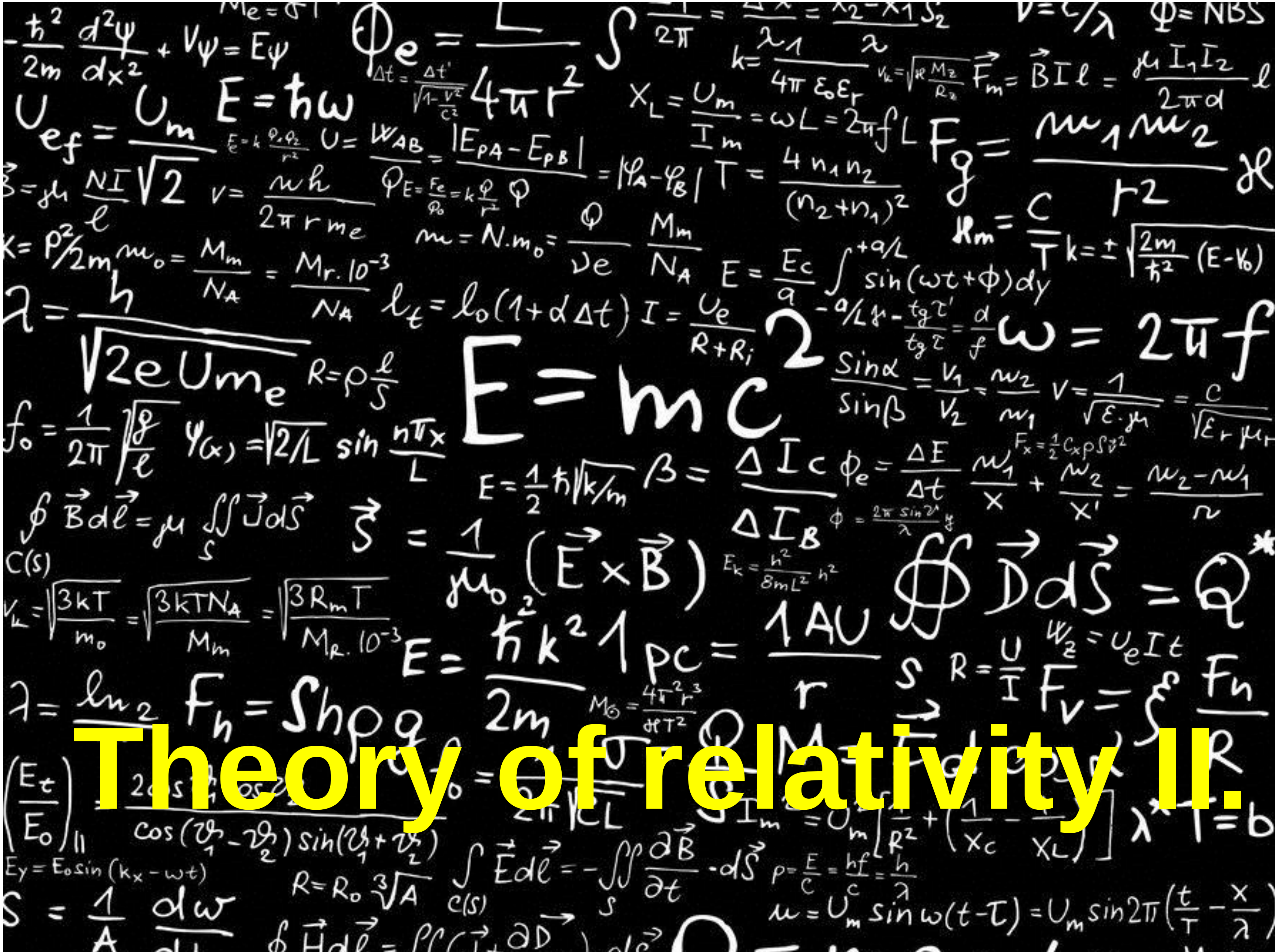
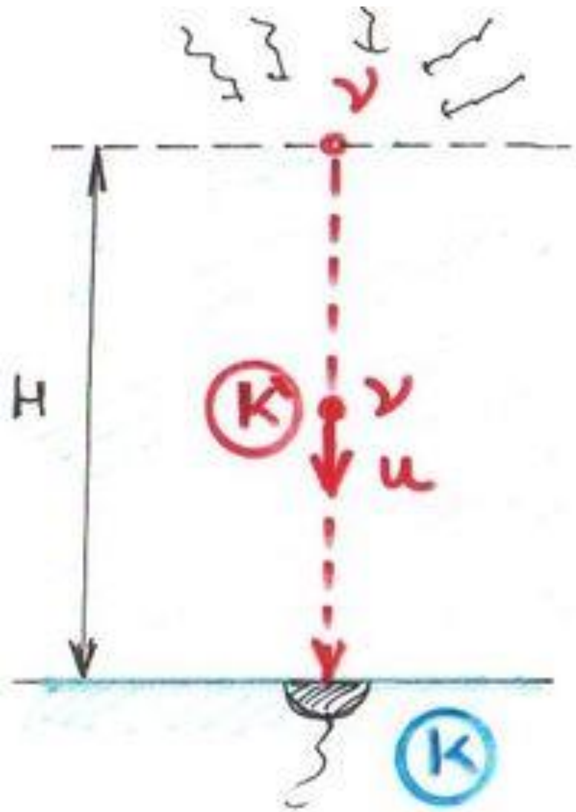
Theory of relativity II.



