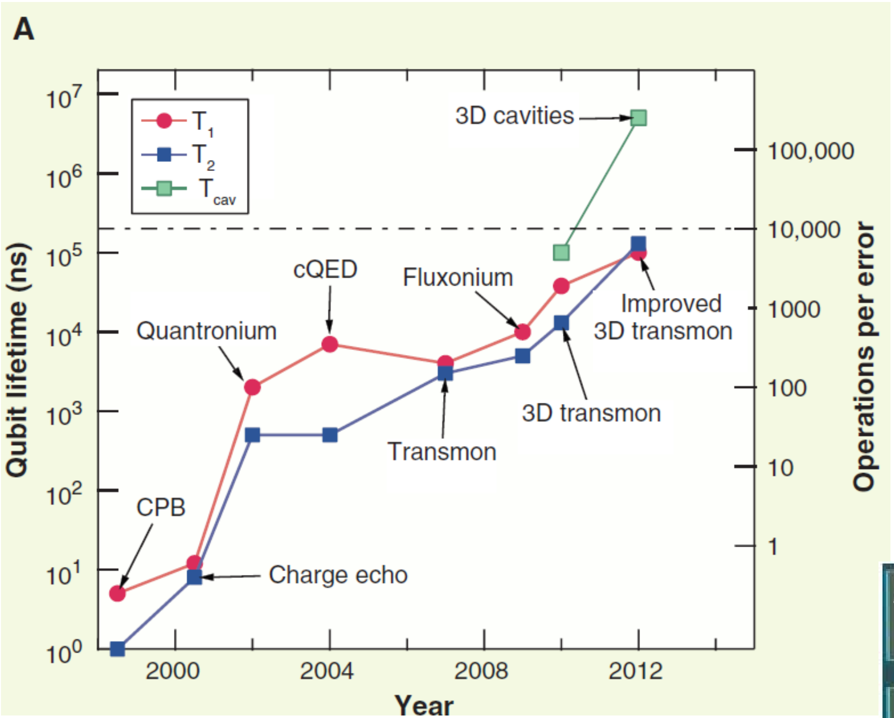
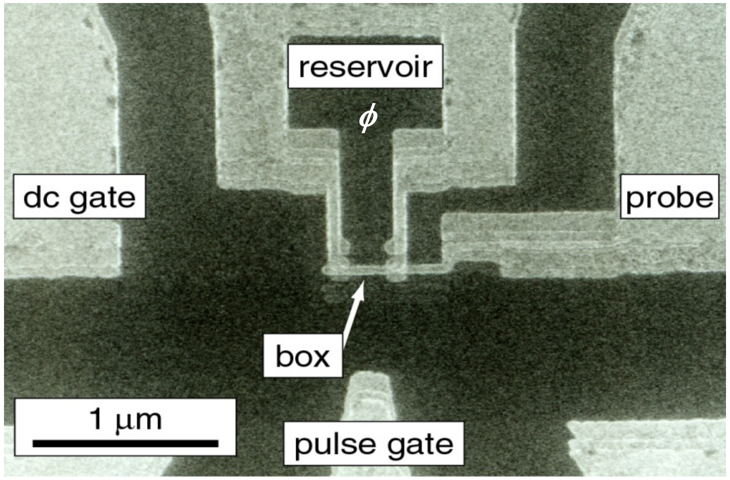
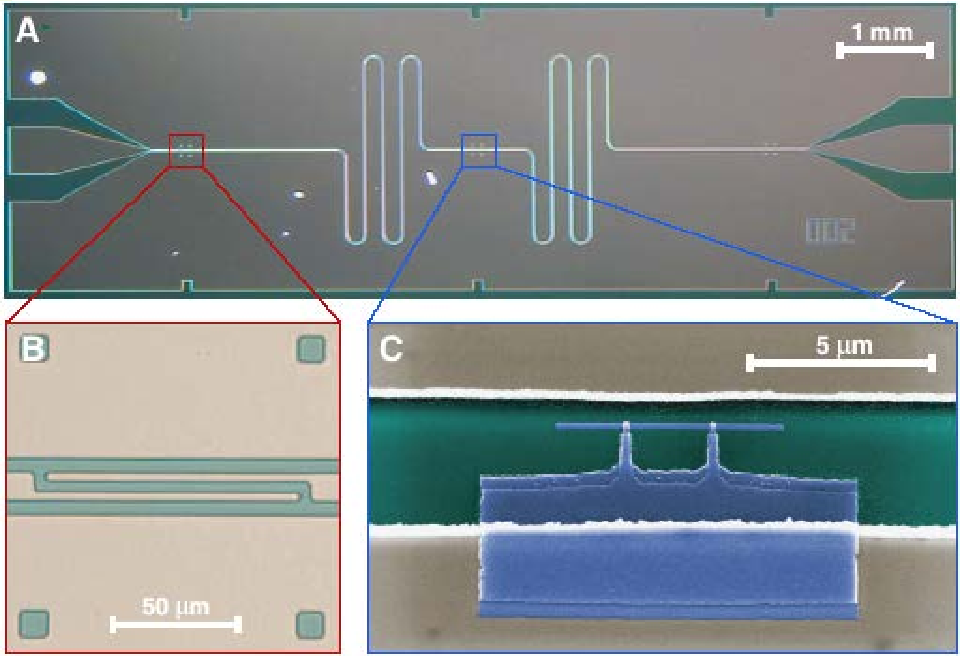
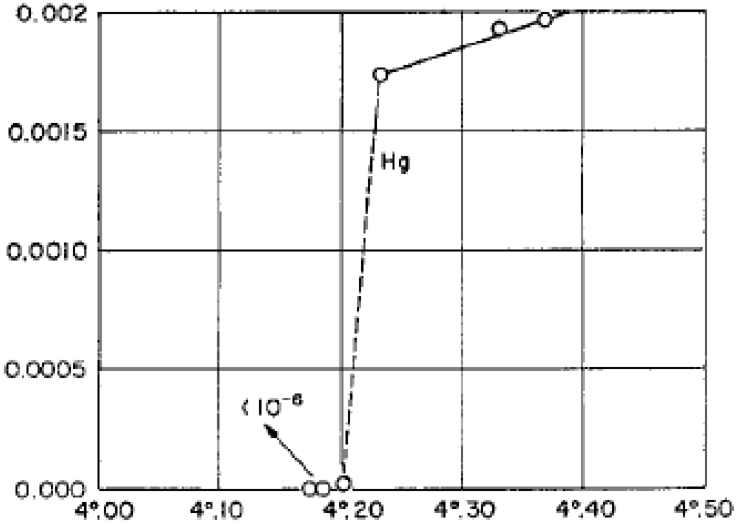
Superconducting qubits



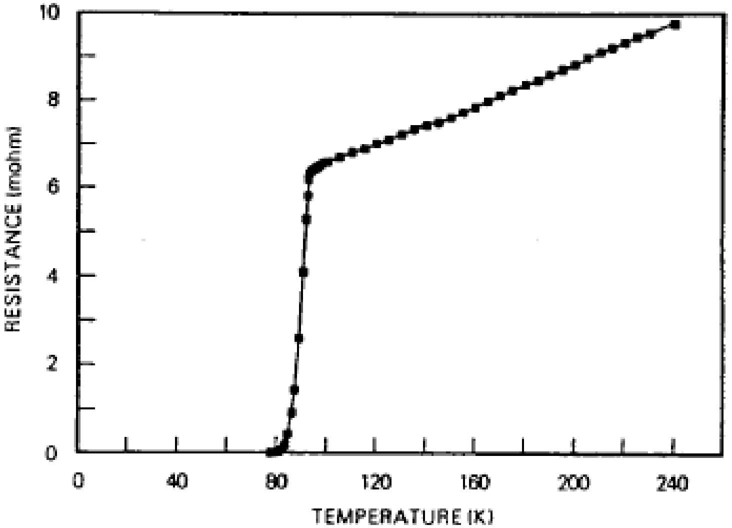




Superconductivity –zero resistance



Comm. Phys. Lab. Univ. Leiden, No. 120b (1911)

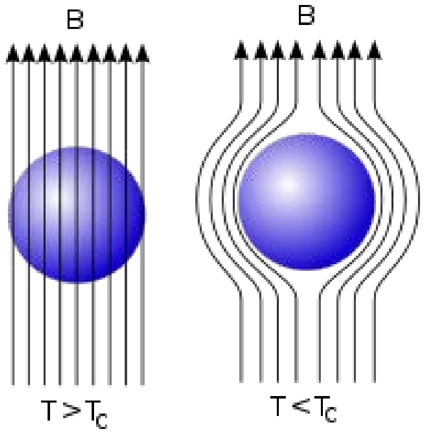


Heike Kamerlingh Onnes
1911: discovery of superconductivity
1913: Nobel prize in Phyics

Below a certain temperature the resistance becomes zero – SC phase

From a high-Tc: Phys. Rev. Lett. 58, 908 (1987).

Superconductivity – Meissner effect

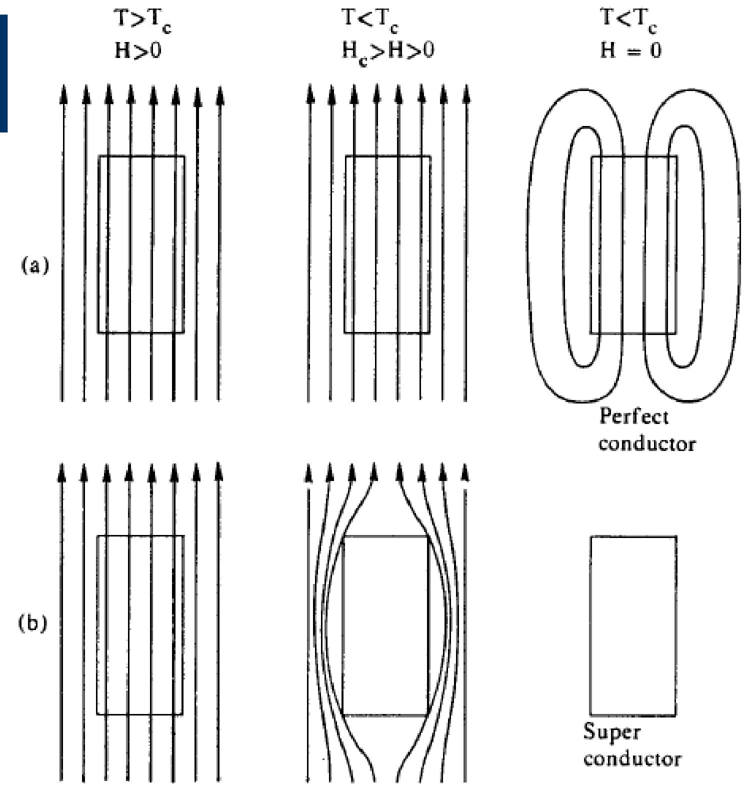


Wikipedia

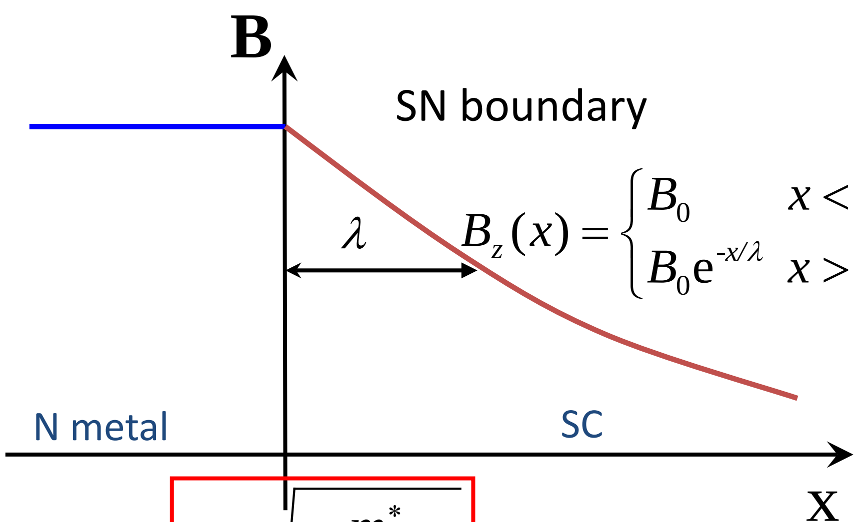
Below a certain temperature the magnetic field is expelled from the sample even in the field cooled case due to screening currents (perfect diamagnet) – SC phase

