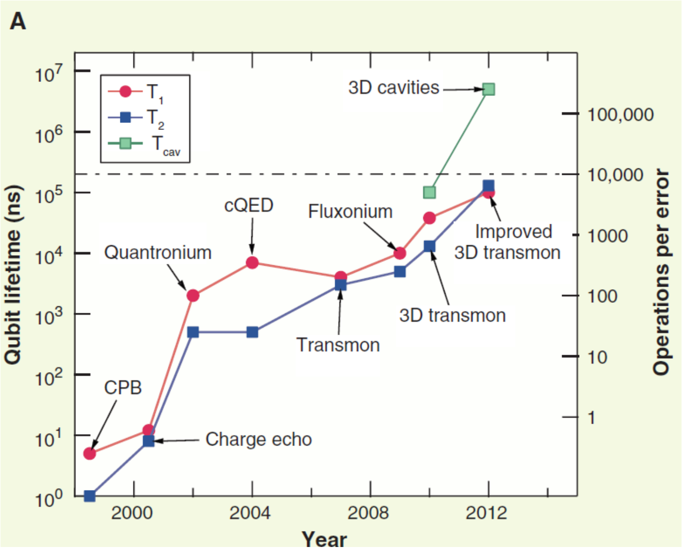
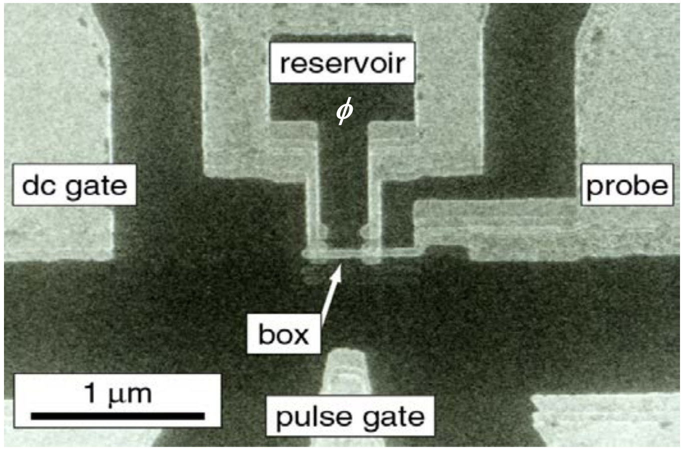
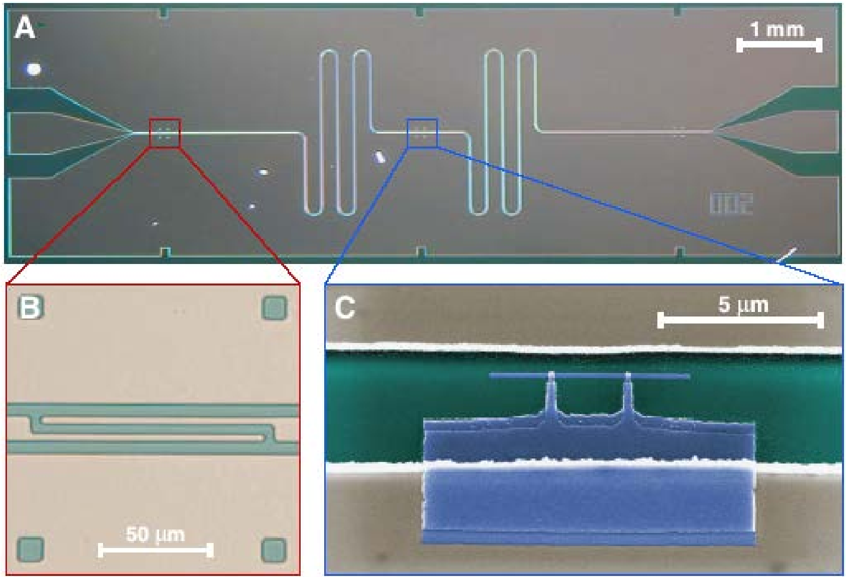
Quantum Computing Architectures

Budapest University of Technology and Economics 2018 Fall



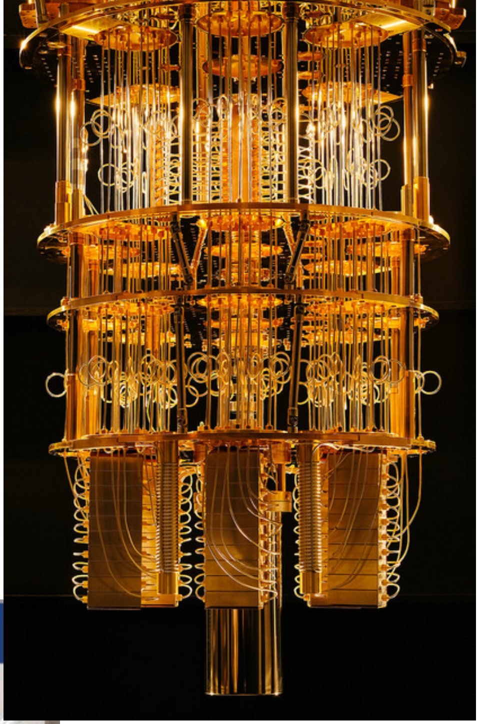


Lecture 8: Cavity QED
Qubit coupling



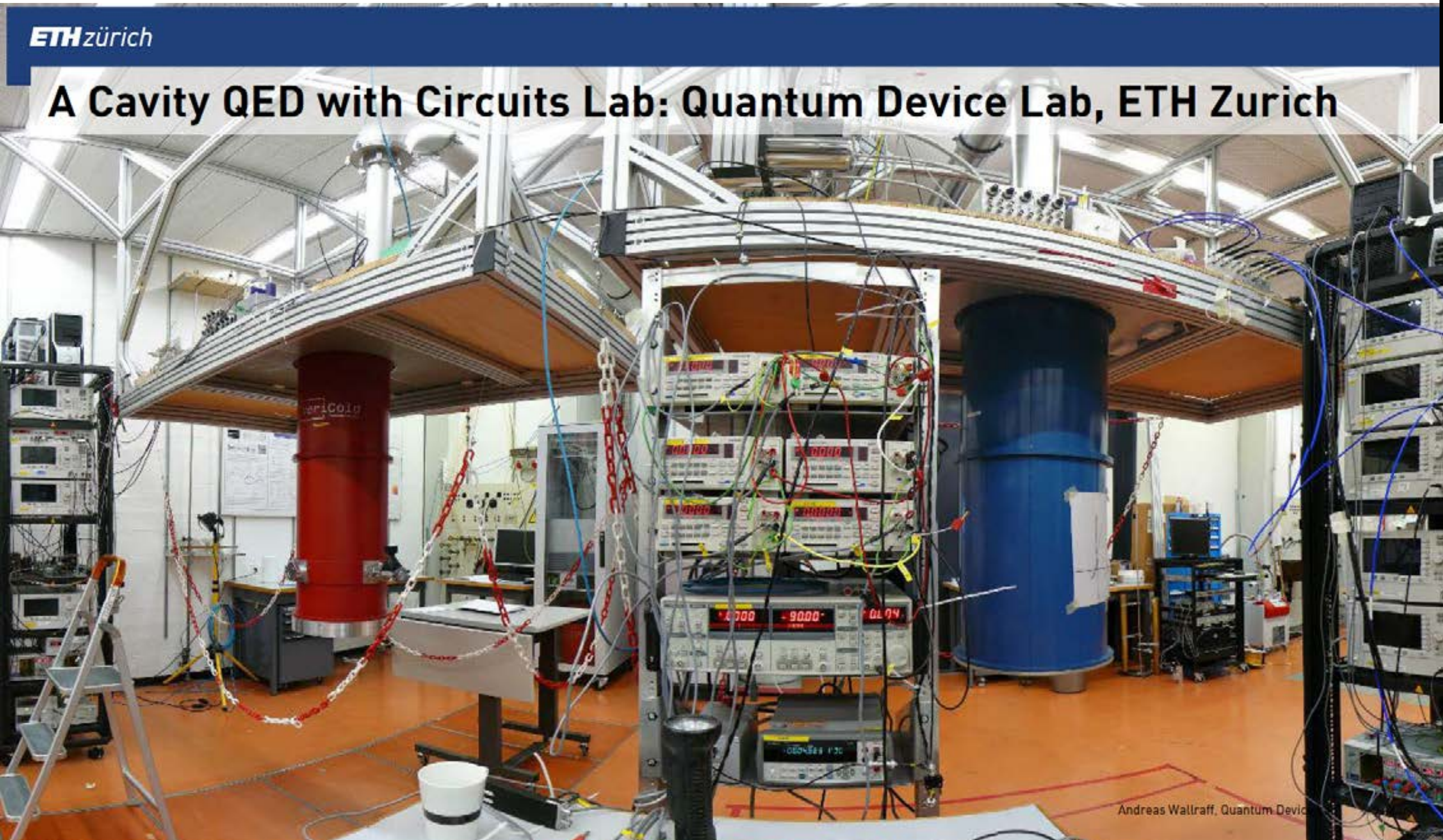
Low temperatures

Fridge: IBM



Ingredients:

- Low temperature (He3-He4 refrigerator)
- Low electrical noise (electron temperature)
- High frequency equipment



ETH zürich

A Cavity QED with Circuits Lab: Quantum Device Lab, ETH Zurich

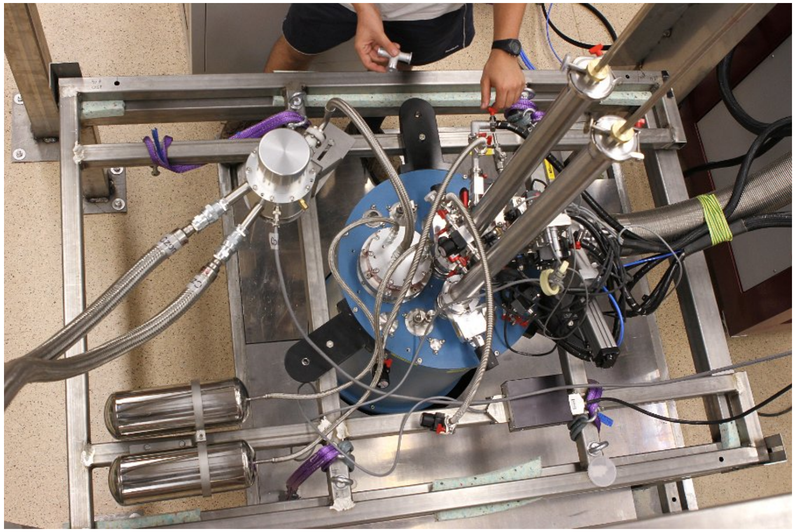
Andreas Wallraff, Quantum Devic

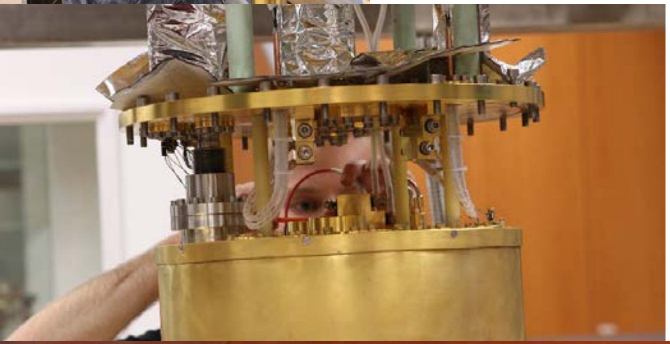
What temperature is needed for a ω_r=5 GHz resonator for average photon number $<n> < 0.05$?