The muon (μ) decay ($q_\mu = q_e$, $S_\mu = \frac{1}{2}\hbar$, $m_\mu > m_e$)



The half-life: $\tau_o \approx 2.2\ \mu s$

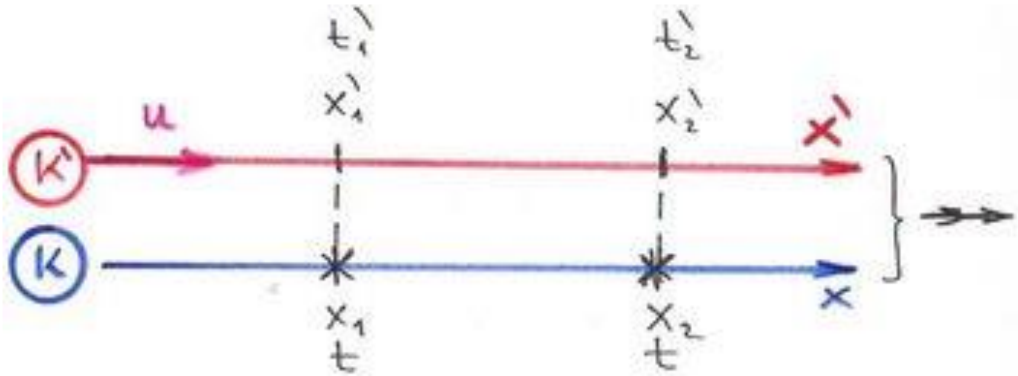
At H = 4700 m are created (the muons)

$$v_\mu = \frac{H}{\tau_o} \approx 7c$$

$$H = u\frac{\tau_0}{\sqrt{1-\frac{u^2}{c^2}}} = u\tau$$

$$u \approx 0.99c$$

Relativity of simultaneity



$$t_1 = t_2 \rightarrow \Delta t = 0$$

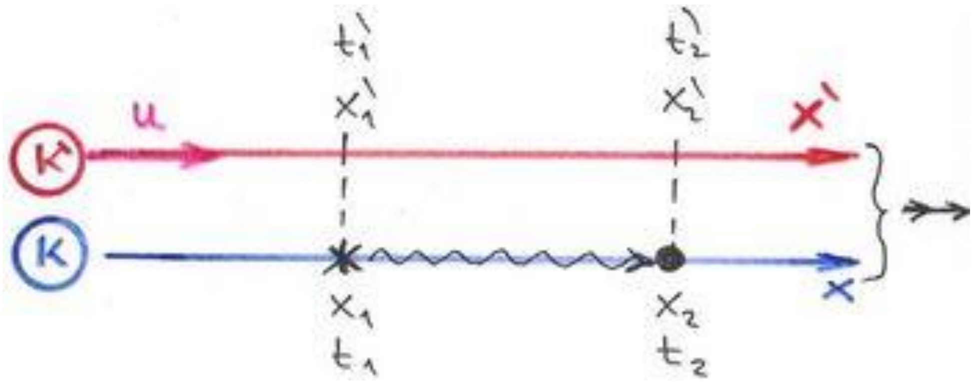
$$\text{(B)} \quad \Delta t' = \Gamma\left(\Delta t - \frac{u}{c^2}\Delta x\right)$$

$$t_2' - t_1' = \Delta t' = \Gamma\left(-\frac{u}{c^2}\Delta x\right) = \frac{-\frac{u}{c^2}(x_2 - x_1)}{\sqrt{1 - \frac{u^2}{c^2}}}$$

There is no absolute simultaneity !!!

Causality

Causality is the **relationship** between **causes** and **effects**.
(The cause-effect relation)



coase(reason) effect

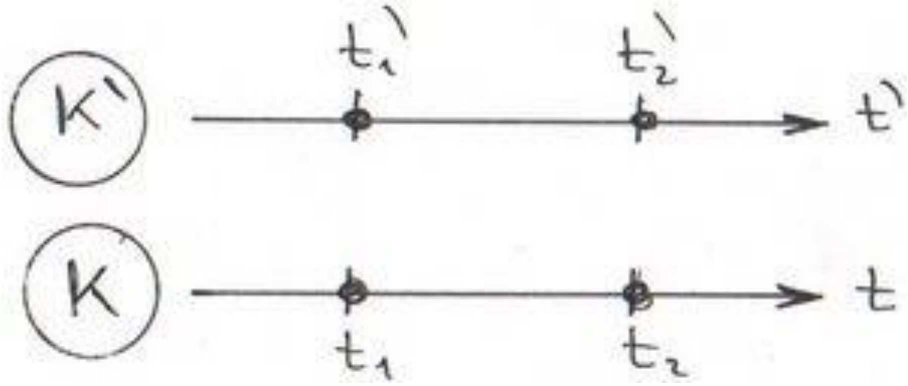
$$t_2 > t_1 \;\Rightarrow\; \Delta t > 0$$