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = 0$$

$$\mathbf{M} = \chi \mathbf{H} \quad \Rightarrow \quad \chi = -1$$



Penetration depth

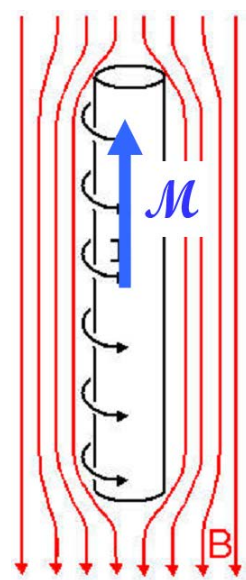


$$B_z(x) = \begin{cases} B_0 & x < 0 \\ B_0 e^{-x/\lambda} & x > 0 \end{cases}$$

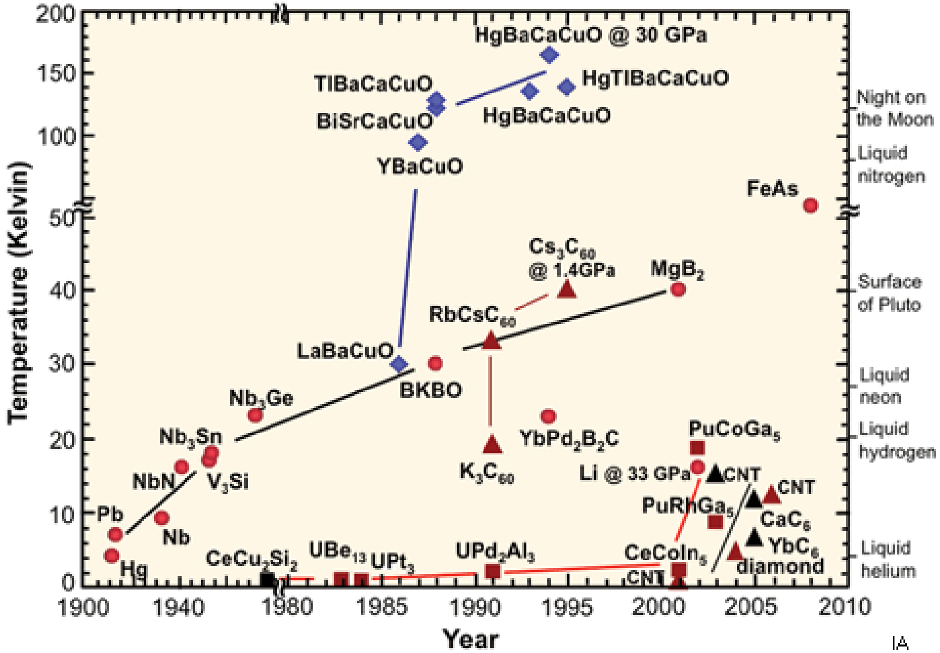
$$\lambda = \sqrt{\frac{m^*}{\mu_0 n_s e^{*2}}}$$

Penetration depth

1933 by
Walter Meissner
Robert Ochsenfeld

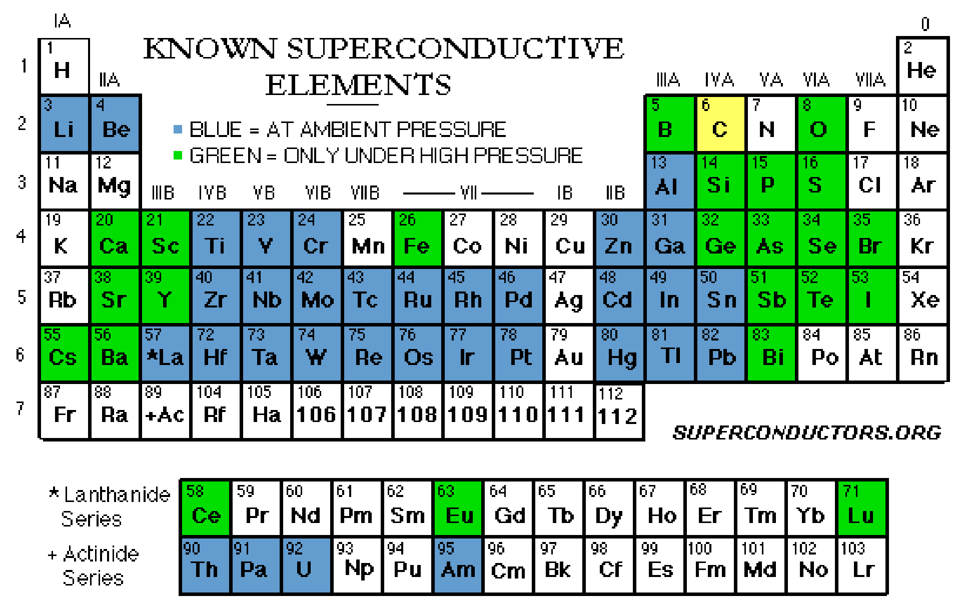


Superconductivity – matierials

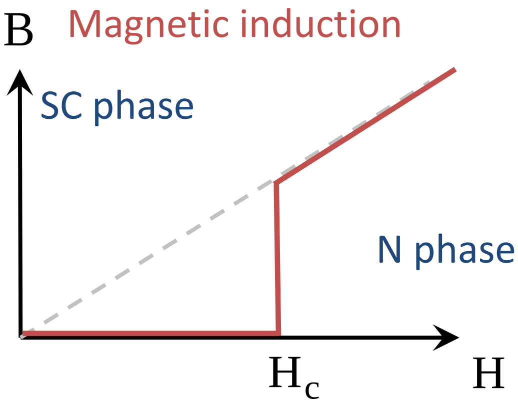


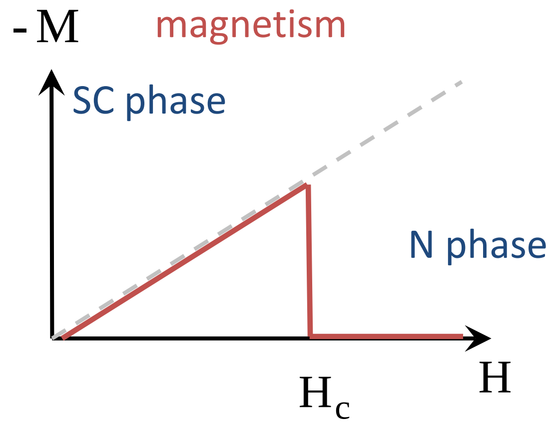
Also claim for RT SC or above
Contraversial yet

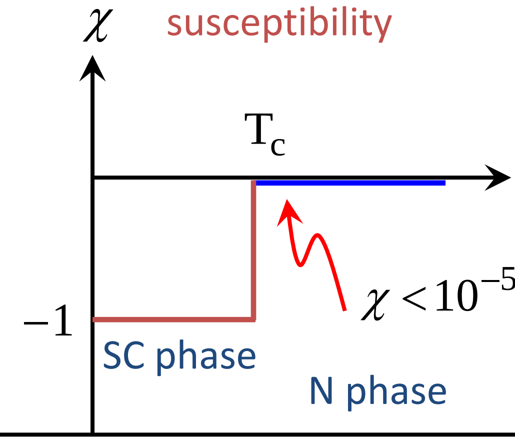
Element	T_c (K)	B_c (mT)	Element	T_c (K)	B_c (mT)
Al	1.18	10.5	Pa	1.4	
Am	0.6		Pb	7.20	80.3
Be	0.03	9.9	Re	1.70	20.1
Cd	0.52	2.8	Rh	3.2×10^{-4}	5×10^{-3}
Ga	1.08	5.9	Ru	0.49	6.9
Hf	0.13	1.3	Sn	3.72	30.5
α-Hg	4.15	41.1	Ta	4.47	82.9
β-Hg	3.95	33.9	Tc	7.8	141
In	3.41	28.2	Th	1.37	16.0
Ir	0.11	1.6	Ti	0.40	5.6
α-La	4.87	80	Tl	2.38	17.6
β-La	6.06	110	U	0.68	10.0
Lu	0.1	35.0	V	5.46	140
Mo	0.92	9.7	W	0.01	0.1
Nb	9.25	206	Zn	0.86	5.4
Os	0.66	7.0	Zr	0.63	4.7



SC – diamagnetism, phase diagram

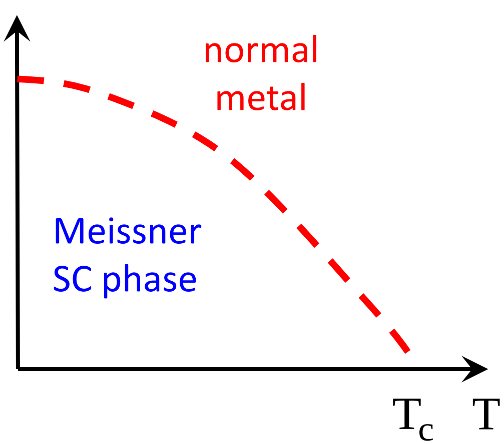




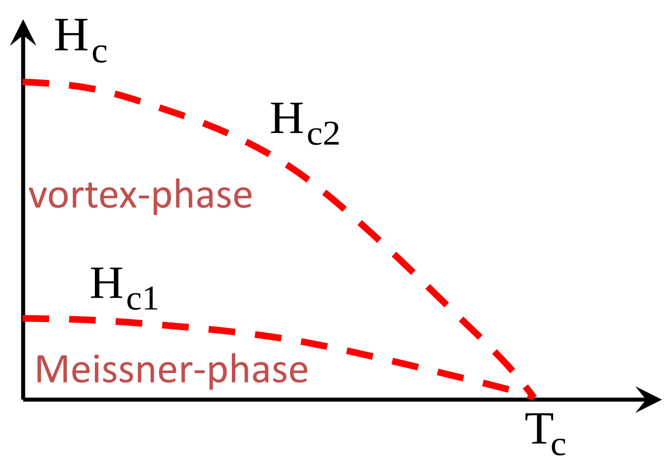


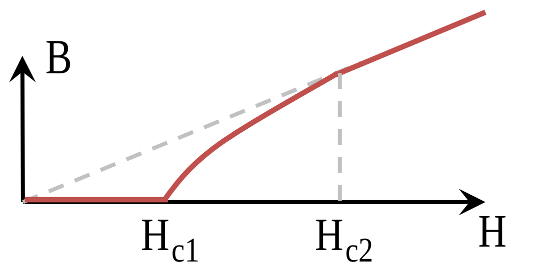
Phase diagram (Type 1)