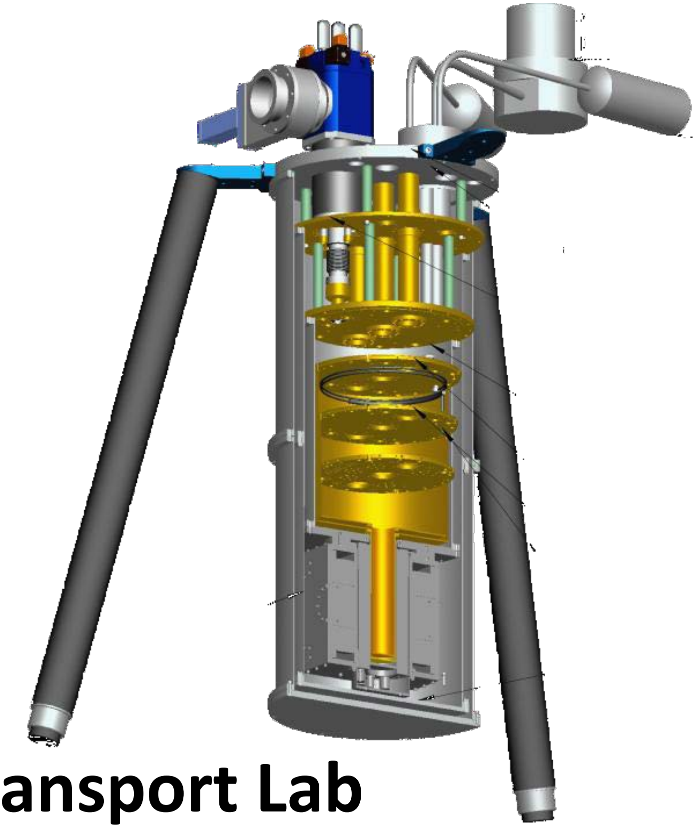
And for a qubit with ω_q=5GHz for a excited state population smaller than 0.05?

Low temperatures

BME quantum transport lab







Transport Lab

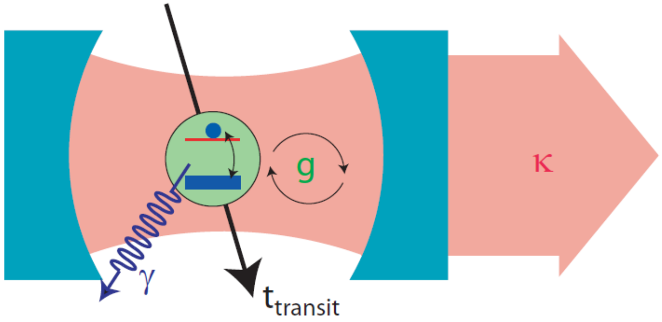
Fridge, $T_{fridge} \approx 7$mK

Vector magnet 9-3T

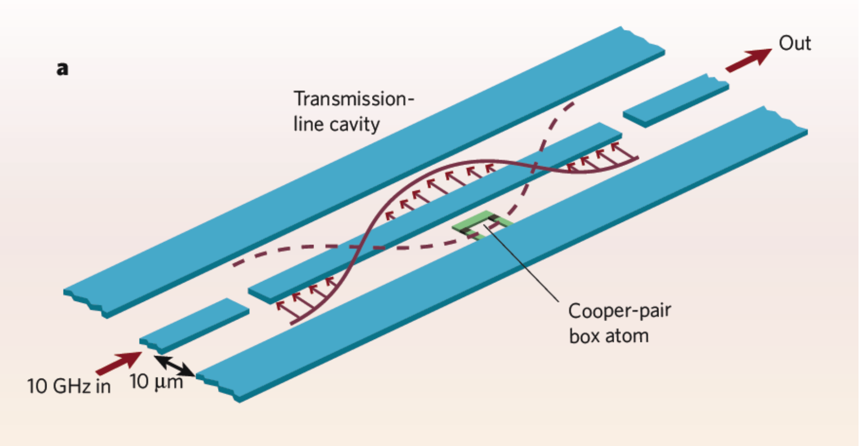
Liquid He facility

Electronics …

SC circuits

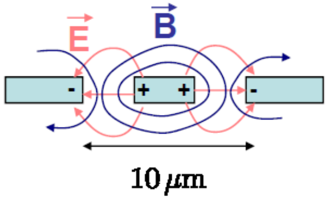


Fabry – Perot cavity for optics – using mirrors



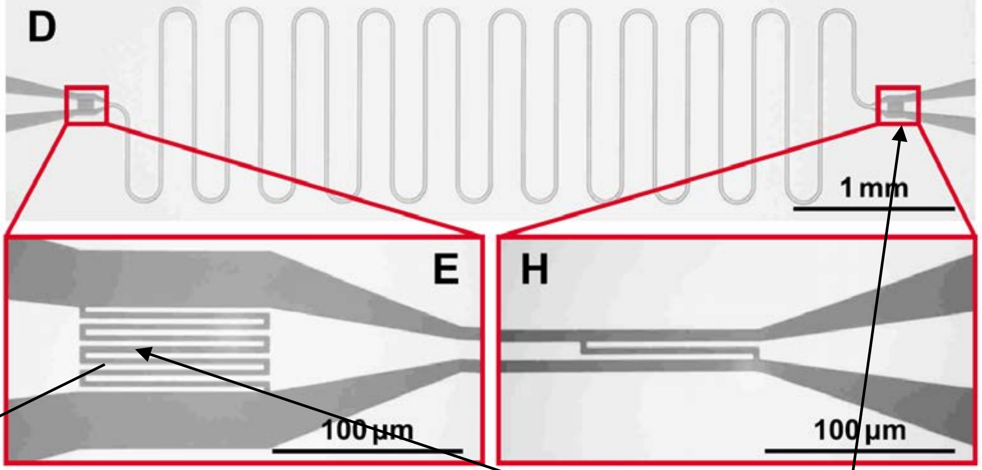
Fabry – Perot cavity for MW photons – capacitive mirrors

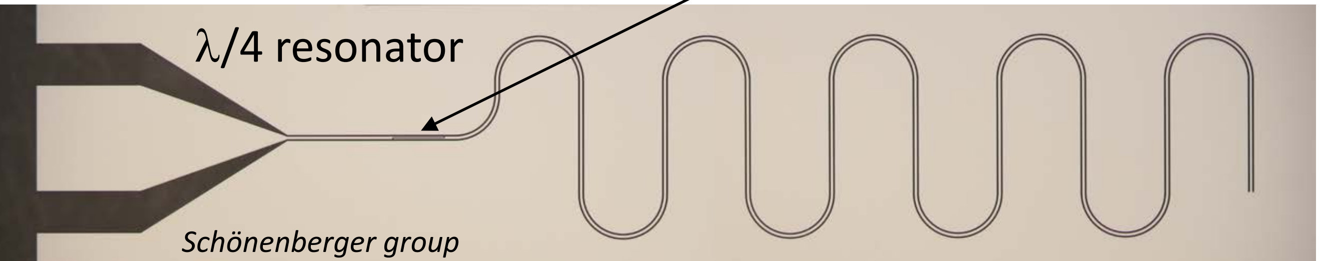
R.J. Schoelkopf et al., Nature 451, 664 (2009)
M. Göppl, : J. Appl. Phys. 104, 113904 (2008)



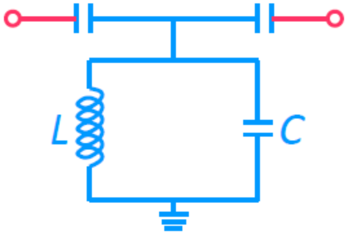
Central conductor and ground plane – essentially a coax

Superconducting circuit to minimize losses (white – SC material, black etched away)
Capacitors: voltage antinodes – zero current – good for electrical dipole coupling
Current antinode (voltage node) - maximal current – good for inductive coupling





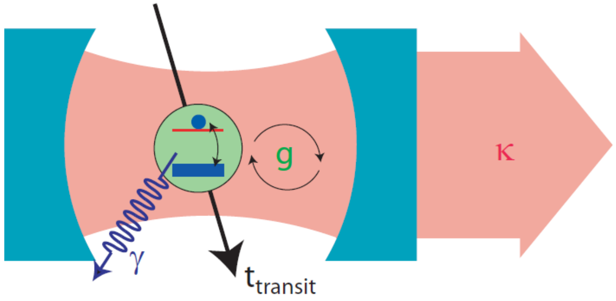
Schönenberger group



Readout: circuit QED

$$\hat{H} = 4E_c\left(N - N_g\right)^2 - E_J\cos\delta + \hbar\omega_r\hat{a}^\dagger\hat{a} + \underbrace{2\frac{C_g}{C_\Sigma}eV^0_{RMS}\hat{N}\left(\hat{a}^\dagger + \hat{a}\right)}$$

Coupling term – electrical coupling to charge (dipole)



Can be mapped to J-C Hamiltonian

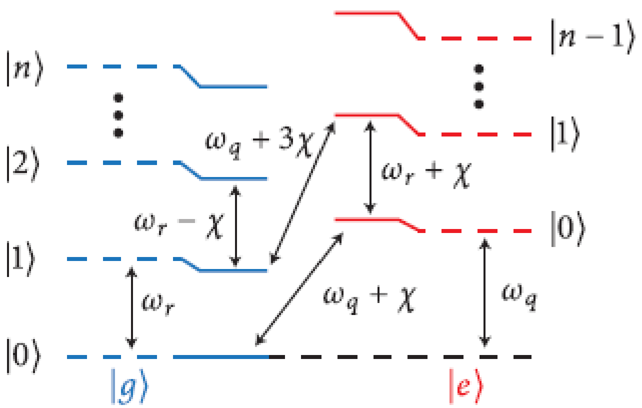
$$\omega_r = \frac{1}{\sqrt{L_r C_r}} \qquad V^0_{rms} = \sqrt{\frac{\hbar\omega_r}{2C_r}}$$

$$\hat{H} = \frac{\hbar\omega_q}{2}\sigma_Z + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar g\left(\hat{a}^\dagger\sigma_- + \hat{a}\sigma_+\right) + H_\kappa + H_\gamma$$