$$\text{(B)}\quad \Delta t' = \Gamma\left(\Delta t - \frac{u}{c^2}\Delta x\right) \longrightarrow \Delta t' = \Gamma \Delta t\left(1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}\right)$$

$$\text{Ha } \Delta t' < 0 \;\rightarrow\; \left(1 - \frac{u}{c^2}\frac{\Delta x}{\Delta t}\right) < 0 \;\rightarrow\; c^2 < u\frac{\Delta x}{\Delta t}$$

The causality means that an effect ***can not occur*** from a cause.

Transformation of velocity (velocity-addition formula):



We have seen:

$$(C)\quad \Delta x = \Gamma(\Delta x' + u\Delta t')$$

$$(D)\quad \Delta t = \Gamma\left(\Delta t' + \frac{u}{c^2}\Delta x'\right)$$

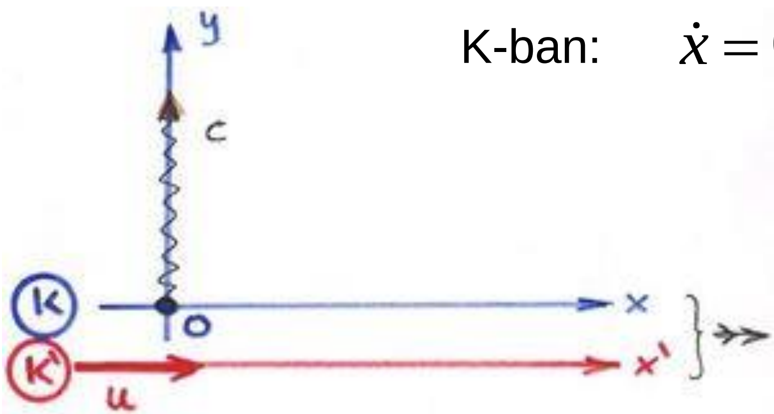
$$v = \frac{\Delta x}{\Delta t} = \frac{\Gamma(\Delta x' + u\Delta t')}{\Gamma\left(\Delta t' + \frac{u}{c^2}\Delta x'\right)} \longrightarrow \boxed{v_x = \frac{v'_x + u}{1 + \frac{uv'_x}{c^2}}}$$

$$v_y = \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{\Gamma\left(\Delta t' + \frac{u}{c^2}\Delta x'\right)} = \frac{v'_y\sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{uv'_x}{c^2}}$$

és

$$v_z = \frac{v'_z\sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{uv'_x}{c^2}}$$

Example for transformation of velocity I.



K-ban: $\dot{x} = 0 \quad \dot{y} = c$

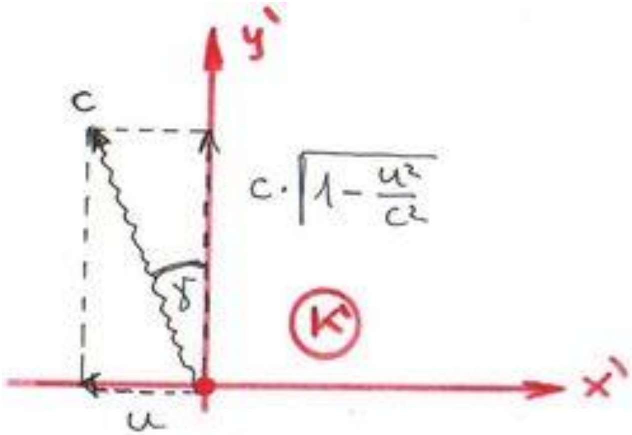
$$\dot{x}' = \frac{\dot{x} - u}{1 - \frac{u\dot{x}}{c^2}}$$

$$\dot{y}' = \frac{\dot{y}}{\Gamma\left(1 - \frac{u\dot{x}}{c^2}\right)} = \frac{c}{\Gamma} = c\sqrt{1 - \frac{u^2}{c^2}}$$

$$c'^2 = u^2 + c^2\left(1 - \frac{u^2}{c^2}\right) = c^2$$

The liht speed in K' is ***c*** !!!,

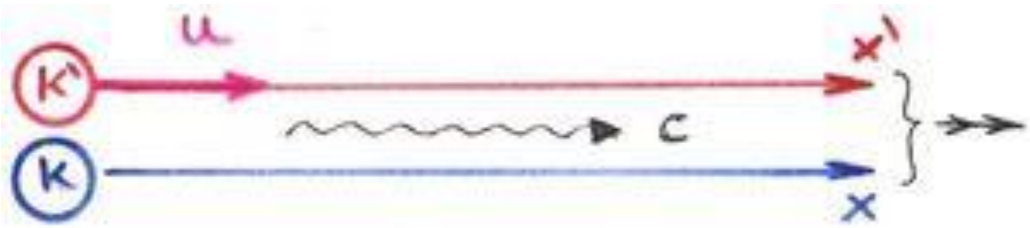
but its direction: γ.