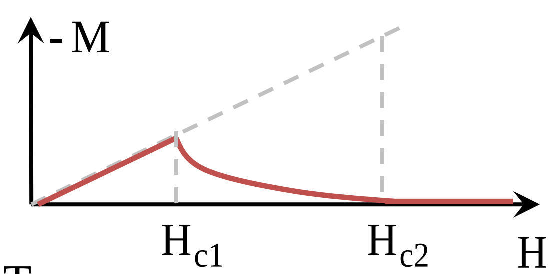
$$H_c(T) \approx H_c(0)\left(1-(T/T_c)^2\right)$$



Phase diagram (Type 2)







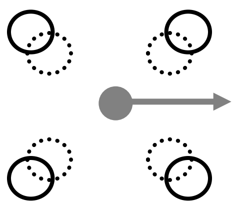
Superconductivity – BCS microscopic theory

- Microscopic BCS theory (Bardeen, Cooper, Schrieffer):

  - The electron-phonon coupling can introduce an attractive interaction between the electrons which may overcome Coulomb repulsion. The phonon mediated attraction is a local interaction, $V_{e\text{-}ph}=-(2\lambda/\nu)\delta(r_1-r_2)$.

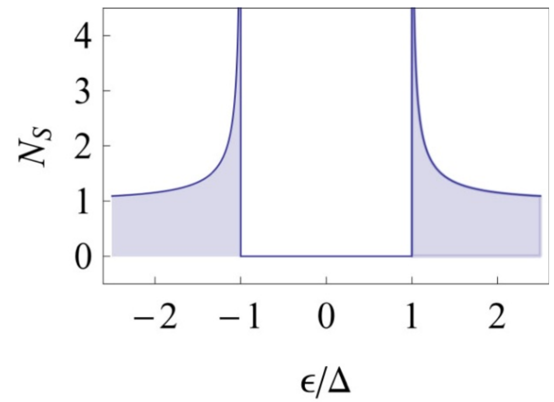
  - The ground state of two electrons with attraction is a bound state with $E=-2\Delta$, where $\Delta=\hbar\omega_D\exp(-1/\lambda)$ is the superconducting energy gap. ($\Delta(T=0)\approx1.76k_BT_C$, approaching T_C it vanishes by $(T_C-T)^{1/2}$.) In the SC state bound states of electron pairs with **k**$\downarrow$ and **-k**$\uparrow$ are formed (Cooper pairs)

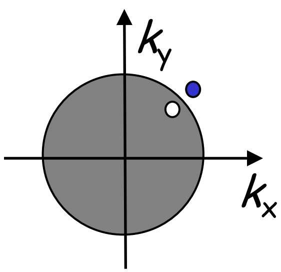
  - The superconducting order parameter is a complex number with the absolute value equal to the gap, and the phase ϕ.



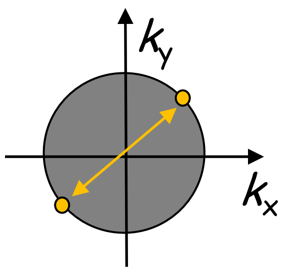
Naive picture: an electron moving in the lattice attracts the ions, which will than attract the next electron passing by.

Energy gap in the excitation spectra

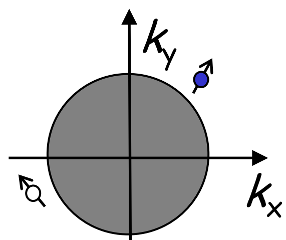




Electron-hole pair excitations in a Fermi liquid



Cooper-pairs
Pairing of electrons on the FS



Bogoliubov quasi-particles: electron hole – excitations of SC

$$\hat{H}_S=\sum_k \xi_k\left(c^+_{k\uparrow}c_{k\uparrow}+c^+_{k\downarrow}c_{k\downarrow}\right)+\underbrace{\Delta c^+_{k\uparrow}c^+_{-k\downarrow}+\Delta^* c_{-k\downarrow}c_{k\uparrow}}_{\text{pairing}},\qquad \Delta=V\left\langle c_{-k\downarrow}c_{k\uparrow}\right\rangle$$

V describes the attractive interaction, ξ is measured from the Fermi energy.

Macroscopic wavefunction

The phase of the macroscopic wave function is important e.g. for Josephson effect

The SC state can be described using a macroscopic wave function:

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})| e^{i\varphi(\mathbf{r})}$$

$$|\psi(\mathbf{r})|^2 = \psi^* \psi = n_s(\mathbf{r})$$

Denisty of SC charge carriers

Current operator and the calculated current (driven by phase gradient):

$$\mathbf{j}_s = \frac{i\hbar e^*}{2m^*}(\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^{*2}}{m^*}\psi^* \psi \, \mathbf{A}$$

$$\longrightarrow \quad \mathbf{j}_s = -\frac{e^*}{m^*}|\psi|^2 (\hbar \nabla \varphi + e^* \mathbf{A})$$

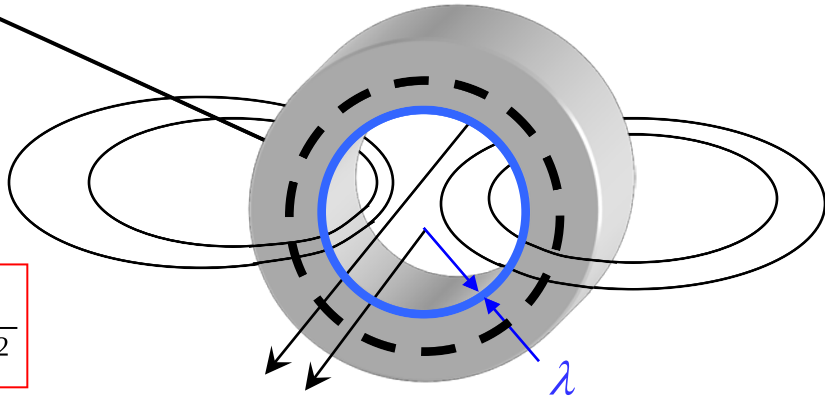
Flux quantization

$$\psi = |\psi| e^{i\varphi} \qquad \mathbf{j}_s = -\frac{e^*}{m^*}|\psi|^2(\hbar\nabla\varphi + e^*\mathbf{A})$$

Integral along the loop – φ should be single valued – same as Bohr-Sommerfeld quantization of momentum

Inside the loop (further than the penetration depth) j_s=0, therefore the integral along contour Γ is zero:

$$\oint_\Gamma \left(\hbar\nabla\varphi + e^*\mathbf{A}\right) d\mathbf{s} = 0$$



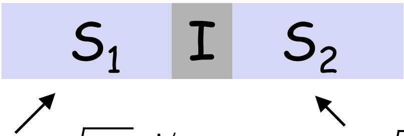
$$\oint_\Gamma \nabla\varphi\, d\mathbf{s} + \frac{e^*}{\hbar}\int_F rot\mathbf{A}\, df = 0 \qquad 2\pi n + \frac{e^*}{\hbar}\Phi = 0$$

The flux threading the Γ contour:

$$\Phi = n\frac{h}{e^*} = n\Phi_0$$

$$\frac{h}{2e} = 2\cdot 10^{-15}\,\frac{\mathrm{T}}{\mathrm{m}^2} = 2\cdot 10^{-7}\,\frac{\mathrm{G}}{\mathrm{cm}^2}$$

Flux quantization



$$\psi_1 = \sqrt{\rho_1}e^{i\phi_1} \qquad \psi_2 = \sqrt{\rho_2}e^{i\phi_2}$$

Josephson effect (traditional approach)

Macroscopic wave functions. $|\psi|^2$~ particle density (ρ)
+ phase difference ($\delta = \phi_2 - \phi_1$)

We apply a voltage of eV on the junction!

$$i\hbar \frac{d\psi_1}{dt} = \frac{2eV}{2}\psi_1 + T\psi_2 \;\Rightarrow\; i\hbar\left(\frac{1}{2\sqrt{\rho_1}}\dot{\rho}_1 e^{i\phi_1} + \sqrt{\rho_1}e^{i\phi_1} i\dot{\phi}_1\right) = \frac{2eV}{2}\sqrt{\rho_1}e^{i\phi_1} + T\sqrt{\rho_2}e^{i\phi_2}$$

$$i\hbar \frac{d\psi_2}{dt} = -\frac{2eV}{2}\psi_2 + T\psi_1 \;\Rightarrow\; \ldots$$

Dividing by $e^{i\phi 1}$ (or $e^{i\phi 2}$) and writing the equations separately for the real and imaginary part:

$$\dot{\rho}_1 = \frac{2T}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta, \quad \dot{\rho}_2 = -\frac{2T}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta$$

$$\dot{\phi}_1 = -\frac{T}{\hbar}\sqrt{\frac{\rho_2}{\rho_1}}\cos\delta - \frac{2eV}{2\hbar}, \quad \dot{\phi}_2 = -\frac{T}{\hbar}\sqrt{\frac{\rho_1}{\rho_2}}\cos\delta + \frac{2eV}{2\hbar}$$

The current is proportional to $d\rho_1/dt = -d\rho_2/dt$:

$$I = I_0 \sin\delta$$

Subtracting the equations for the phase:

$$\dot{\delta} = \frac{2eV}{\hbar} \;\Rightarrow\; \delta(t) = \delta_0 + \frac{2e}{\hbar}\int V(t)dt$$