Jaynes Cummings Hamiltonian

Qubit Resonator Coupling Cavity decay Qubit lifetime

b



$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta}\right)\sigma_Z + \left(\hbar\omega_r + \hbar\frac{g^2}{\Delta}\sigma_Z\right)\hat{a}^\dagger\hat{a}$$

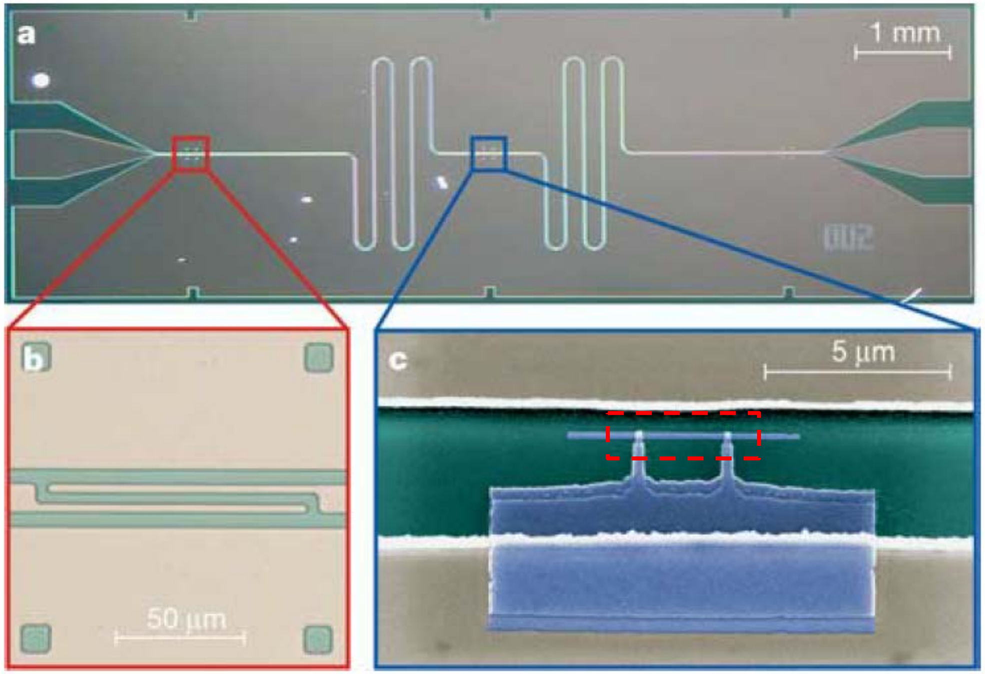
Lamb-shift

Qubit-state dependent resonance shift

$$\Delta = \omega_q - \omega_r$$

Readout: circuit QED

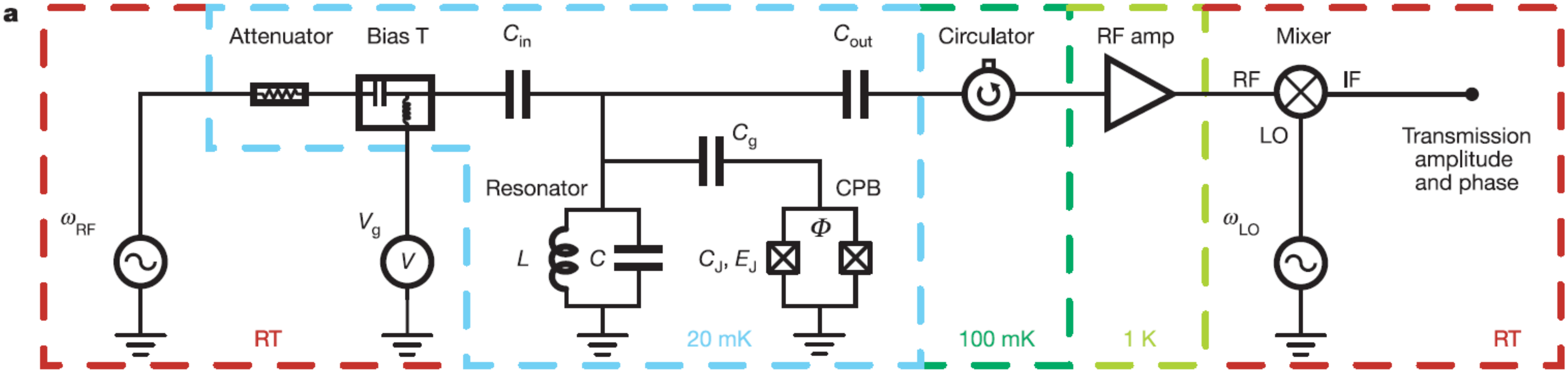
Spectroscopy on resonator



CP-box coupled (capacitively) to a MW cavity

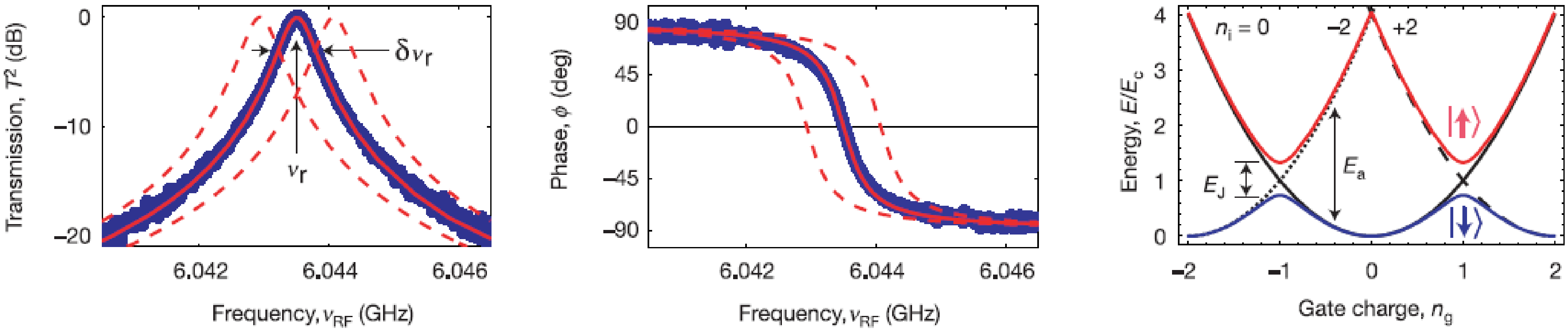
External B field tunes E_J

In the circuit model the qubit is a tunable capacitance which shifts the resonator



Many circuit elements are at low T (amplifier, circulator etc.)

A .Walraff et al., Nature 431, 162 (2004)



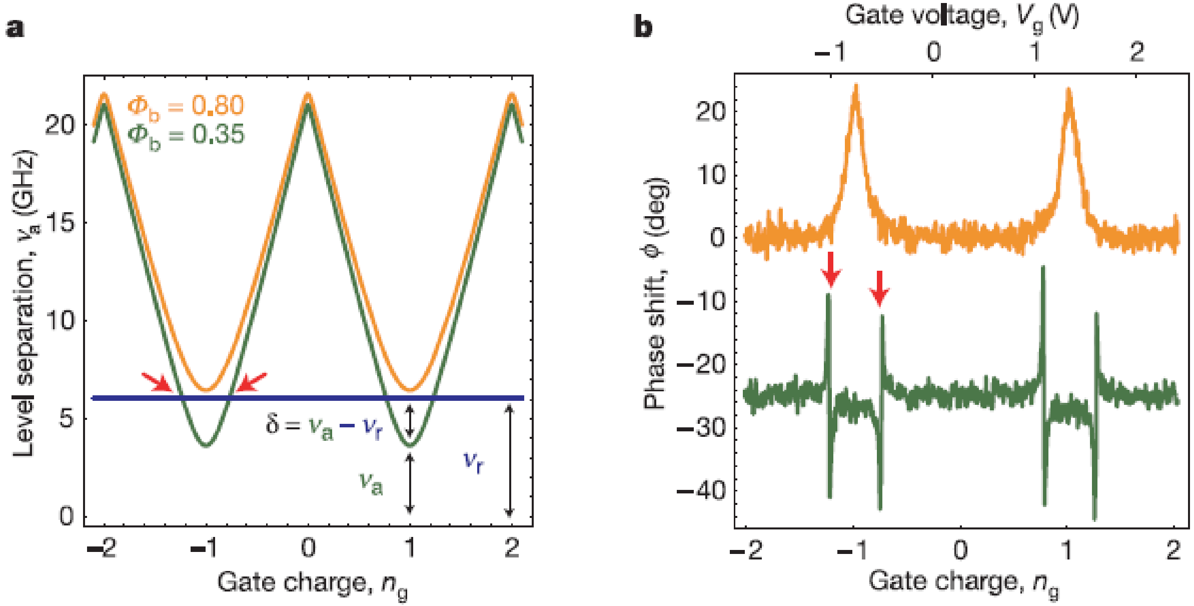
Resonator: Lorentz-like resonance curve with high Q. Phase response is more sensitive

Simulation: shifted curves for the two different qubit states. Idea: measurement at fixed frequency – measure phase response

Reminder:

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta}\right)\sigma_Z + \left(\hbar\omega_r + \boxed{\hbar\frac{g^2}{\Delta}\sigma_Z}\right)\hat{a}^\dagger\hat{a}$$

$$\chi = \frac{g^2}{\Delta}$$

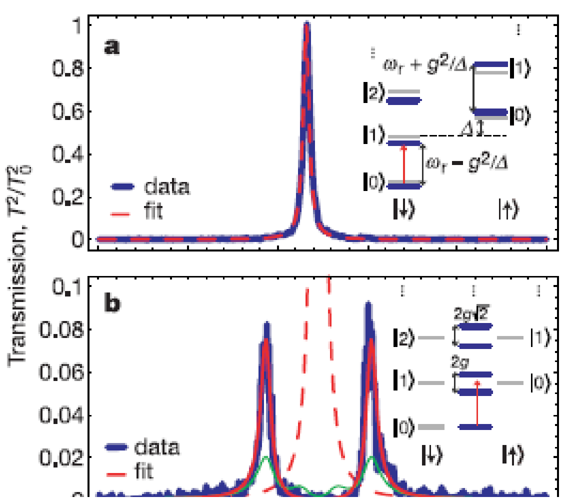


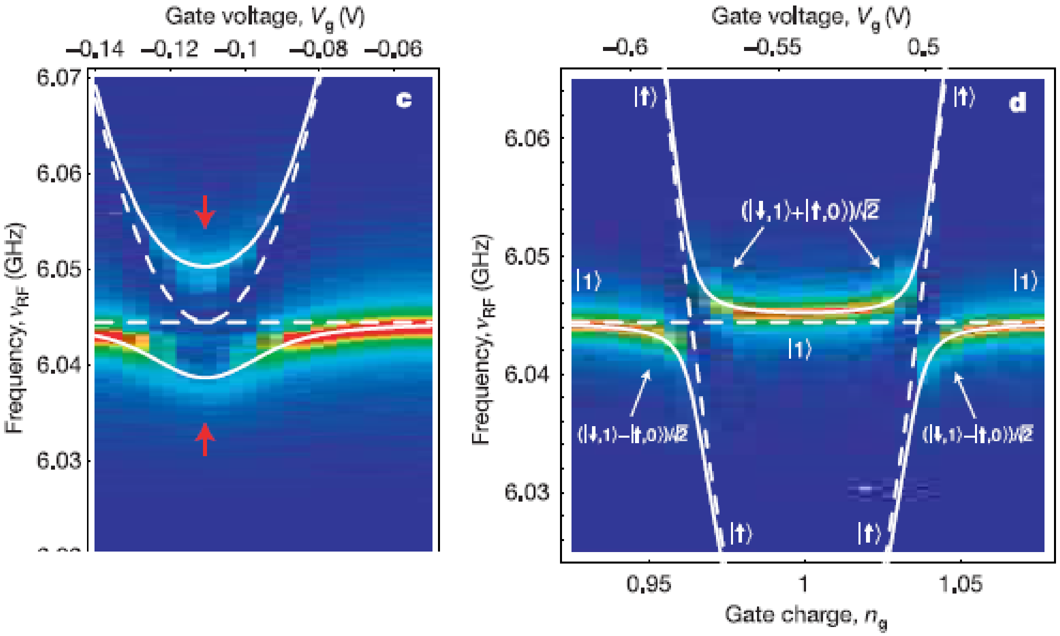
Here Qubit is in the ground state, and resonator is probed for different parameters

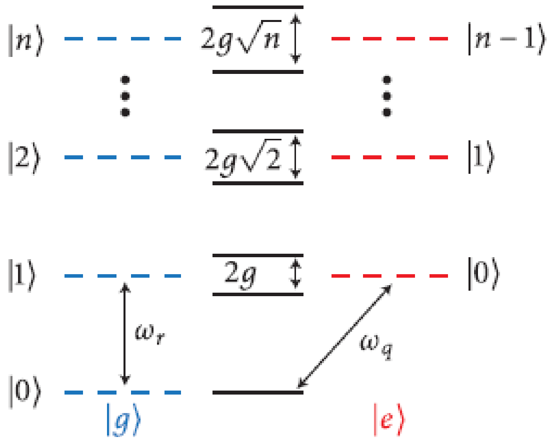
2 different flux biases: for one it goes through the resonance with the resonator (green), for the other not (orange).

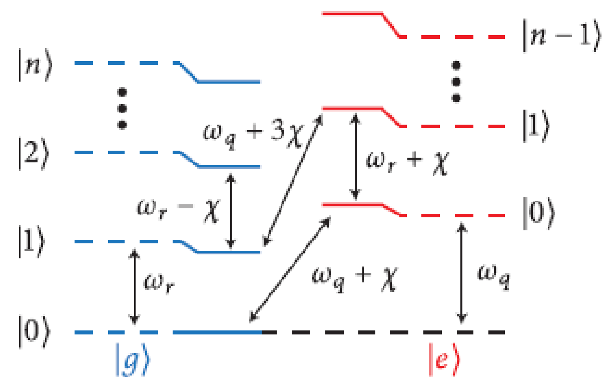
Phase shift decreases by increasing detuning from resonance

Strong coupling –
Spectroscopy measurement







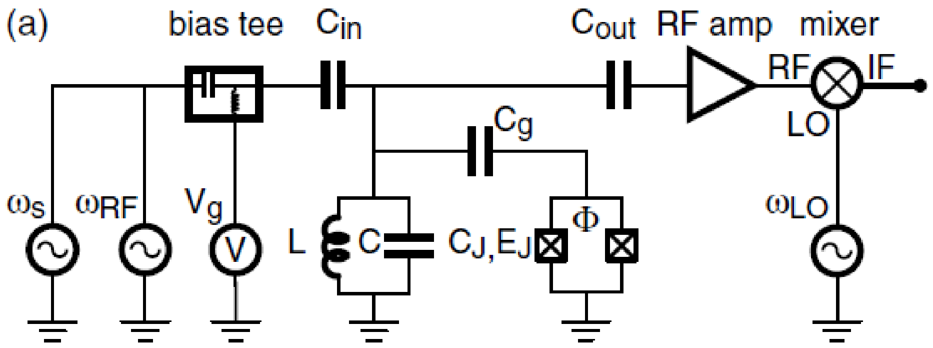


Far away from crossing pure resonator states. Close to resonance an avoided crossing is seen. Bonding and anti-bonding states – entangled states with both photon and qubit character – „phobit” and „quton”.

Here the photon number is small n<<1. Vacuum Rabi oscillation with frequency 2g. Continous photon emission and absorbtion.

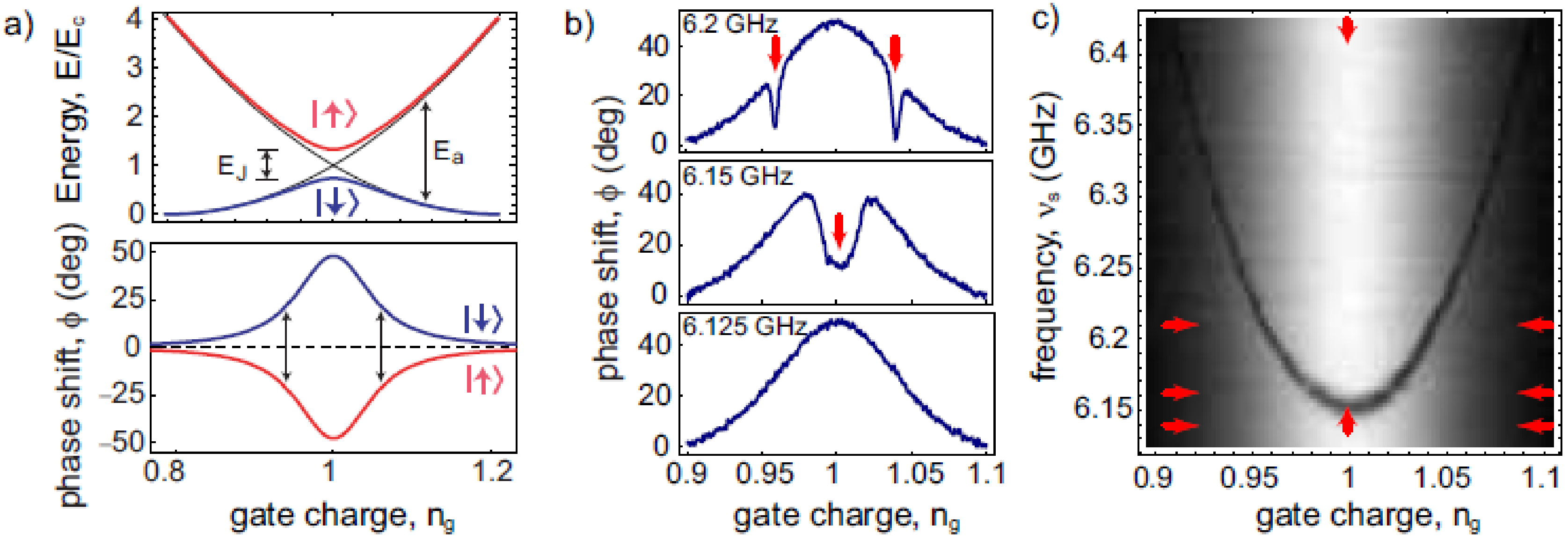
Spectroscopy

2-tone measurements



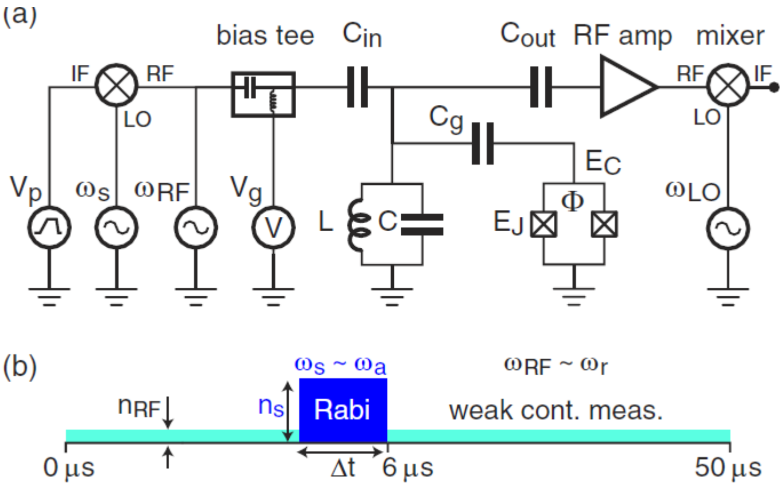
2 tone:
ω_S: qubit frequency (here continuous)
ω_{RF}: cavity frequency (here continuous)

Phase shift: opposite for the two states. If ω_S excites cavity than reduction in phase shift (red arrows). For high power, both states are equally populated and the shift averages to zero.
6.125 GHz- no resonance with qubit, just phase shift observed
6.15 GHz - at N_g=1 the qubit is driven. Reduction in the phase shift is seen. Similarly at 6.2 GHz.
For Rabi etc. pulsing at ω_S is needed (see later).