Example for transformation of velocity II.

The light is traveling along the x’ (and x) axis.

$$\dot{x}' = c$$

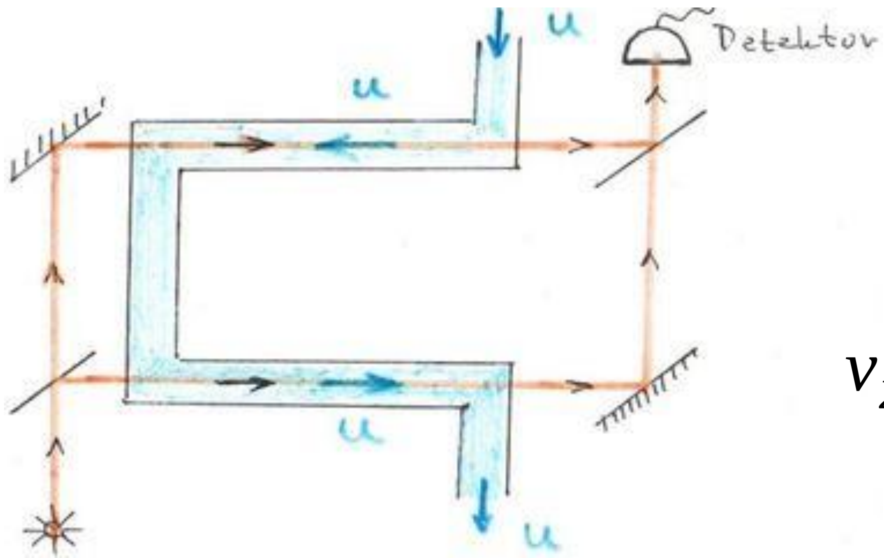


$$\dot{x} = \frac{\dot{x}' + u}{1 + \frac{u\dot{x}'}{c^2}} = \frac{c + u}{1 + \frac{u}{c}} = c$$

Example for transformation of velocity III.

Hippolyte Fizeau (1851)

The light in moving media (fluid).



$$v_x = \frac{v'_x + u}{1 + \frac{u v'_x}{c^2}} \quad \text{, de} \quad \frac{u v'_x}{c^2} << 1$$

$$v_x = \frac{v'_x + u}{1 + \frac{u v'_x}{c^2}} \approx (v'_x + u)\left(1 - \frac{u v'_x}{c^2}\right)$$

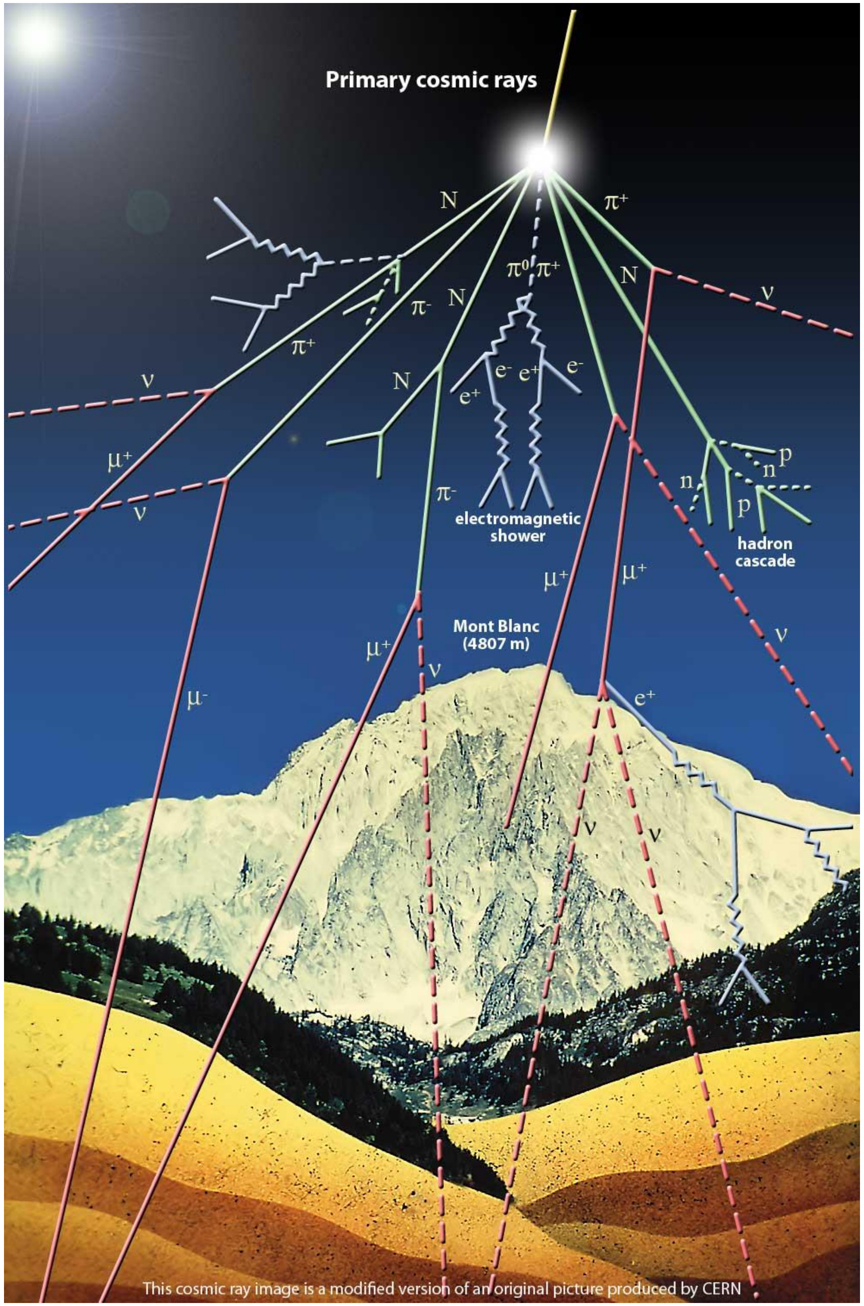
$$v_x \approx v'_x + u - \frac{u v'^2_x}{c^2} - \frac{u^2 v'_x}{c^2} \approx v'_x + u - \frac{u v'^2_x}{c^2}$$