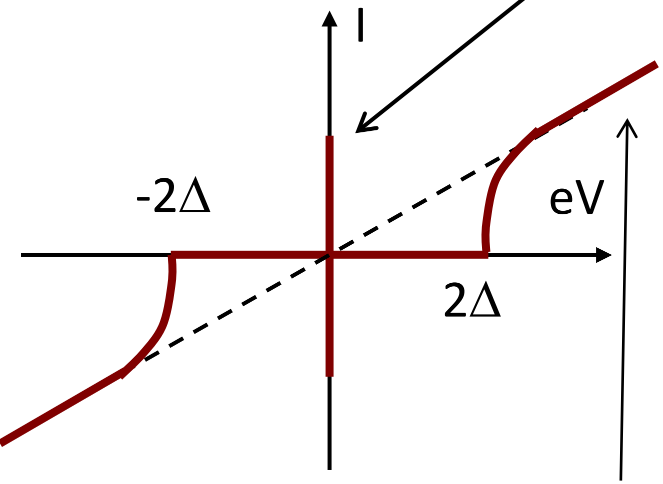
Josephson equations

S_1 I S_2

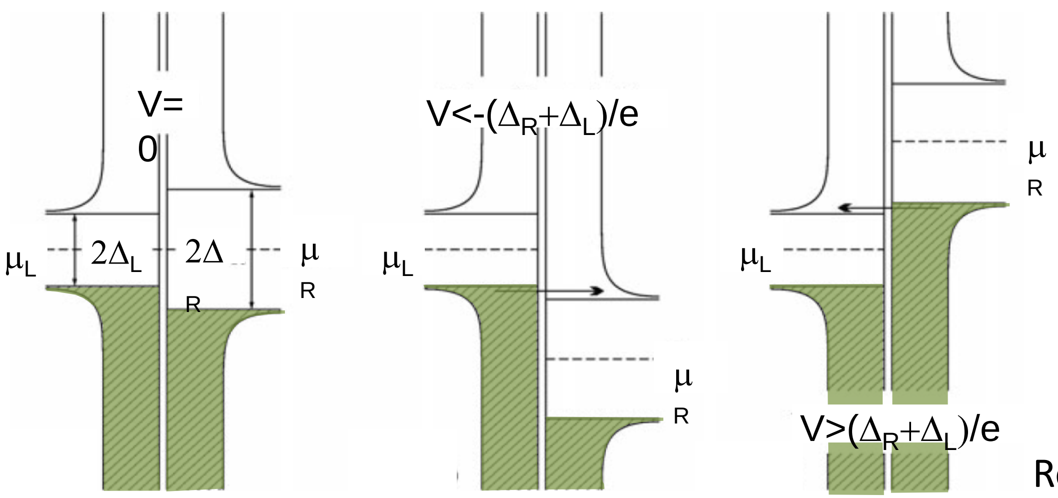
Josephson effect (traditional approach)

Supercurrent → tunelling of Cooper pairs

$$I = I_0 \sin\delta$$



Quasiparticle tunneling



AC-Josephson effect

Applying a constant bias voltage:

$$I = I_0 \sin\left(\delta_0 + \frac{2eVt}{\hbar}\right)$$

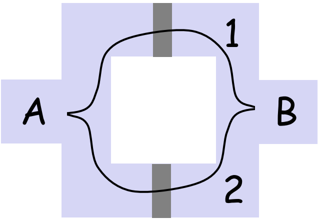
An AC current with ω=2eV/ħ is flowing.
The DC current averages to zero.

Remark: This is for a tunnel junction (could be different for large transmission)

DC SQUID

Superconducting quantum interferometer device (SQUID):

Two Josephson junctions in parallel in a "loop" geometry. The loop encloses a magnetic flux of Φ



The superconductor has a well-defined phase at every position. -> The pase difference between A and B is constant for all trajectories.

$$\left(\phi_B-\phi_A\right)_1=\delta_1+\frac{2e}{\hbar}\int_1 \mathbf{A}d\mathbf{s} \quad = \quad \left(\phi_B-\phi_A\right)_2=\delta_2+\frac{2e}{\hbar}\int_2 \mathbf{A}d\mathbf{s}$$

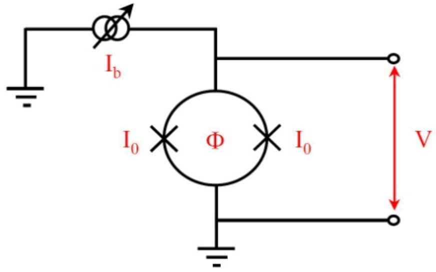
$$\Rightarrow \delta_2-\delta_1=\frac{2e}{\hbar}\oint \mathbf{A}d\mathbf{s}=\frac{2e}{\hbar}\Phi=2\cdot 2\pi\frac{\Phi}{\Phi_0}$$

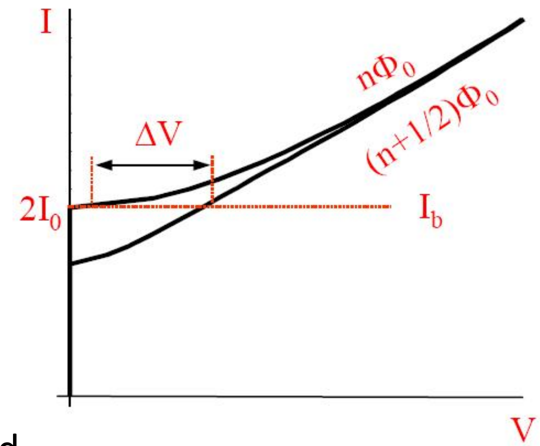
Let us take: $\delta_1=\delta_0+\frac{e}{\hbar}\Phi,\ \ \delta_2=\delta_0-\frac{e}{\hbar}\Phi$

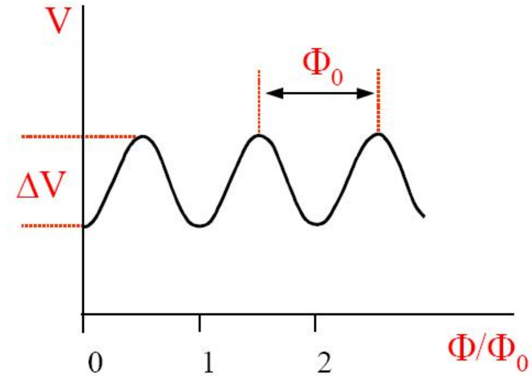
$$I=I_1+I_2=I_0\left[\sin\left(\delta_0+e\Phi/\hbar\right)+\sin\left(\delta_0-e\Phi/\hbar\right)\right]=2I_0\sin\delta_0\cos\left(e\Phi/\hbar\right)$$

The maximal value of the critical current is tuned by the magnetic flux: $I_{\max}=2I_0\left|\cos\left(e\Phi/\hbar\right)\right|$

Here we neglected the self-inductance of the ring



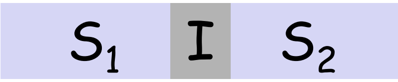




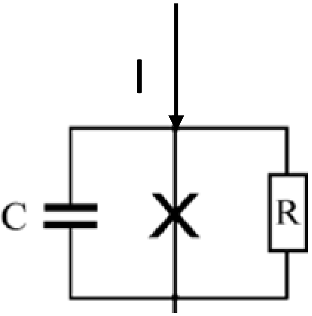
Measurement: current bias at treshold
Measure switching voltage

Source: Wikipedia

RCSJ model



$$\delta = \phi_2 - \phi_1$$



$$\frac{d\delta}{dt} = \frac{2eV}{\hbar}$$

$$I_J = I_c \sin(\delta)$$

$$I_D = C\frac{dV}{dt}$$

$$I_N \leq \frac{V}{R_N}$$

$$I = I_D + I_N + I_J = I_c \sin(\delta) + C\frac{dV}{dt} + \frac{V}{R}$$

$$I = I_c \sin(\delta) + \frac{\hbar C}{2e}\frac{d^2\delta}{dt^2} + \frac{\hbar}{2eR}\frac{d\delta}{dt}$$

$$\frac{\hbar C}{2e}\frac{d^2\delta}{dt^2} + \frac{\hbar}{2eR}\frac{d\delta}{dt} + \frac{d}{d\delta}(I_c(1-\cos(\delta)) - I\delta) = 0$$

$$\left(\frac{\hbar}{2e}\right)^2 C\frac{d^2\delta}{dt^2} + \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R}\frac{d\delta}{dt} + \frac{d}{d\delta}\underbrace{E_{J0}(1-\cos(\delta)) - I\delta}_{U(\delta)} = 0$$

Similar to the motion of a particle in potential. with friction

$$M\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \nabla U(x) = 0$$

$$E_{J0} = I_C\frac{\hbar}{2e}$$

$$U(x) = \frac{kx^2}{2}$$

In case of a harmonic oscillator

$$U(\delta) = E_{J0} - E_{J0}\cos(\delta) - E_{J0}I\delta$$

$$Q = \frac{1}{2\gamma}\sqrt{kM}$$

Quality factor

$$M = \left(\frac{\hbar}{2e}\right)^2 C$$

$$\omega_0^2 = \frac{k}{m}$$

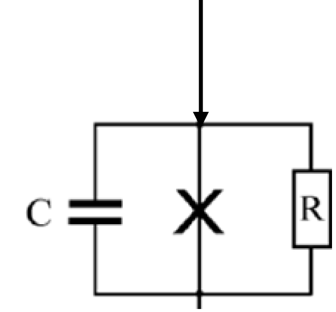
Resonancy frequency without

$$\gamma = \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R}$$

$$\omega_r = \sqrt{\omega_0^2 - \frac{\gamma^2}{4M}}$$

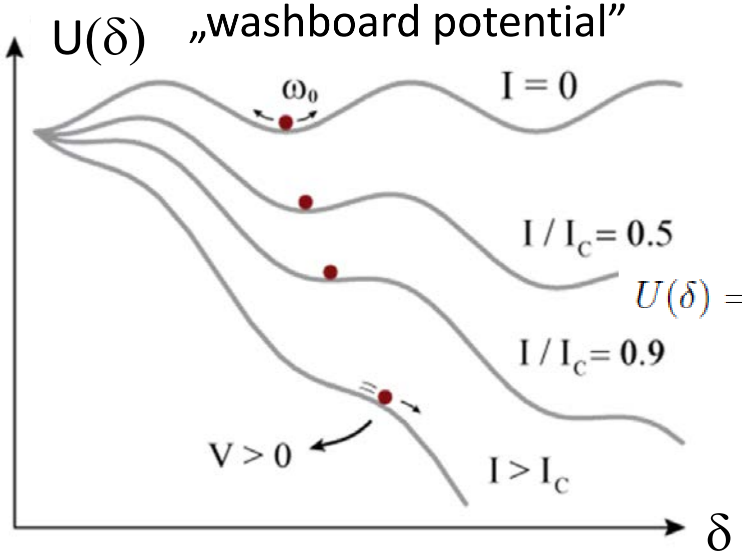
with damping

$$\left(\frac{\hbar}{2e}\right)^2 C\frac{d^2\delta}{dt^2} + \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R}\frac{d\delta}{dt} + \frac{d}{d\delta}E_{J0}(1-\cos(\delta)) - I\delta) = 0$$



$$M\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \nabla U(x) = 0$$

„washboard potential"



The equation decribes the motion of the phase in a potential

If the particle manages to get out of a minimum of the potential, (happens for $I>I_c$, when the potential have an inflection) the phase changes, and DC voltage appears on the junction (Josephson relation)

$$U(\delta) = E_{J0} - E_{J0}\cos(\delta) - E_{J0}I\delta \approx E_{J0}(-1+\delta^2) + E_{J0} - E_{J0}I\delta \approx E_{J0}\delta^2$$