D. I. Schuster et al., PRL 94, 123602 (2005)

Qubit rotations
Two tone measurements

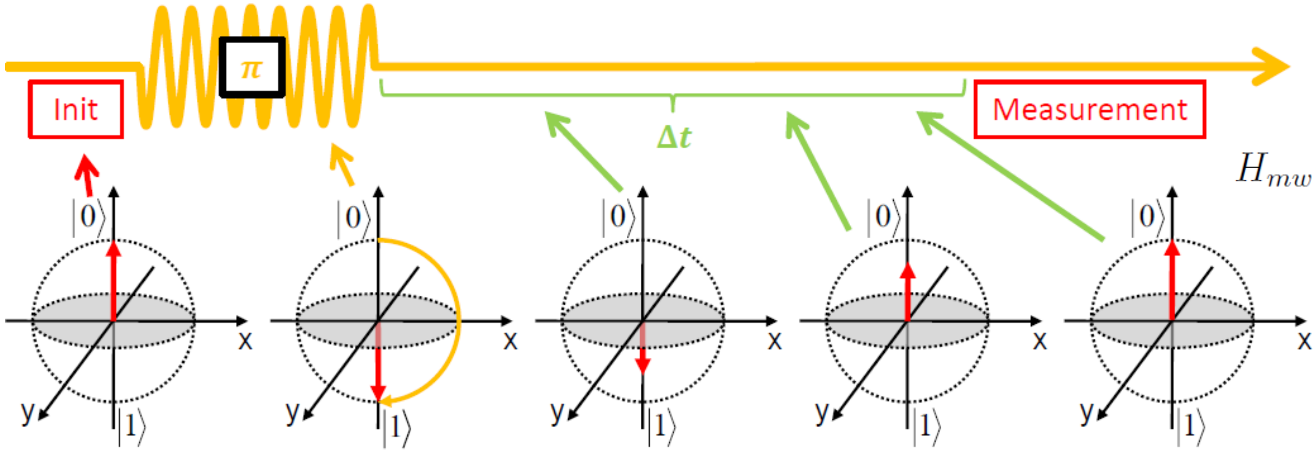


2 tone:

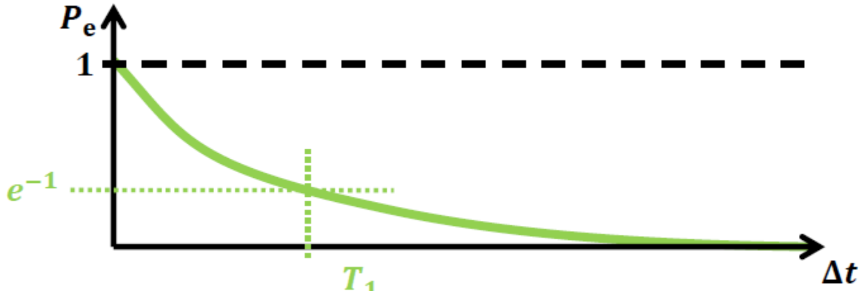
ω_S: qubit frequency (here pulsed)

ω_{RF}: cavity frequency (here continuous)

T1 measurement



Rotating frame & no detuning ($\Delta\omega = \omega - \omega_q = 0$) → no xy-evolution



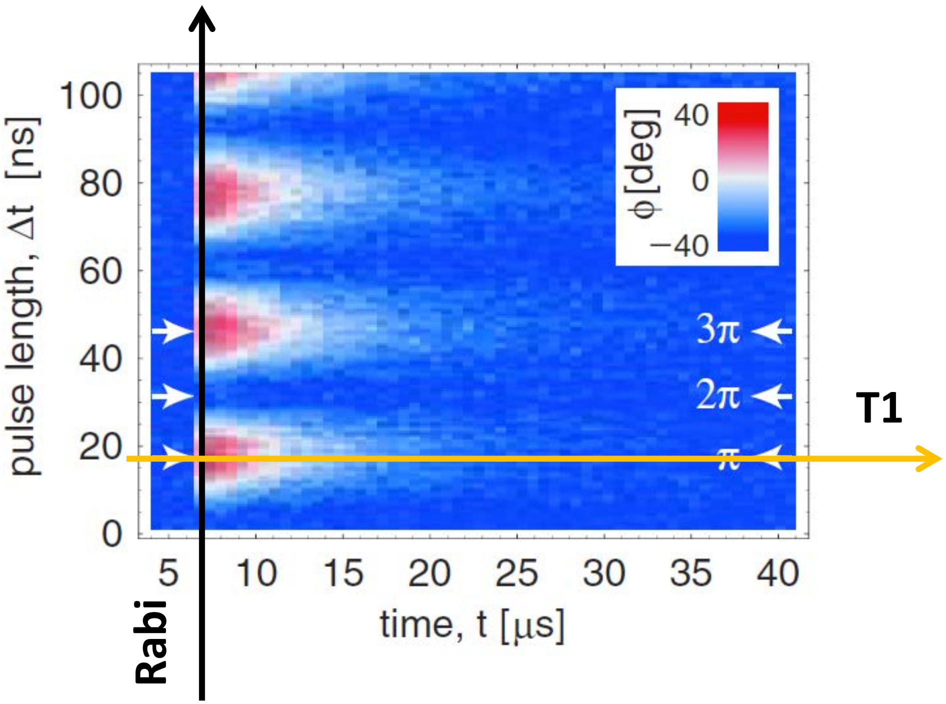
$$H_{mw}(t) = \hbar\epsilon(t)\left(\hat{a}^\dagger e^{i\omega_{mw}t} + \hat{a}e^{-i\omega_{mw}t}\right)$$

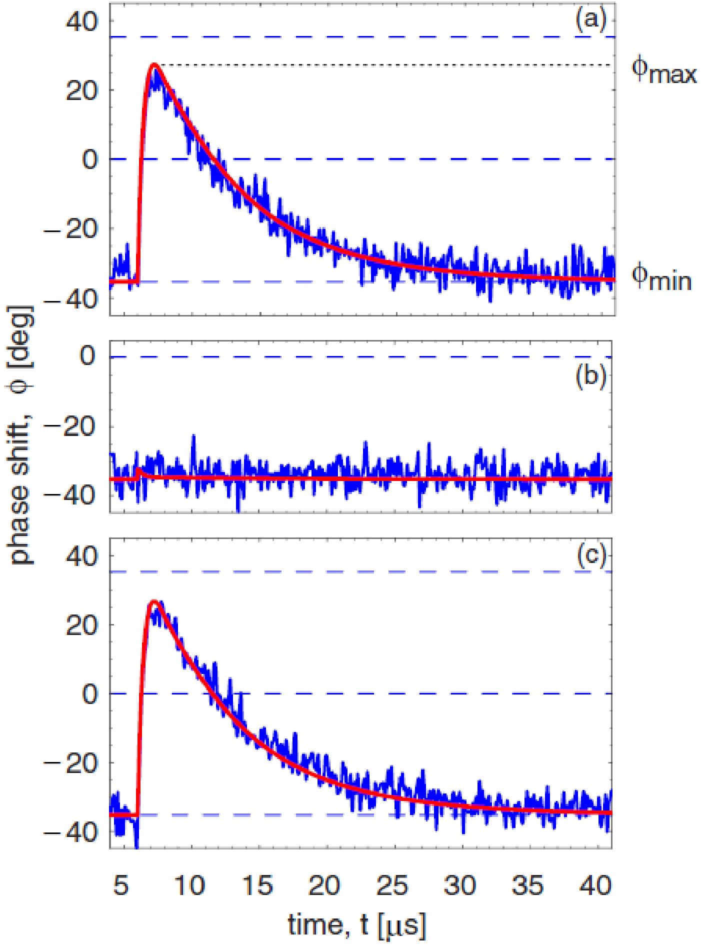
Rotating frame

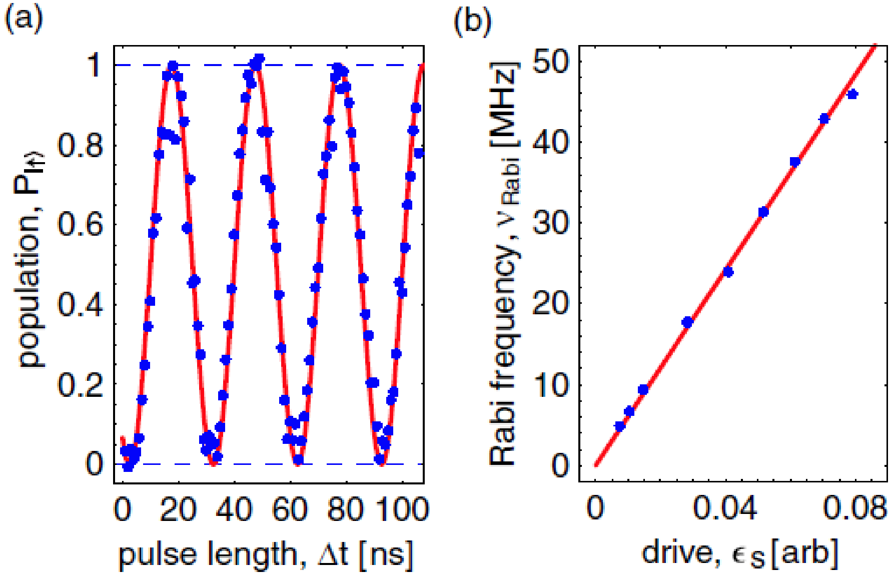
$$\hat{H} = \frac{1}{2}\hbar\left(\omega_q + \frac{2g^2}{\Delta}\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) - \omega_{mw}\right)\sigma_Z + \hbar(\omega_r - \omega_{mw})\hat{a}^\dagger\hat{a} + \hbar\epsilon(t)\left(\hat{a}^\dagger + \hat{a}\right) + \hbar\frac{g\epsilon(t)}{\Delta}\sigma_X$$

R. Gross, A. Marx, Applied Superconductivity, Lecture notes (Walter-Meissner Institute)

A. Walraff et al., PRL 95, 060501 (2005)







Charge qubit

0 phase: if the qubit was not there

GS and ES has opposite shift. In ES does not reach maxima due to finite cavity lifetime

2 π : no relaxation should occur

T1~ 7 μs

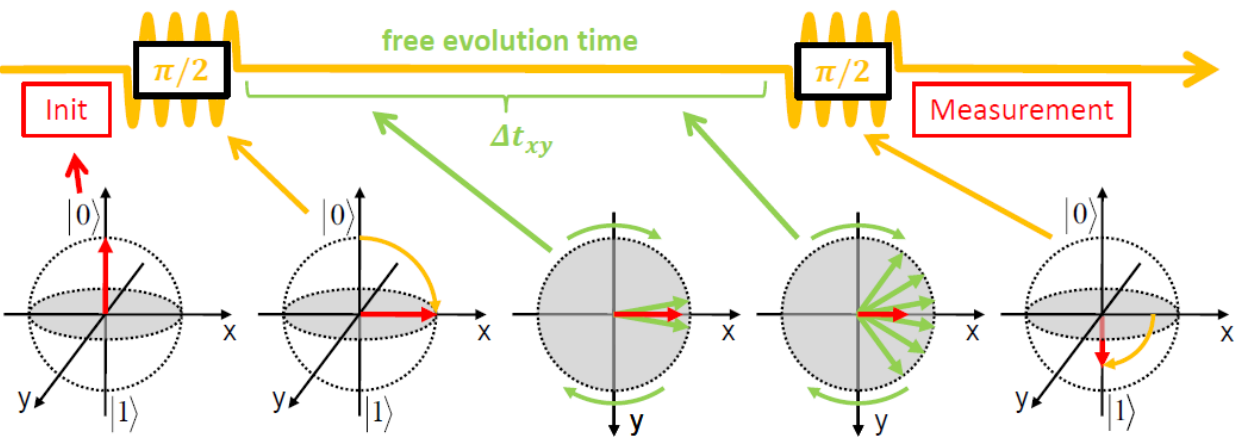
A. Walraff et al., PRL 95, 060501 (2005)