$$v'_x = \frac{c}{n}$$

$$v_x \approx \frac{c}{n} + u - \frac{u}{n^2} = \frac{c}{n} + u\left(1 - \frac{1}{n^2}\right)$$

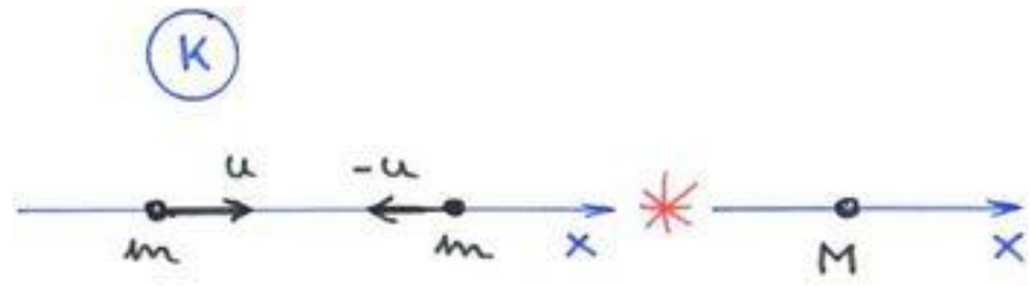
Fizeau’s experimental result

Dynamics



Primary cosmic rays
N
π+
π0
π-
e-
e+
ν
μ+
μ-
n
p
electromagnetic shower
hadron cascade
Mont Blanc (4807 m)
This cosmic ray image is a modified version of an original picture produced by CERN

Conservation of linear momentum I.



before the collision

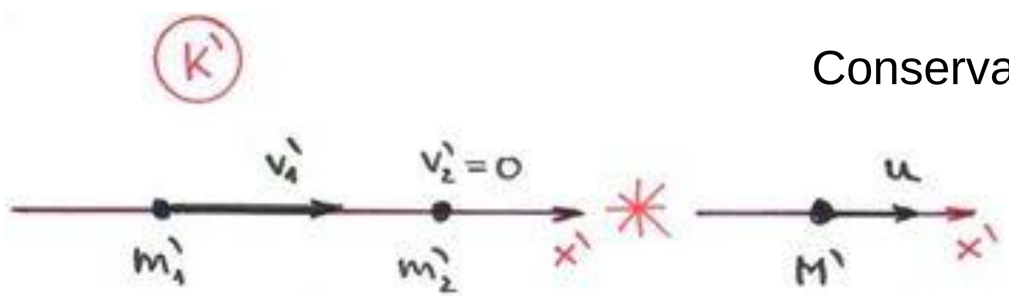
after the collision

$$mu + m(-u) = 0$$

↑

"From ***K***…"

Let's fix the ***K'*** to the particle moving at the velocity of ***-u*** .



before the collision

after the collision

$$m_1' v_1' = M' u$$

***u**

Conservation of mass: $m_1' + m_2' = M'$

$$m_1'(v_1' - u) = m_2' u$$

$$\frac{m_1'}{m_2'} = \frac{u}{v_1' - u} = \frac{1}{\frac{v_1'}{u} - 1}$$

Conservation of linear momentum II.

$$\frac{m_1'}{m_2'} = \frac{u}{v_1' - u} = \frac{1}{\frac{v_1'}{u} - 1} \qquad (*)$$

$$m_1' = m_2' = m \longrightarrow 1 = \frac{u}{v_1' - u} \rightarrow v_1' = 2u$$

But we have seen: $$v_1' = \frac{2u}{1 + \frac{u^2}{c^2}} = \frac{2u}{1 + \beta^2}$$

Newton (class. mech.): $$m_1' = m_2' = m \rightarrow M = 2m$$

$$mv_1' = Mu \longrightarrow m\frac{2u}{1+\beta^2} \neq 2mu$$

?