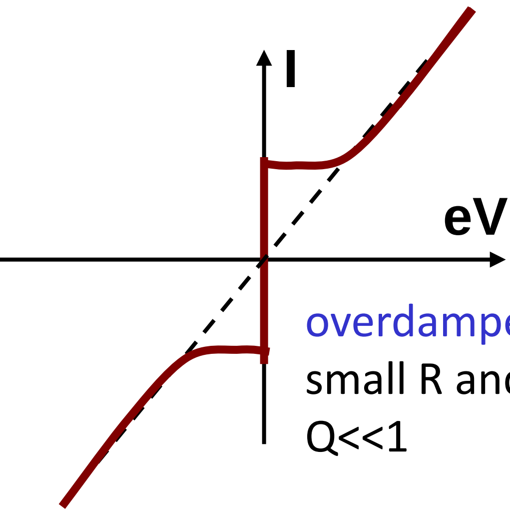
$$\omega_0^2 = \omega_{pl}^2 = \frac{k}{m} = I_c\frac{2e}{\hbar C}$$

For no extenal current and weak damping oscillations in the potential well

$I > I_c$: part of the current must flow as I_N or I_D -> finite junction voltage $|V| > 0$ → time varying I_s → $I_N + I_D$ is varying in time → complicated non-sinusoidal oscillations of I_s

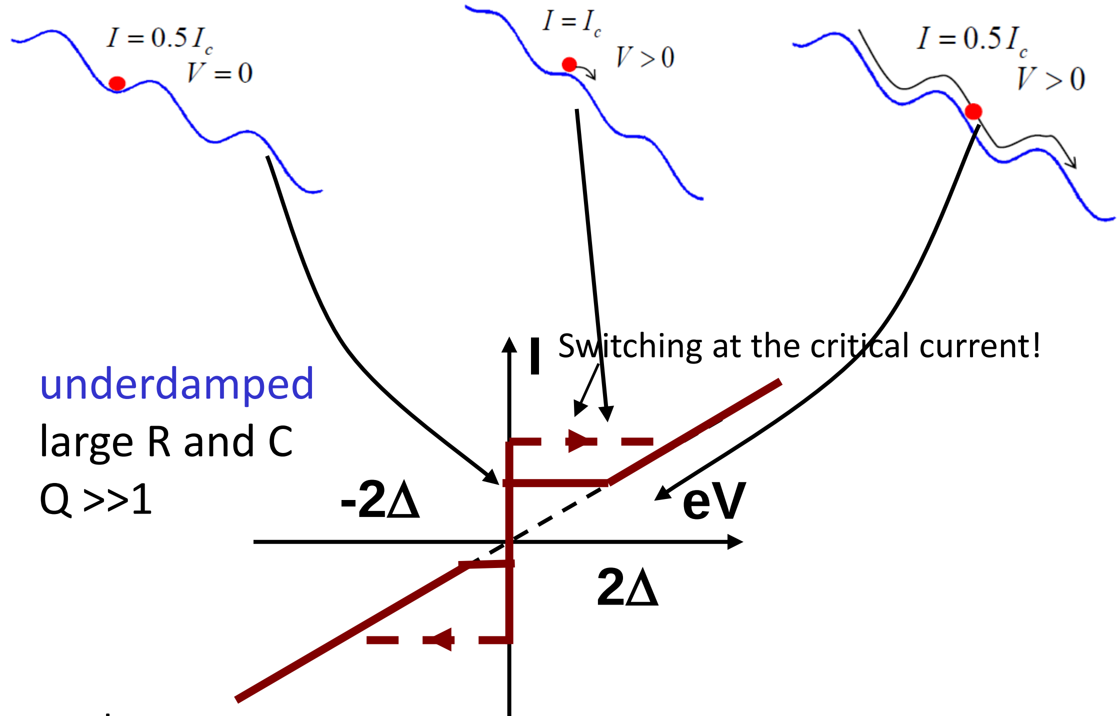
I » Ic – almost all current flows on the resistor →V is ~ constant → sinusoidal oscillation with time average 0

$$Q^2 = \frac{2e}{\hbar} I_c R^2 C$$



overdamped
small R and C
Q<<1

underdamped
large R and C
Q >>1



Overdamped: Q<<1 -- second derivative can be omitted.
Viscous drag dominates – velocity proportional slope of washboard
For $I>I_c$, $\delta=\sin^{-1}(I)$ solution, V=0
If $I>I_c$ it escapes the potential, however, at $I<I_c$ retraped immediately, no hysteresis

$$\langle V(t)\rangle = I_c R \sqrt{\left(\frac{I}{I_c}\right)^2 - 1}$$

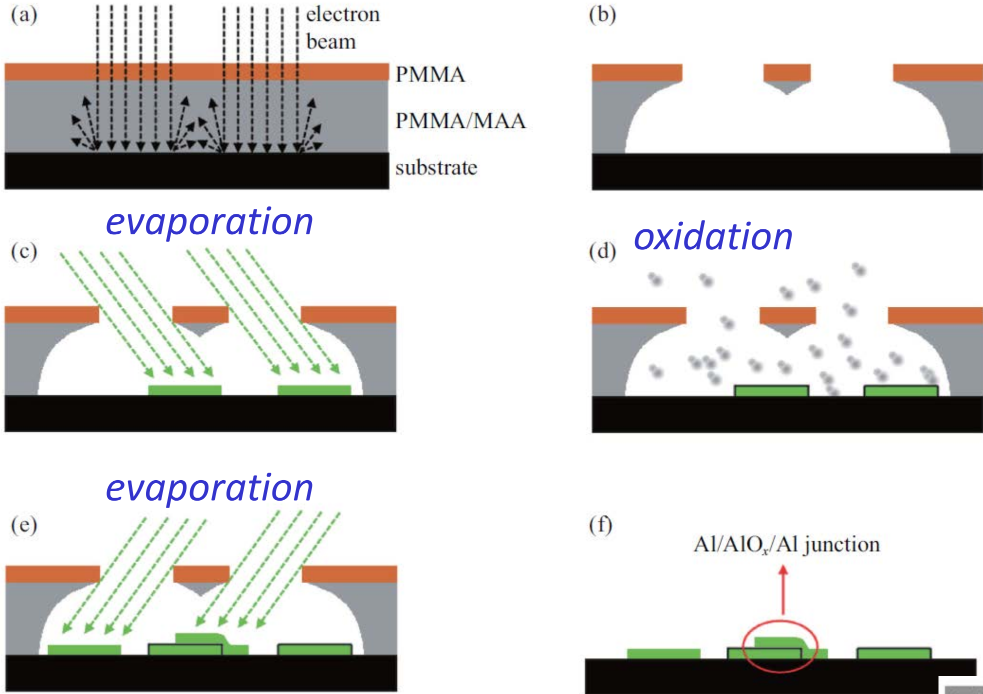
Underdamped Q>>1 -- if I goes over I_c than the inertia is bigger than the damping, it will roll down continuously. Hysteresis – only traps at smaller current when kinetic energy=damping. For zero damping only traps at 0 current.
Large C → shunt oscillating part of V → <I>=0
Down to ω_{RC}

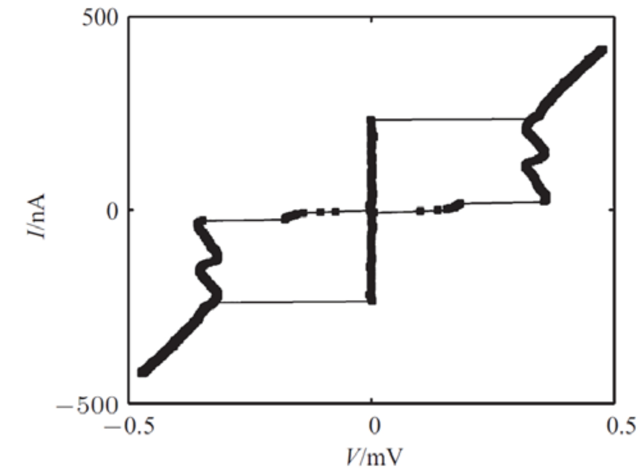
$$\langle V(t)\rangle \sim \frac{\hbar}{eRC} \ll I_c R_N \qquad \langle I\rangle = I_N(\langle V\rangle) = \frac{\langle V\rangle}{R_N}$$

Sinusoidal supercurrent → averages to zero
Normal current flows → hysteretic behaviour

Josephson junctions
Fabrication

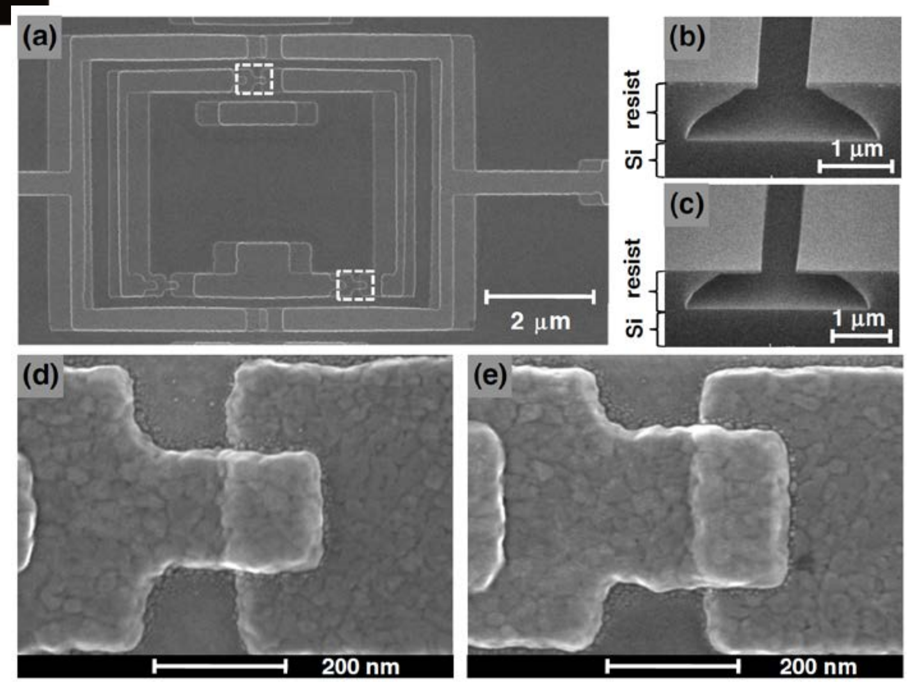
Fabrication: e-beam lithography and shadow evaporation





Real JJ measurement

Shadow, not used structures remain



T Niemczyk et al., Supercond. Sci. Technol. 22, 034009 (2009)
Wu Yu-Lin et al., Chin. Phys. B. 22, 060309(2013)

RCSJ model
Thermal or quantum escape

Thermal escape: Due to the phase motion at higher temperature and/or larger current particle can escape:

$$\Gamma_t = \omega_{pl} \frac{\left(1-(I/I_c)^2\right)^{1/4}}{2\pi} \exp\left(-\frac{U_0}{k_B T}\right)$$

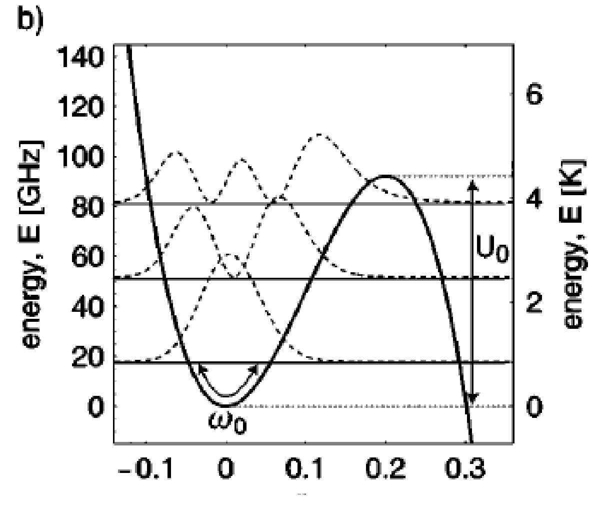
Here $U_0(E_{J0}, I/I_c)$

This is a stochastic process, the switching current varies.