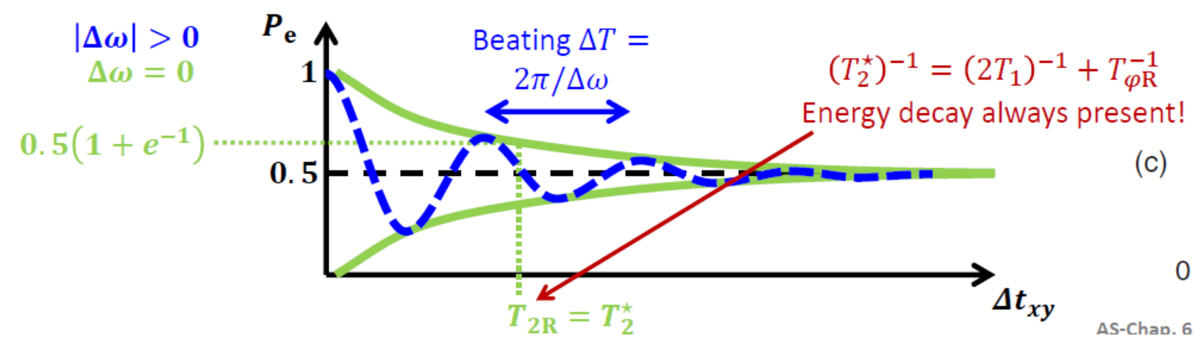
T2 measurement

2 tone:

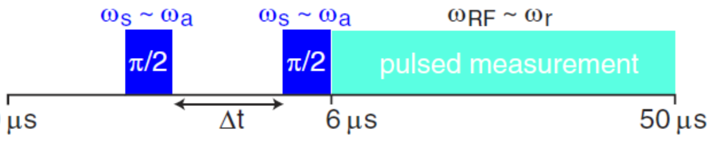
ω_S: qubit frequency (here pulsed)

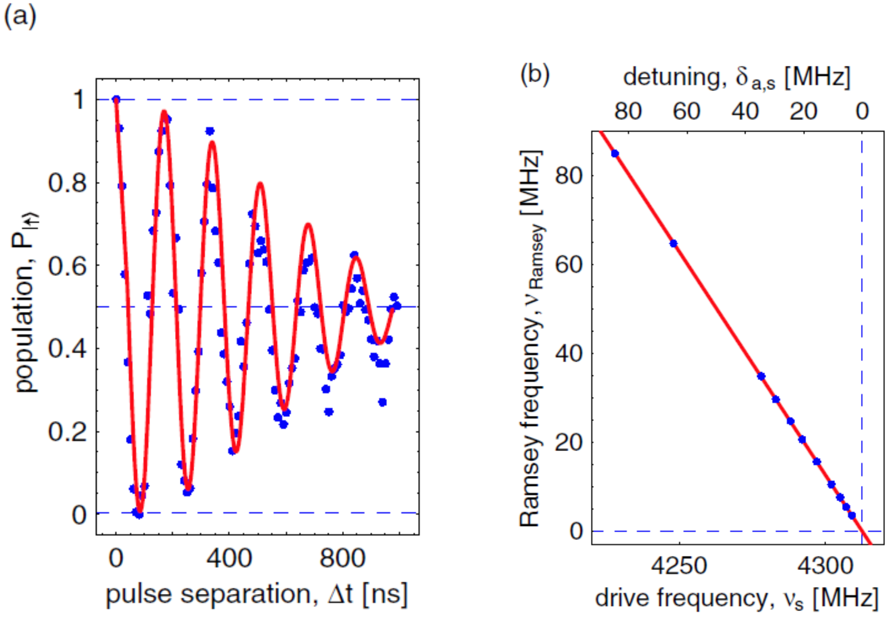
ω_{RF}: cavity frequency (here pulsed)





(c)





Ramsey measurement for different detunings (detuning – small precession compared to the rotating frame) – decay: T2 ~500 ns

A. Walraff et al., PRL 95, 060501 (2005)

R. Gross, A. Marx, Applied Superconductivity, Lecture notes (Walter-Meissner Institute)

Transmon cQED

Mostly the same, gate voltage not a useful parameter

Using the transmon wave function, RWA only the following relevant terms remain:

$$\hat{H} = \hbar \sum_j \omega_j |j\rangle \langle j| + \hbar\omega_r \hat{a}^\dagger \hat{a} + \left[\hbar \sum_i g_{i,i+1} |i\rangle \langle i+1| \hat{a}^\dagger + \text{H.C.}\right]$$

Multi level Jaynes Cummings Hamiltonian, where

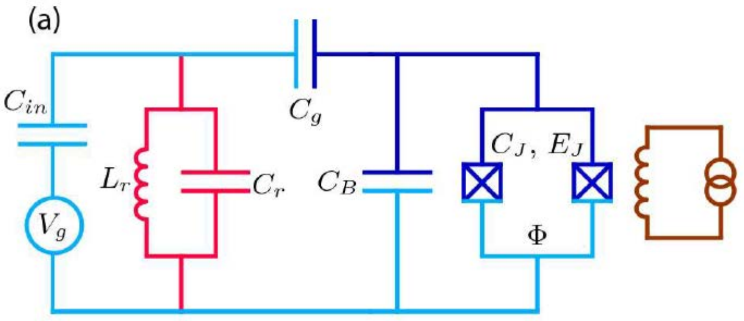
$$\hbar g_{i,i+1} = 2e\frac{C_g}{C_\Sigma} e V^0_{rms} \langle i|\hat{N}|i+1\rangle \qquad \langle i|\hat{N}|i+1\rangle \sim \left(\frac{E_j}{8E_C}\right)^{1/4}$$

g – coupling term is large, even increases with increasing E_J

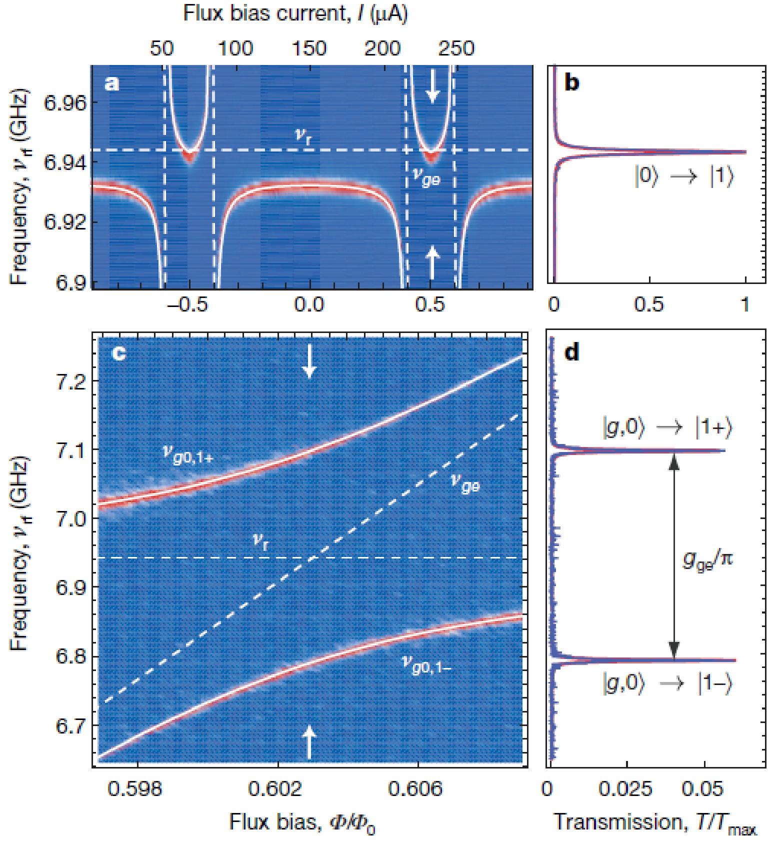
$$\hat{H} = \frac{1}{2}\left(\hbar\omega_{01} + \hbar\chi_{01}\right)\sigma_Z + \left(\hbar\omega_r - \hbar\chi_{12} + \hbar\chi\sigma_Z\right)\hat{a}^\dagger \hat{a}$$

$$\chi = \chi_{01} - \chi_{12}/2 \qquad \chi_{ij} = \frac{g_{ij}}{\omega_{ij} - \omega_r}$$

Higher levels matter a bit, otherwise the same



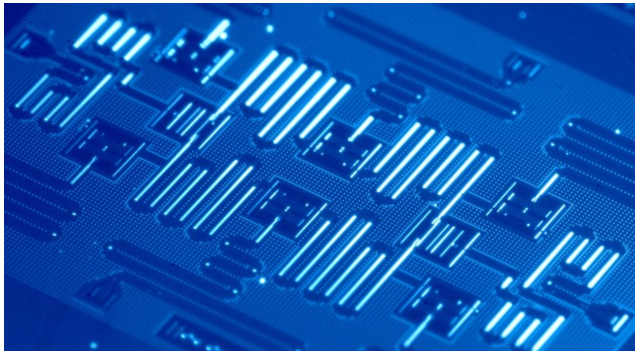
Strong coupling



Strong coupling achieved
For 0-1 state 2g Rabi frequency
For 1-2 state √2*2g as J-C says

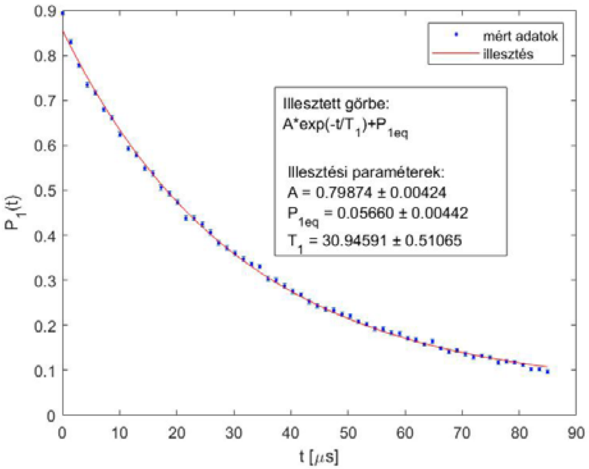
J. M. Fink et al., Nature 454, 315 (2008)

Transmon cQED



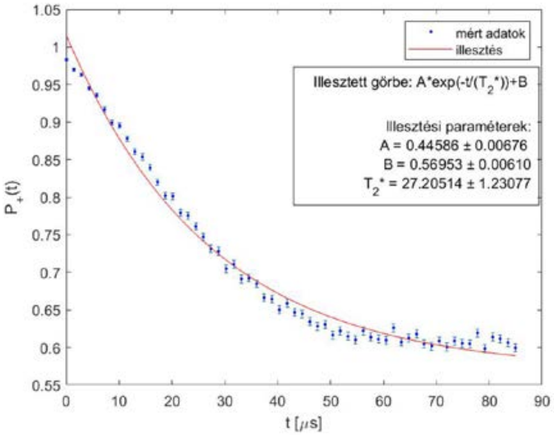
Timescales have evolved
Measurements on IBM experience

T1 measurement



T_1~ 31 μs

T2 measurement



T_2^*~ 27 μs

T2 is T1 limited – relaxation not by decoherence.
Claim T1 comes from spontaneous emission to the cavity – Purcell effect.