Conservation of linear momentum III.

$$\frac{v_1'}{u} = \frac{2}{1+\beta^2}$$

Put in (∗):

$$\frac{m_1'}{m_2'} = \frac{1}{\frac{v_1'}{u} - 1} = \frac{1}{\frac{2}{1+\beta^2} - 1} = \frac{1+\beta^2}{1-\beta^2}$$

$$\beta_1 = \frac{v_1'}{c} = \frac{2u/c}{1+\beta^2} = \frac{2\beta}{1+\beta^2}$$

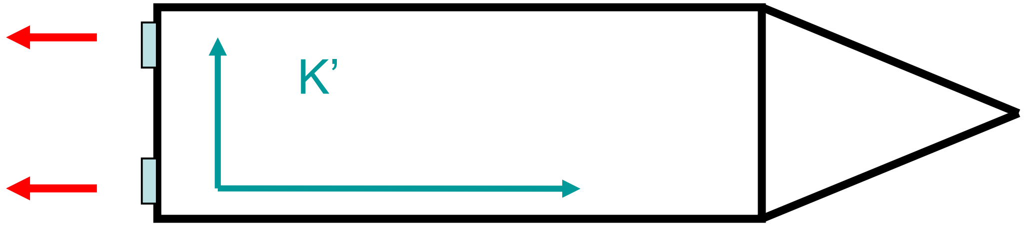
$$1-\beta_1^2 = 1 - \frac{4\beta^2}{\left(1+\beta^2\right)^2} = \frac{1+2\beta^2+\beta^4-4\beta^2}{\left(1+\beta^2\right)^2} = \frac{\left(1-\beta^2\right)^2}{\left(1+\beta^2\right)^2}$$

$$\frac{m_1'}{m_2'} = \frac{1}{\sqrt{1-\beta_1^2}} \longrightarrow m_1' = \frac{m_2'}{\sqrt{1-\left(v_1'/c\right)^2}} \longrightarrow m = \frac{m_0}{\sqrt{1-\left(v/c\right)^2}}$$

Rest mass (m_0)

Relativistic mass (m)

Relativistic moment (without relativistic mass !!!)



in K': $a = \frac{F}{m} = const.$

$$m dv_o = F d\tau$$

in K:

$$m dv_o = F d\tau = F\sqrt{1-\frac{v^2}{c^2}}dt$$

$$dv : dv_o = \frac{d\ell}{dt} : \frac{d\ell_o}{d\tau}$$

$$d\ell = d\ell_o\sqrt{1-\frac{v^2}{c^2}}$$ és $$dt = \frac{d\tau}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$dv = dv_o\left(1-\frac{v^2}{c^2}\right)$$

$$\frac{dv}{dt} = \left(1-\frac{v^2}{c^2}\right)^{3/2}\frac{F}{m}$$

$$\frac{1}{\left(1-v^2/c^2\right)^{3/2}}\frac{dv}{dt} = F$$

$$\frac{dp}{dt} = F$$

$$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

Relativistic equation of motion

$$\dot{\vec{p}} = \vec{F}$$ where $$p = \frac{m_o v}{\sqrt{1-\frac{v^2}{c^2}}}$$

Example: the motion of a particle under constant force I.

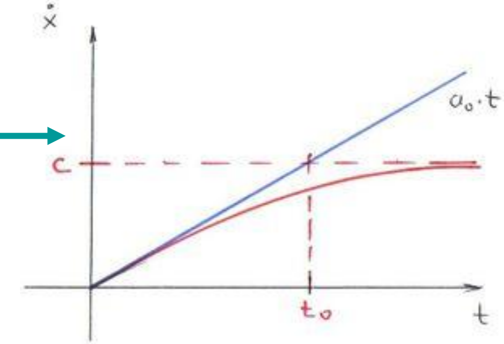
$$m_o \ddot{x} = F_o \qquad\qquad \ddot{x} = \frac{F_o}{m_o} \equiv a_o$$

Example: the motion of a particle under constant force II.

$$\frac{d}{dt}\left[\frac{m_o\dot{x}}{\sqrt{1-(\dot{x}/c)^2}}\right]=F_o \quad\longrightarrow\quad \left[\frac{m_o\dot{x}}{\sqrt{1-(\dot{x}/c)^2}}\right]_0^{\dot{x}}=F_o t$$