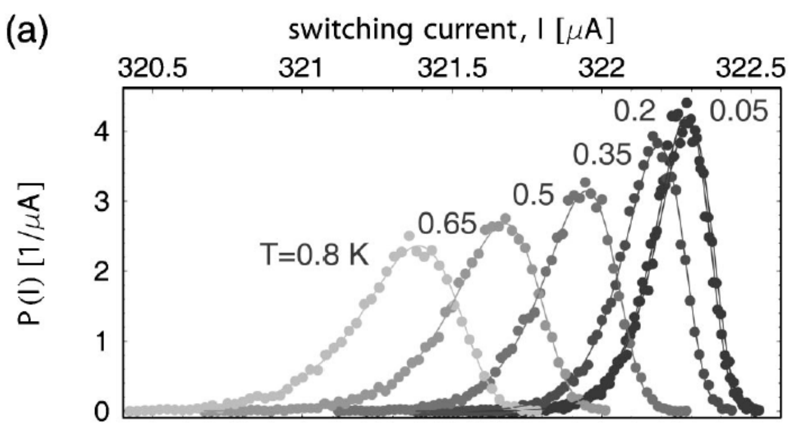
The distribution of I_c can be known.

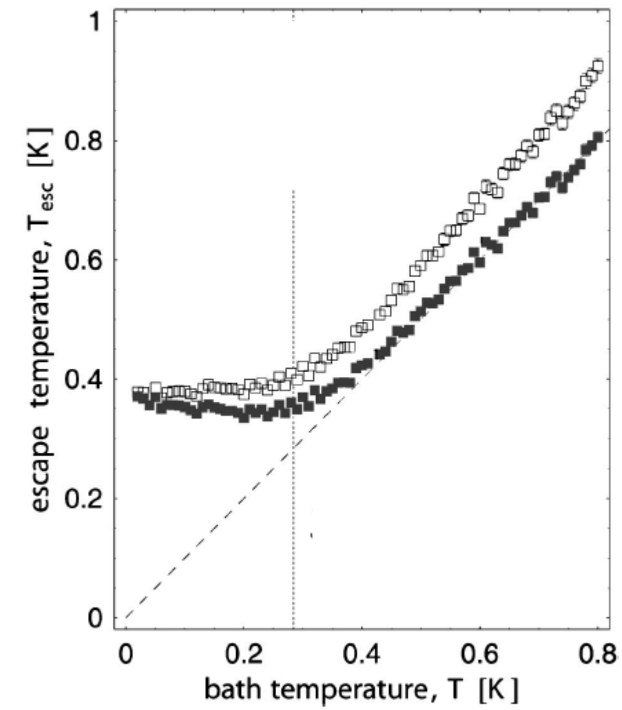
For low temperatures the phase particle can tunnel out: **macroscopic quantum tunneling** – finite voltage appears on the junctions (if it is underdamped enough)

$$\Gamma_q = A \exp\left(-\frac{U_0}{\hbar\omega_{pl}} B\right) \qquad B\sim 1$$

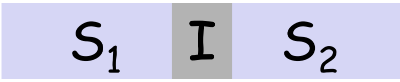


Nb JJs
At high T, thermal escape, at low T, macroscopic quantum tunneling dominates





Walfraff et al., Rev. of Sc. Instr, 7, 3740 (2003)

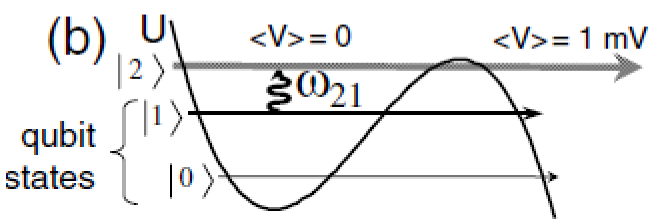


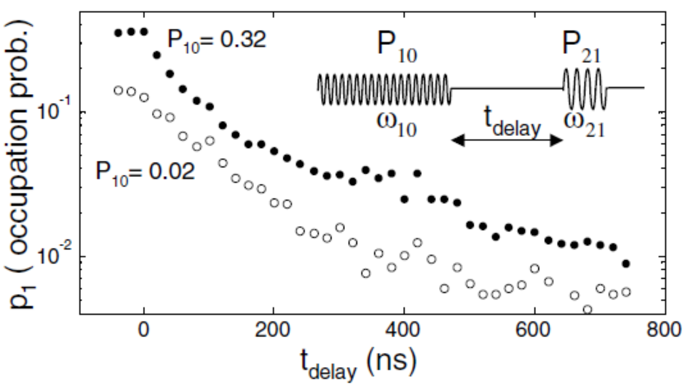
$$\delta = \phi_2 - \phi_1$$

Phase qubit – current biased JJ

By tuning the potential of a single Josephson junction (washboard potential) , such, that it is asymmetric, close to the critical current

If, $I \approx I_c$ and it only houses 2-3 levels, the lowest two forms a qubit





Operation:
Anharmonic oscillator, qubit states are separated
Make transitions with microwave pulses ω_{01} and prepare state – AC current pulses
Readout: A pulse with frequency ω_{12} is applied.
As the barrier for state 2 is small, the state can tunnel out → changing phase → finite voltage appears

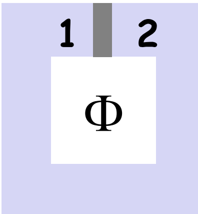
If the qubit was in state 1 it will be resonant for the readout pulse ω_{12}, if in state 0 not.

For superpositions, it will tunnel out with a probability corresponding to state 1. To measure these probabilities, multiple measurements on the qubit prepared in the same way is needed.

T_1 measurement
Populate state 1 and wait before readout
The measured signal will decay as the waiting time increased – measure of T_1

J. Martinis et al., Phys. Rev. Lett., 89, 117901 (2002)

RF SQUID



L loop inductance

Similarly to DC squid the phase difference equals the flux inside the loop

$$\frac{\Phi_0}{2\pi}(\phi_1 - \phi_2) = \int_1^2 \vec{A}d\vec{l} \qquad \delta = \phi_1 - \phi_2 = \frac{2\pi\Phi}{\Phi_0}$$

The flux inside the loop will be partially screened by and induced circulating current

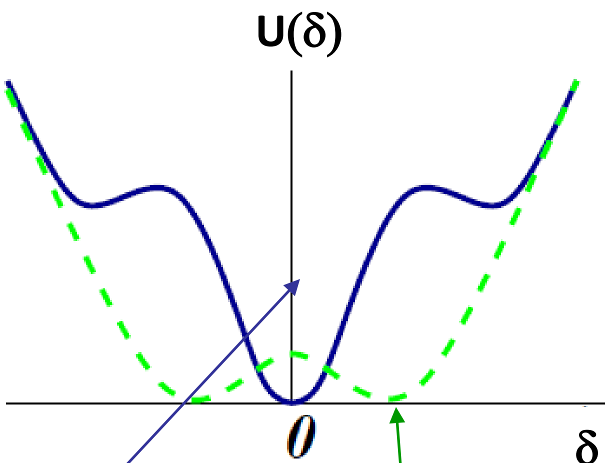
$$\Phi = \Phi_{ext} - LI_{circ}$$

Equation of motion, with the calculated current:

$$0 = C\frac{\Phi_0}{2\pi}\frac{d^2\delta}{dt^2} + \frac{\Phi_0}{2\pi}\frac{1}{R}\frac{d\delta}{dt} - I_c\sin(\delta) - \frac{1}{L}(\delta - \Phi_{ext})$$

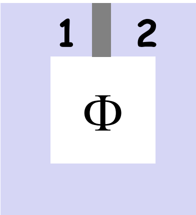
$$U(\delta) = \frac{\Phi_0}{2\pi}I_c\left(1 - \cos(\delta)\right) + \frac{1}{2L}\left(\frac{\hbar}{2e}\delta - \Phi_{ext}\right)^2$$

Potential – junction + magnetic energy



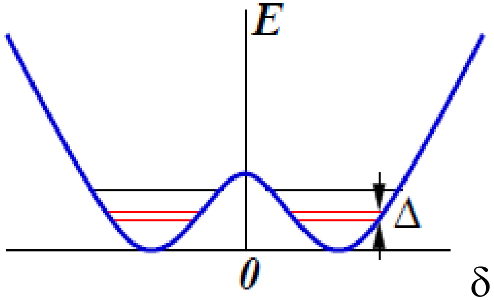
At half quanta the circulating current changes sign and a flux quanta jumps into the loop

For half integer quantum, two minima:
two persistent current states, circulating in different direction



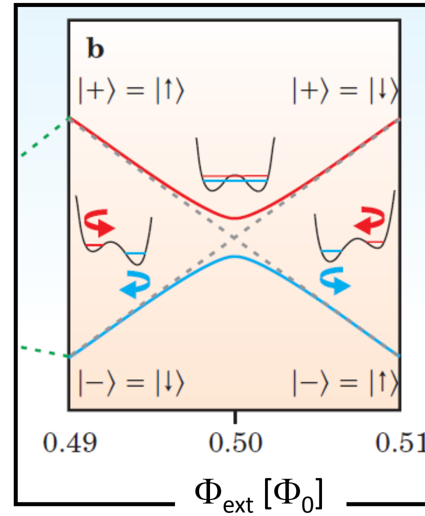
Flux qubit

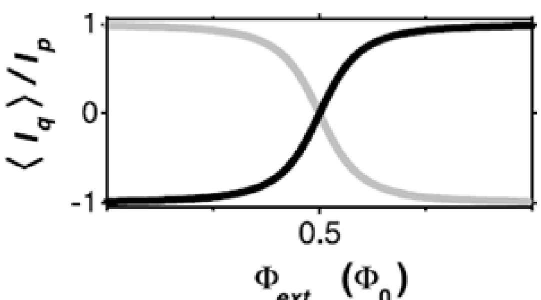
$$U(\delta) = \frac{\Phi_0}{2\pi} I_c \left(1 - \cos(\delta)\right) + \frac{1}{2L}\left(\frac{\hbar}{2e}\delta - \Phi_{ext}\right)^2$$



potential for half integer flux bias

Two wells → two levels – for symmetric potential degenerate flux states
The two states correspond to oppositely circulating persistent current
If tunneling is possible between the two wells (Δ), states hybridize and split up and the macroscopic tunneling determines the separation