Akos Budai, BSc thesis

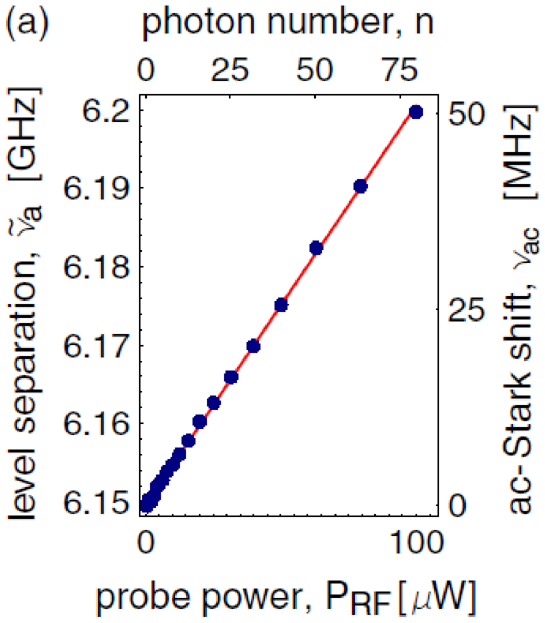
Back-action
Stark-shift

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta}\right)\sigma_Z + \left(\hbar\omega_r + \hbar\frac{g^2}{\Delta}\right)\hat{a}^\dagger\hat{a}$$

$$\downarrow$$

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta} + \boxed{\hbar\frac{g^2}{\Delta}\hat{a}^\dagger\hat{a}}\right)\sigma_Z + \hbar\omega_r\hat{a}^\dagger\hat{a}$$

Stark shift

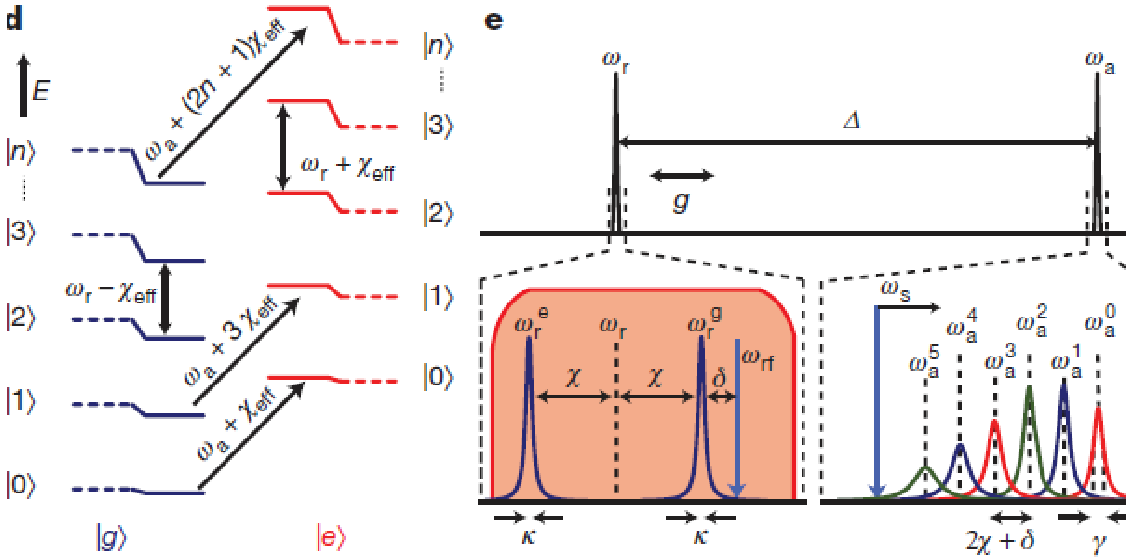


By increasing the resonator power, hence the average photon number, the qubit frequency shifts.

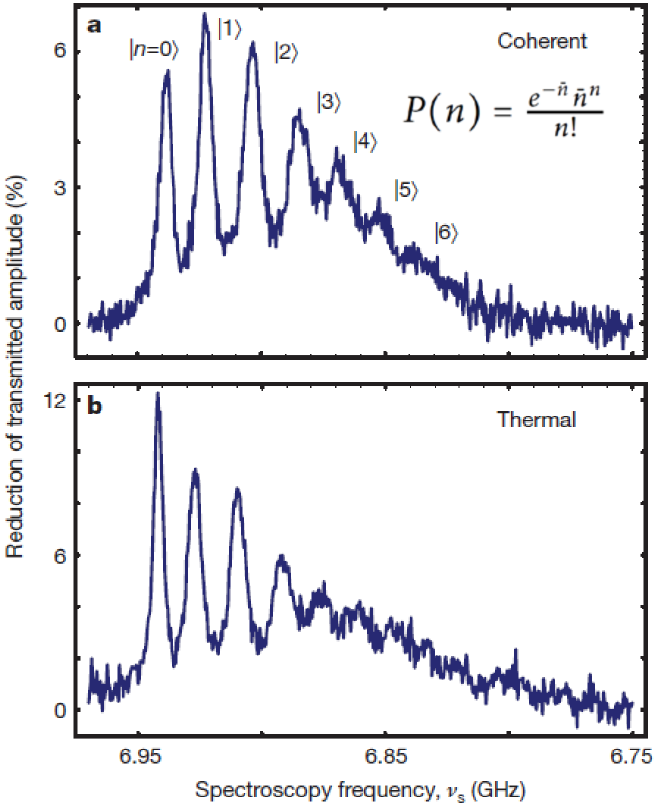
D. I. Schuster et al., PRL 94, 123602 (2005)

$$\hat{H} = \frac{1}{2}\left(\hbar\omega_q + \hbar\frac{g^2}{\Delta} + \boxed{\hbar\frac{g^2}{\Delta}\hat{a}^\dagger\hat{a}}\right)\sigma_Z + \hbar\omega_r\hat{a}^\dagger\hat{a} \qquad \chi = \frac{g^2}{\Delta}$$

In the strong dispersive regime ($\chi >> \gamma, \kappa$) individual photon states resolved:
Populate resonator at wrf. Than sweep ws (qubit frequncy). If there were n photons in the cavity the resonance will be at $2n\chi$. If the qubit gets excited can be seen from the resonator frequency shift.
Individual photon states resolved.
Under usual drive close to coherent states observed.
Addig large thermal noise – thermal distribution.



D. I. Schuster et al., Nature, 445, 515 (2007)

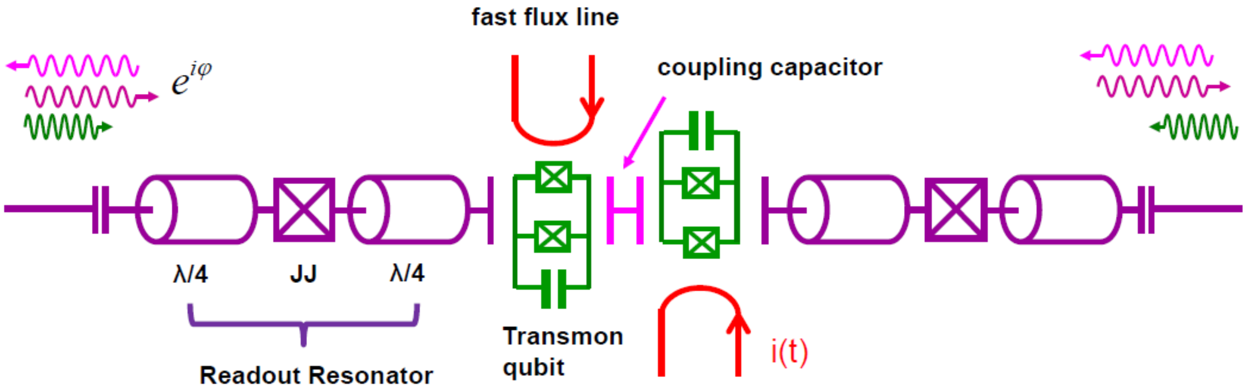


Coupling qubits
Capacitive coupling

Fix coupling – not tuneable

Separate readout resonator for both of them

Can perform swap operation



Coupling term

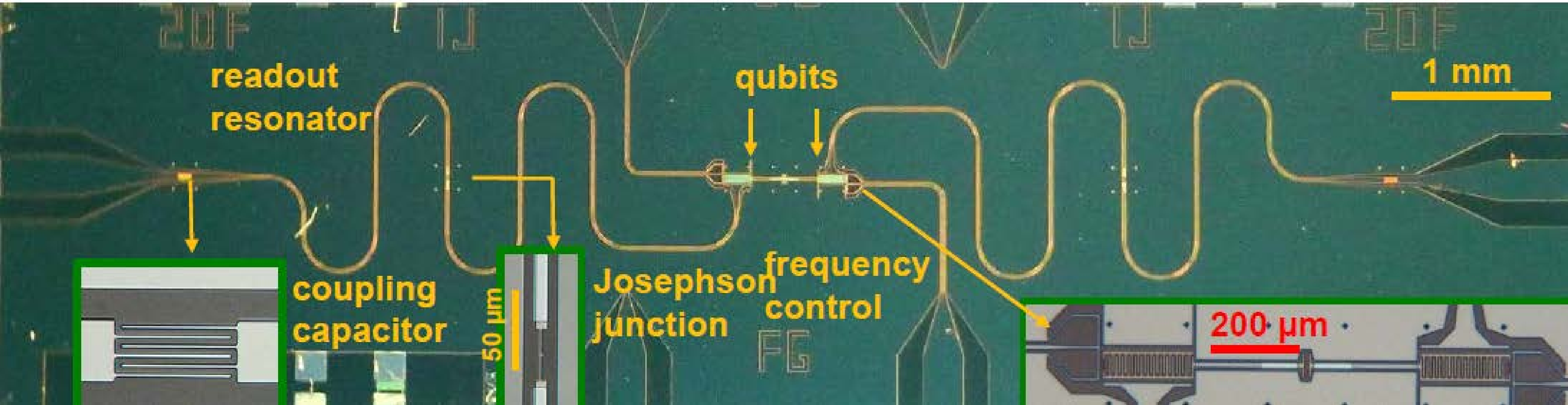
$$2\frac{E_{c,I}E_{c,II}}{E_{cc}}(\hat{N}_I - N_{g,I})(\hat{N}_{II} - N_{g,II})$$