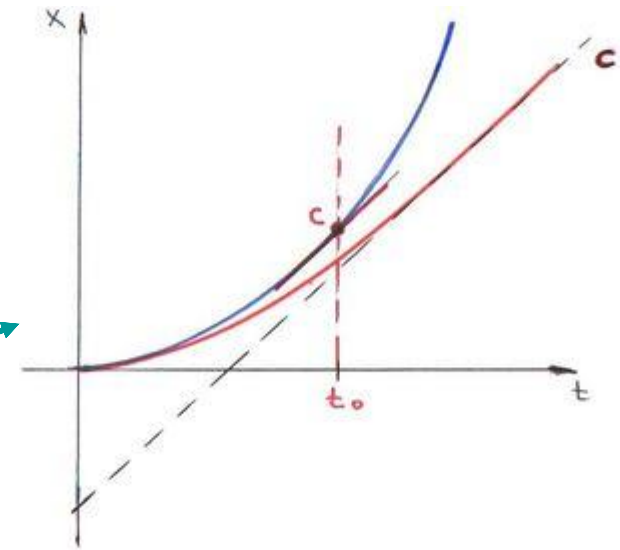
$$\dot{x}=a_o t\sqrt{1-(\dot{x}/c)^2} \quad\longrightarrow\quad \dot{x}=\frac{a_o t}{\sqrt{1+(a_o t/c)^2}}$$

$$\lim_{t\to\infty}\dot{x}=c$$



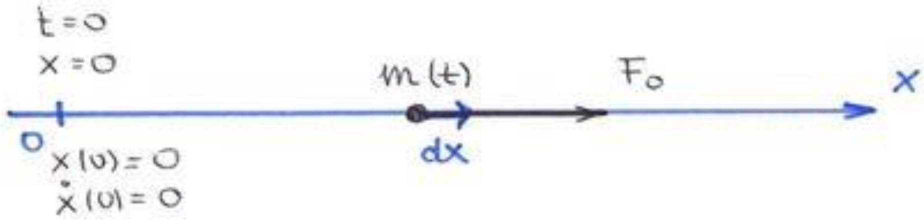
$$[x]_0^t=\int_0^t\frac{a_o t}{\sqrt{1+(a_o t/c)^2}}dt=\left[\frac{c^2}{a_o}\sqrt{1+(a_o t/c)^2}\right]_0^t$$

$$x(t)=\frac{c^2}{a_o}\left(\sqrt{1+(a_o t/c)^2}-1\right)$$



The mass energy equivalence

$$W=\int_0^x F\,dx$$



$$\frac{dp}{dt}=F\rightarrow p=mv=\frac{m_o v}{\sqrt{1-(v/c)^2}}$$

$$W=\int_0^x F\,dx=\int_0^x \frac{dp}{dt}dx=\int_0^p \frac{dx}{dt}dp=\int_0^p v\,dp=\int_0^v v\frac{dp}{dv}dv \longrightarrow W=[pv]_0^v-\int_0^v p\,dv$$

$$W=[pv]_0^v-m_o\int_0^v \frac{v}{\sqrt{1-(v/c)^2}}dv \rightarrow W=m_o\frac{v^2}{\sqrt{1-(v/c)^2}}-m_o\left[-c^2\sqrt{1-(v/c)^2}\right]_0^v$$

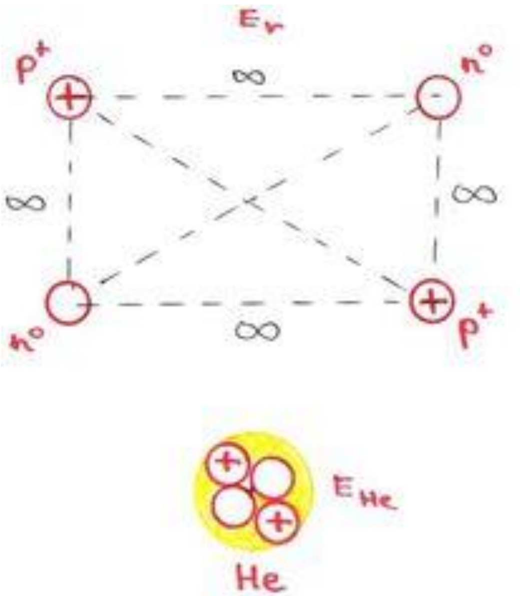
$$W=m_o\frac{v^2}{\sqrt{1-(v/c)^2}}+m_o c^2\sqrt{1-(v/c)^2}-m_o c^2$$