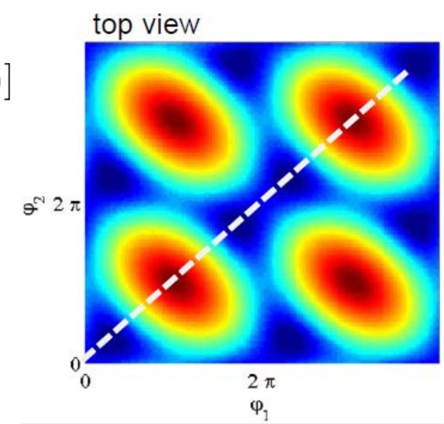
The expectation value of the current as a function of the flux
Away from half flux quanta, pure flux states

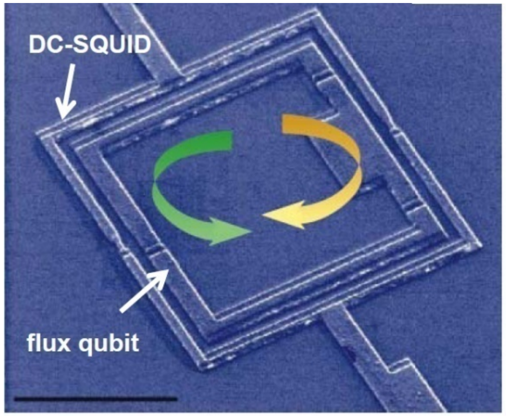
$$U = -E_j\left[\cos(\varphi_1) + \cos(\varphi_2) + \alpha\cos(\varphi_1 - \varphi_2 - 2\pi\Phi_{ext}/\Phi_0)\right]$$

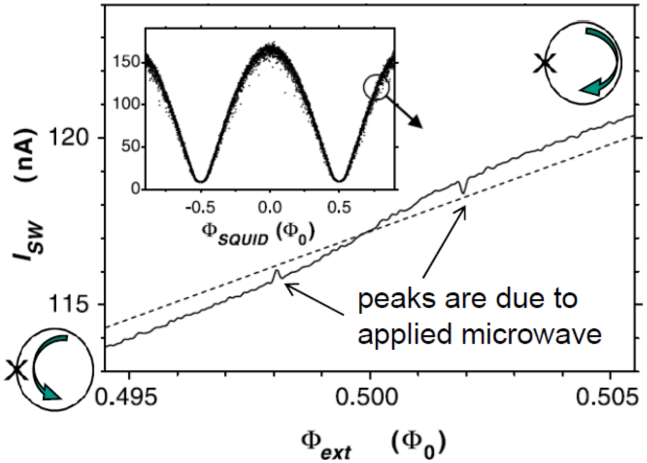
Hard to fabricate, big loop is needed for inductance matching (large noise pickup possible big decoherence)→ 3 JJ-s qubit (effectively the same).

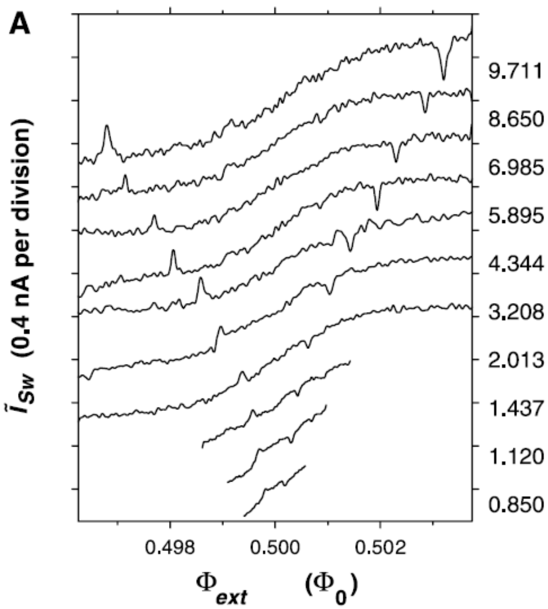
$$\varphi_1 + \varphi_2 + \varphi_3 + 2\pi\Phi/\Phi_0 = 2\pi n$$

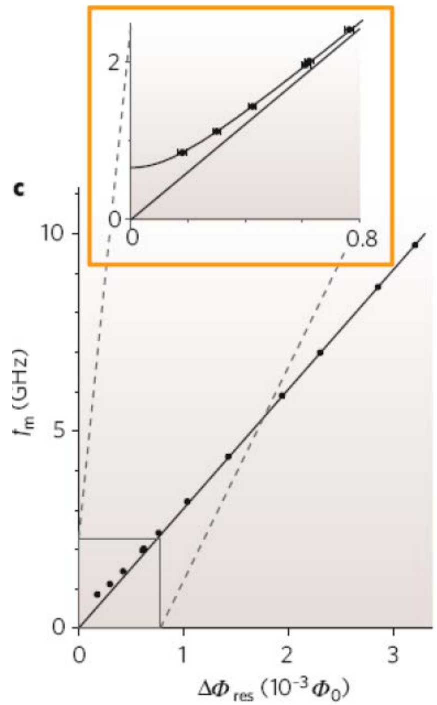
the potential is parabolic on the white intersection α tunes the macroscopic quantum tunneling.







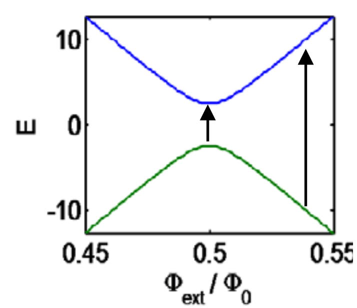


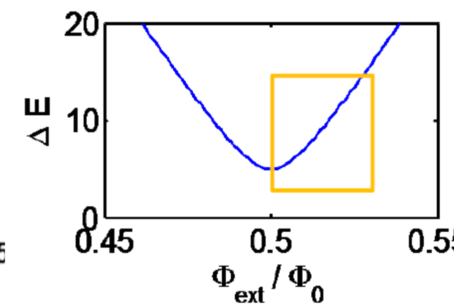


Readout – by DC squid measuring the opposite supercurrents in the qubit. Measurement with squid – measure the switching currents

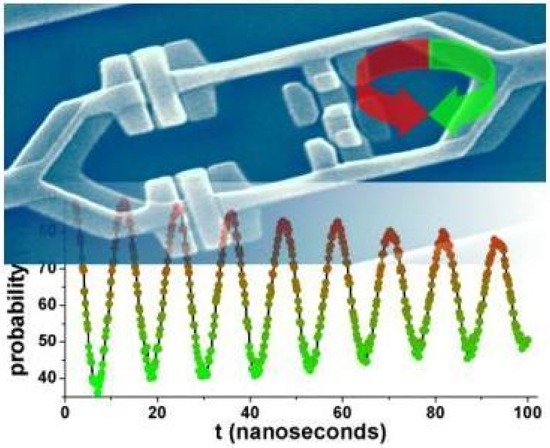
During the sweeping of the magnetic field, microwave applied. transition causes supercurrent flowing opposite direction → change in field measured by squid (change in switching current)

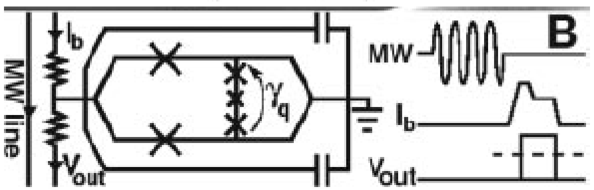
- the resonance seen for different frequencies at different flux points.
- peaks indicate switching between flux states
-the excitation spectra is nicely reproduced
- at zero detuning the avoided crossing of the two levels is extrapolated

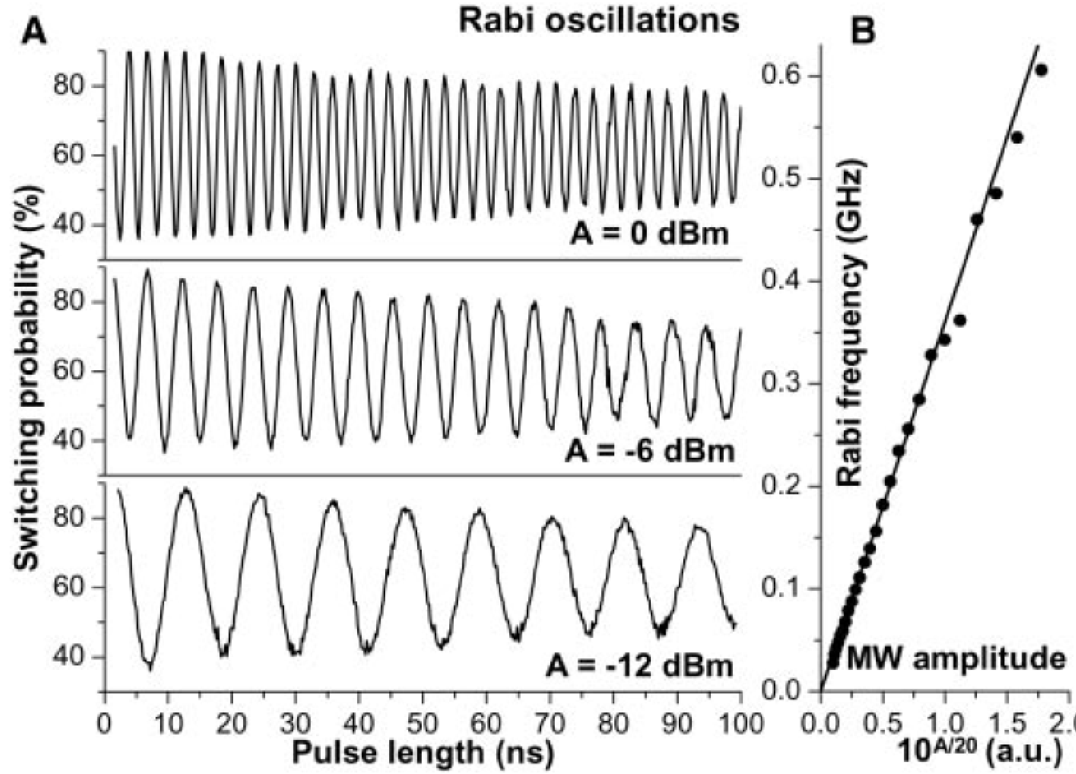




Caspar H. van der Wal et al., Science 290, 773 (2000)







Other design: squid is directly coupled to achieve higher sensitivity

T_1~900ns, T_2~20-30 ns

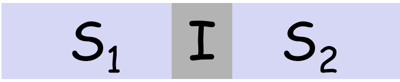
Dephasing: likely flux noise → changes the qubit frequency randomly

Ideal opeation would be at $\Phi=\pi$, however this did not work for this devices.

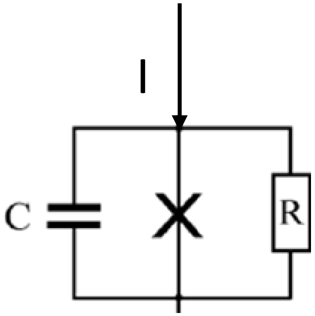
There $\delta E \sim \Phi^2$, less sensitive to flux noise →sweet spot

I. Chiorescu et al., Science 299, 1869 (2003)

RCSJ model – energy terms



$$\delta = \phi_2 - \phi_1$$



Neglect damping, SC state, R=0

E=K+U

$$K = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}C\left(\frac{\hbar}{2e}\right)^2\left(\frac{d\delta}{dt}\right)^2$$

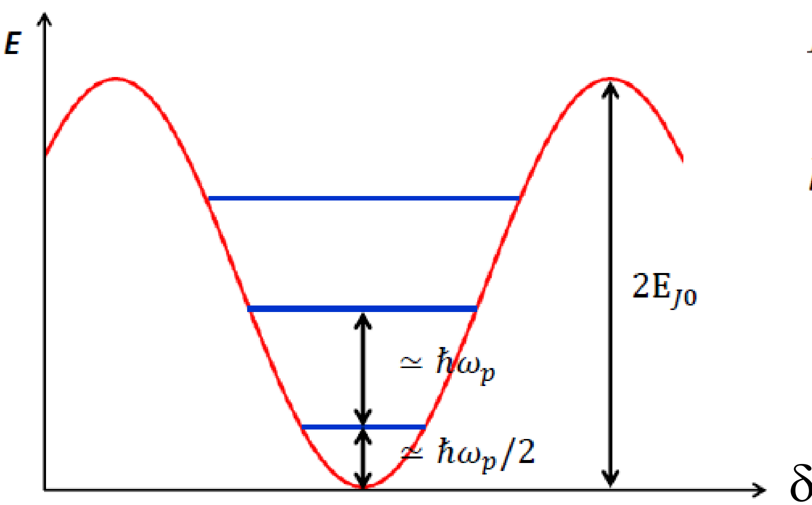
or using

$$M = \frac{\hbar C}{2e}$$

$$U = E_{J0}(1-\cos(\delta))$$

$$E_{J0} = I_C\frac{\hbar}{2e}$$

Josephson energy



$$E_C = \frac{e^2}{2C}$$

Charging energy

$$\hbar\omega_{pl} = \sqrt{8E_C E_{J0}}$$

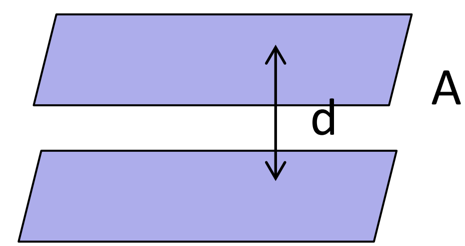
$$\hbar\omega_{pl} \ll E_{J0}$$

Classical treatment valid:
Oscillation only in the bottom of the potential well

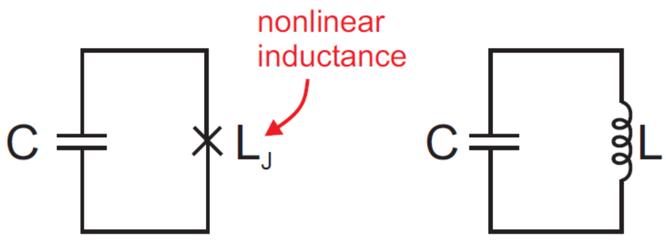
$$\downarrow$$

$$E_C \ll E_{J0}$$

Homework: How to enter the quantum regime? Investigate scaling with the junction area. Suppose d=1nm, ε=10, I_c= 100 A/cm^2. What is the temperature range where the measurement should be done?



Energy terms
Why JJ, not a simple inductor?



LC - oscillator

$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 \qquad V = L\frac{dI}{dt} \qquad \Phi = LI$$

$$I = C\frac{dV}{dt} \qquad Q = CV$$

Josephson junction

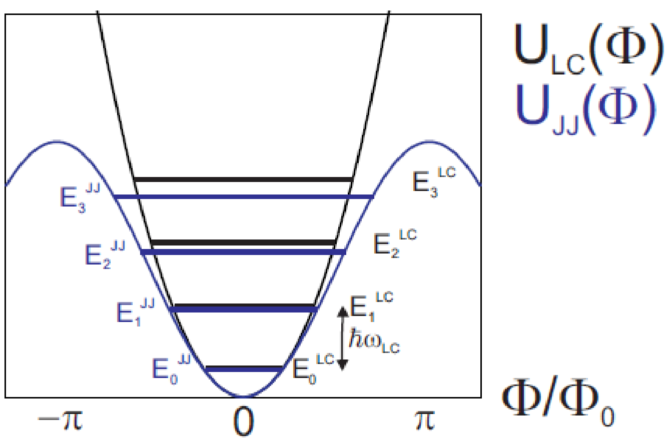
$$I = I_c \sin(\delta) = I_c \sin(2\pi\Phi/\Phi_0)$$

$$\frac{dI}{dt} = L_J^{-1}V \qquad L_J^{-1} = \frac{2\pi I_c}{\Phi_0}\cos(2\pi\Phi/\Phi_0)$$

for small Φ $\qquad L_J = \frac{\Phi_0}{2\pi I_c} \qquad I \simeq \frac{\Phi}{L_J}$

$$H = \frac{1}{2}CV^2 + \frac{1}{2}L_J I^2 = \frac{Q^2}{2C} + \frac{1}{2L_J}\Phi^2$$

Josephson junctions is a *non-linear inductance:* the energy spectra is anharmonic. The qubit can be separated from excited states



Why else superconductors?

-Single non-degenerate macroscopic ground state
- no low energy excitations

Quantization of EM circuits

$$H = E + K = \frac{p^2}{2m} + \frac{1}{2} m\omega_{pl}^2 = \frac{Q^2}{2C} + \frac{1}{2L_J}\Phi^2$$ Energy of a harmonic oscillator

$$H = E + K = \frac{1}{2} C \left(\frac{\hbar}{2e}\right)^2 \left(\frac{d\delta}{dt}\right)^2 + E_{J0}(1 - \cos(\delta))$$ JJ: nonlinear Harmonic oscillator

$$p = mv = C\left(\frac{\hbar}{2e}\right)^2 \frac{d\delta}{dt}$$ Knowing the mass, identify momentum $M = \left(\frac{\hbar}{2e}\right)^2 C$

Quantization – using the momentum and position operators

$$\hat{p}_\delta = \frac{\hbar}{i}\frac{d}{d\delta} \qquad \hat{x} = \hat{\delta} \longrightarrow \left[\hat{\delta}, \hat{p}_\delta\right] = i\hbar$$

$$\boxed{\hat{H} = -4E_c \frac{d^2}{d\delta^2} + E_{J0}(1 - \cos(\delta))}$$ Quantized JJ Hamiltonian

Charge, Cooper pair number, flux basis

Homework:

$$Q = C\frac{\hbar}{2e}\frac{d\delta}{dt}$$

$$\downarrow$$

$$\hat{Q} = -2ei\frac{d}{d\delta}$$

charge

$$\hat{N} = -i\frac{d}{d\delta}$$

CP number

$$\hat{\Phi} = \frac{\hbar}{2e}\hat{\delta}$$

Flux

$$\longrightarrow \left[\hat{\delta}, \hat{N}\right] = i$$

$$\left[\hat{\Phi}, \hat{Q}\right] = i\hbar$$

$$\boxed{\Delta N \Delta \delta \geq 1}$$

Either phase (flux) or number of Cooper pairs (charge) is well defined
→ Phase or charge regime

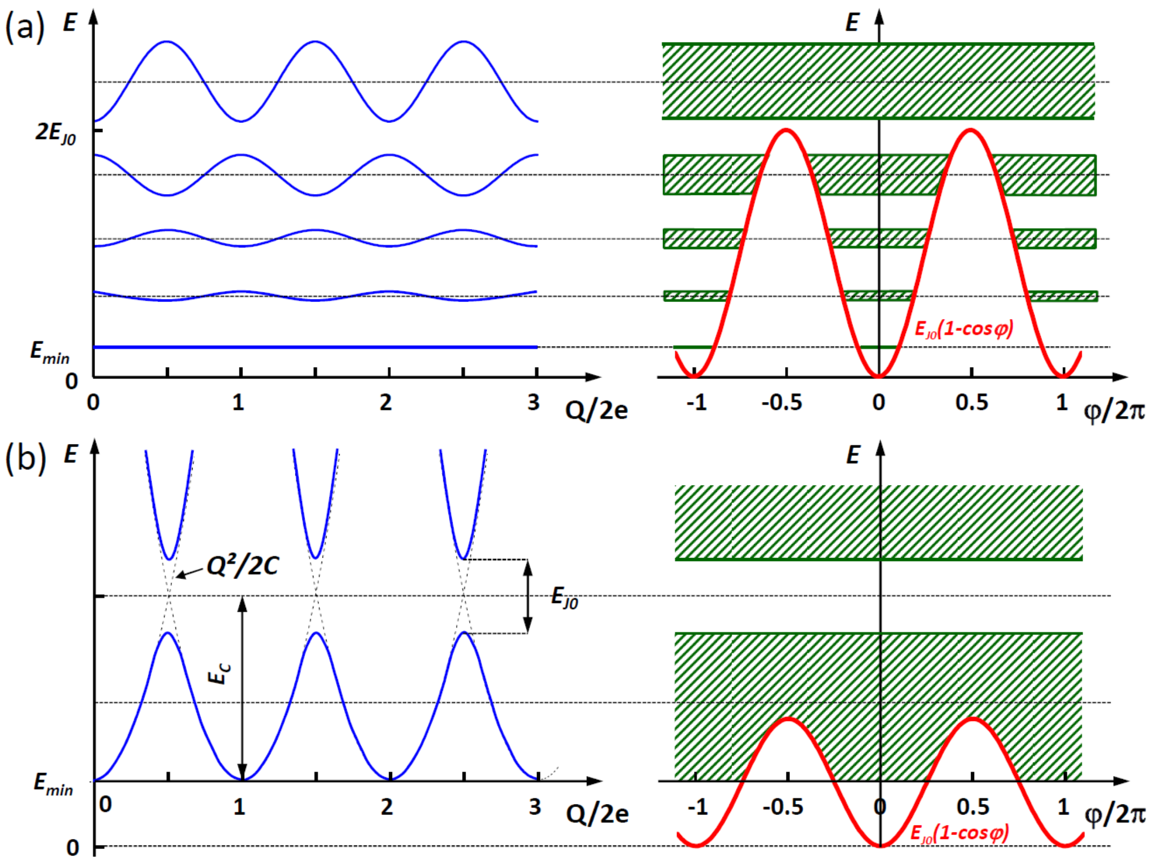
$$\Delta N \Delta \delta \geq 1$$

1) Phase regime $\hbar\omega_{pl} \ll E_{J0}$ and $E_C \ll E_{J0}$

phase is well localized in one of the minima, large charge fluctuations are possible (small E_c)

2) Charge regime $\hbar\omega_{pl} \gg E_{J0}$ and $E_C \gg E_{J0}$

e.g. a small island tunnel coupled, number of states well localized (Coulomb blockade), phase fluctuations are large



Analogous to the problem of electrons in a periodic potential

Strong phase potential → localized states (in phase)

Weak phase potential → extended states in phase space

R. Gross, A. Marx, Applied Superconductivity, Lecture notes (Walter-Meissner Institute)