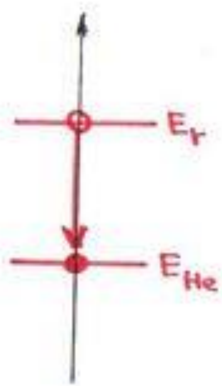
$$\Delta E_k=W=\frac{m_o c^2}{\sqrt{1-(v/c)^2}}-m_o c^2$$

$$E=mc^2$$

$$\frac{v}{c}<<1\rightarrow E_k=\frac{1}{2}mv^2+\ldots$$

The mass defect

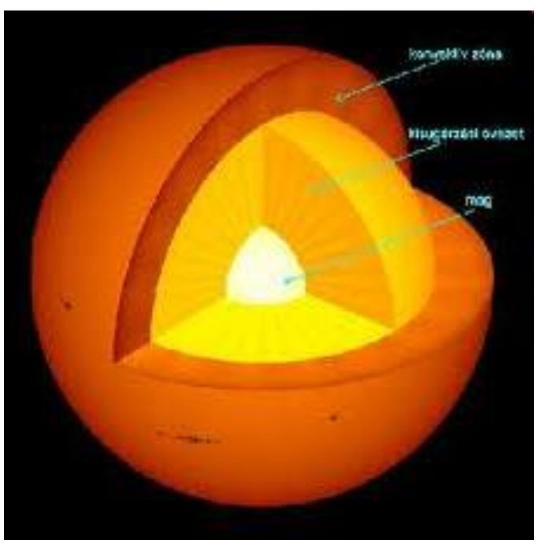


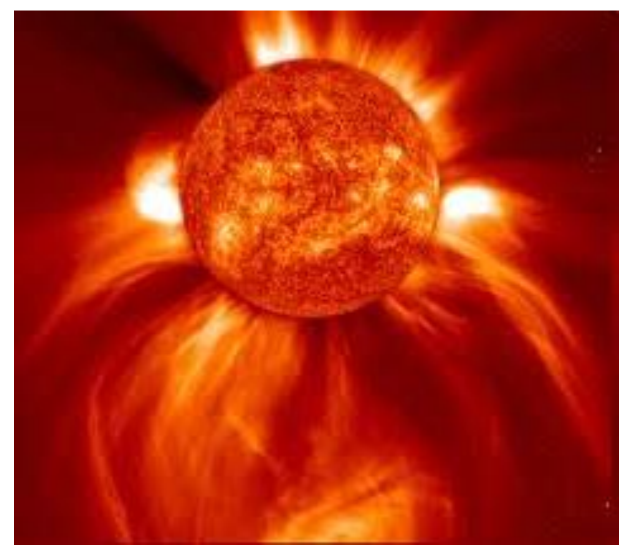


$$2m_p + 2m_n > m_{He}$$

$$\Delta E = 28 MeV$$

$$1\,mol \rightarrow 10^{11}\,J$$

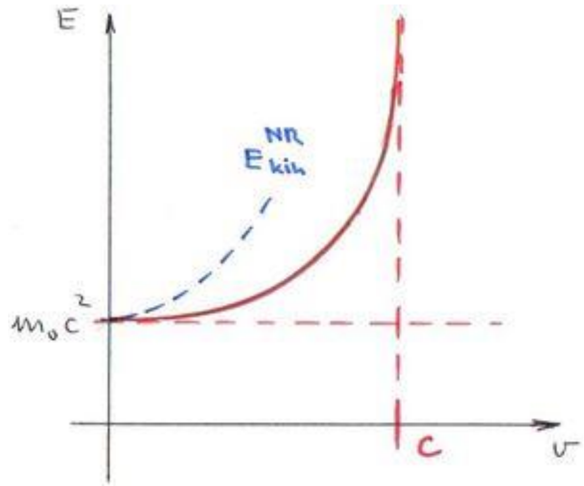




Energy momentum relation

Class. mechanics: $E = \frac{1}{2} m v^2$

$p = mv$ → $E = \frac{p^2}{2m}$

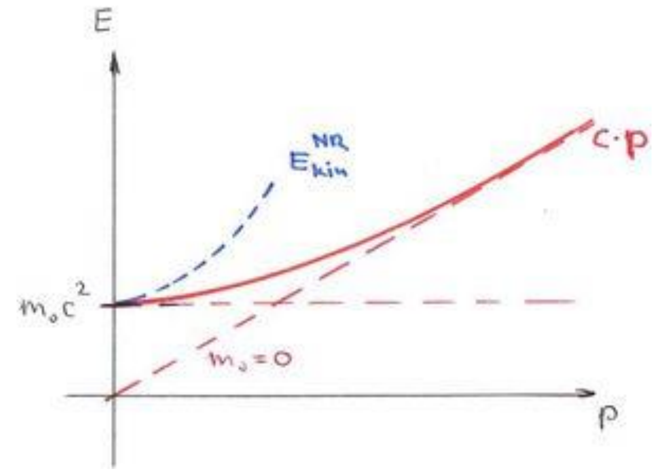


$$E = mc^2 = \frac{m_o c^2}{\sqrt{1-(v/c)^2}}$$

$$p = \frac{m_o v}{\sqrt{1-(v/c)^2}}$$

→ **?**

$$E^2 - p^2c^2 = m_o^2 c^4$$



$$E = \sqrt{m_o^2 c^4 + p^2 c^2}$$ → $$E_k = \sqrt{m_o^2 c^4 + p^2 c^2} - m_o c^2 \approx \frac{p^2}{2m} \quad \text{ha} \quad p << m_o c$$