…

$$\boxed{\hbar g(\sigma_I{}^{+}\sigma_{II}{}^{-} + \sigma_I{}^{-}\sigma_{II}{}^{+})}$$

$$\boxed{\hbar g = (2e)^2\frac{C_c}{C_I C_{II}}\left|\langle 0_I|\hat{N}_I|1_I\rangle\langle 0_{II}|\hat{N}_{II}|1_{II}\rangle\right|}$$

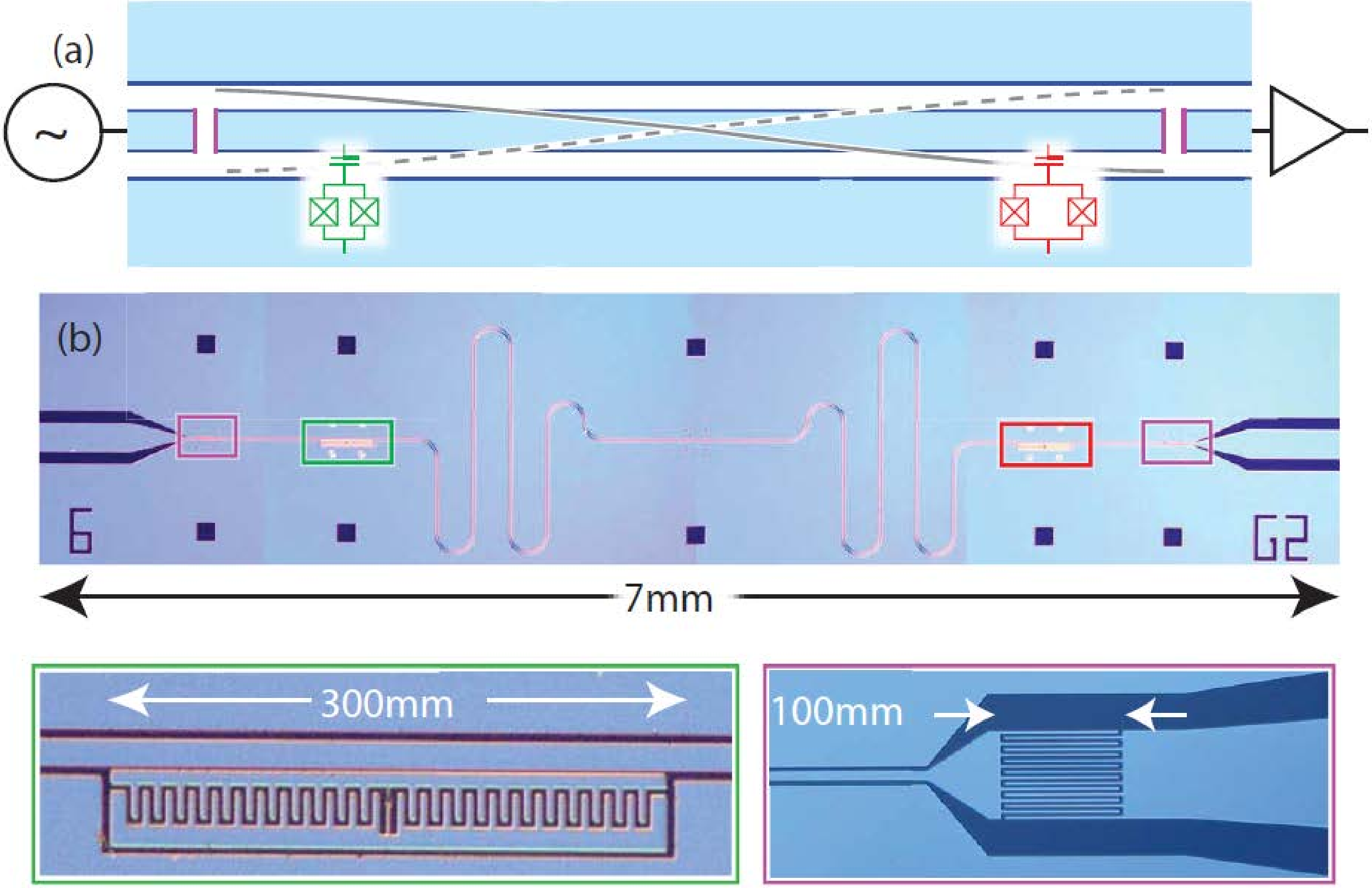
A. Dewes et al., Phys. Rev. Lett. 108, 057002 (2012)



Coupling qubits

Quantum bus

Two qubits at opposite sides of the resonator
λ/2 mode



Different loop area – different EJ

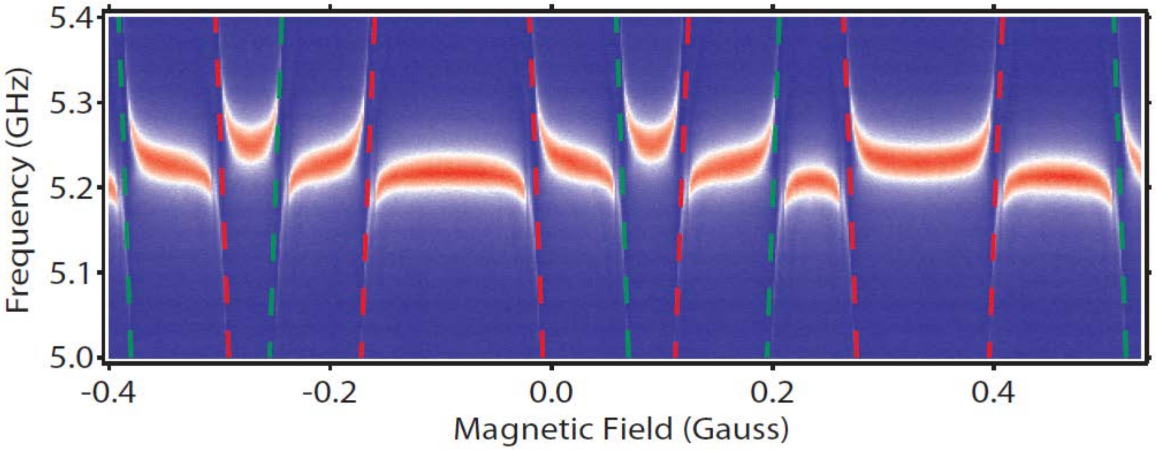
Device parameters:

$$E_{C1}/h = 424\text{ MHz}$$
$$E_{C2}/h = 442\text{ MHz}$$
$$E_{J1}^{\max}/h = 14.9\text{ GHz}$$
$$E_{J2}^{\max}/h = 18.9\text{ GHz}$$

Resonator:

$$\omega_C/2\pi = 5.22\text{ GHz},\ \kappa/2\pi = 33\text{ MHz}$$

J. Majer et al., Nature 449, 443 (2007)
J. M. Chow Phd thesis

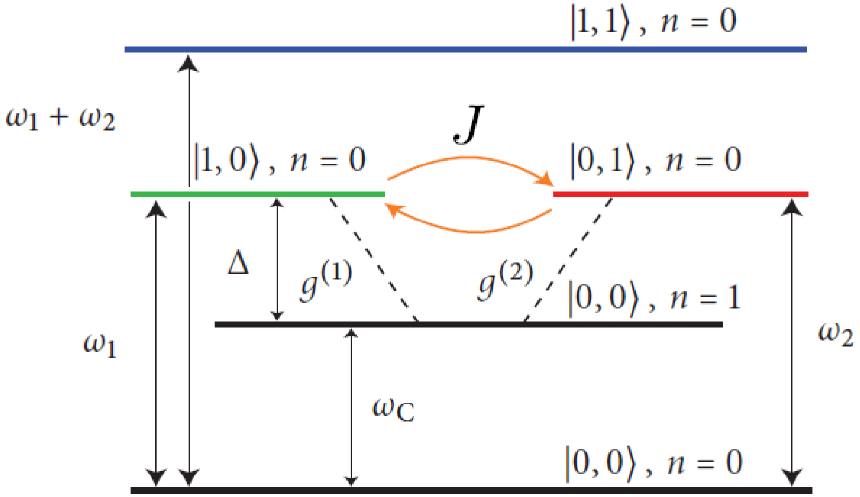


Single tone spectroscopy on resonator
Avoided crossings for both resonors suggest strong coupling – theory curve dashed lines
→ $g^{(1),(2)}/\pi$= 105 MHz

$$\hat{H} = \frac{1}{2}\hbar\omega_q^{(1)}\sigma_Z^1 + \frac{1}{2}\hbar\omega_q^{(2)}\sigma_Z^2 + \hbar\left(\omega_r + \chi^{(1)}\sigma_Z^{(1)} + \chi^{(2)}\sigma_Z^{(2)}\right)\hat{a}^\dagger\hat{a}$$

$$\boxed{+\hbar J\left(\sigma_-^{(1)}\sigma_+^{(2)} + \sigma_-^{(2)}\sigma_+^{(1)}\right)}$$

Lecture 2: Perturbation theory and in the rotating frame: 2 qubit + interaction term



$$J = \frac{g^{(1)}g^{(2)}}{2}\left(1/\Delta^{(1)} + 1/\Delta^{(2)}\right)$$

$$\left|\Delta^{(1),(2)}\right| = \left|\omega^{(1),(2)} - \omega_r\right| \gg g^{(1),(2)}$$

Virtual exchange of photons via the cavity with rate g1 and g2, if they are on resonce with each other (off to the cavity)

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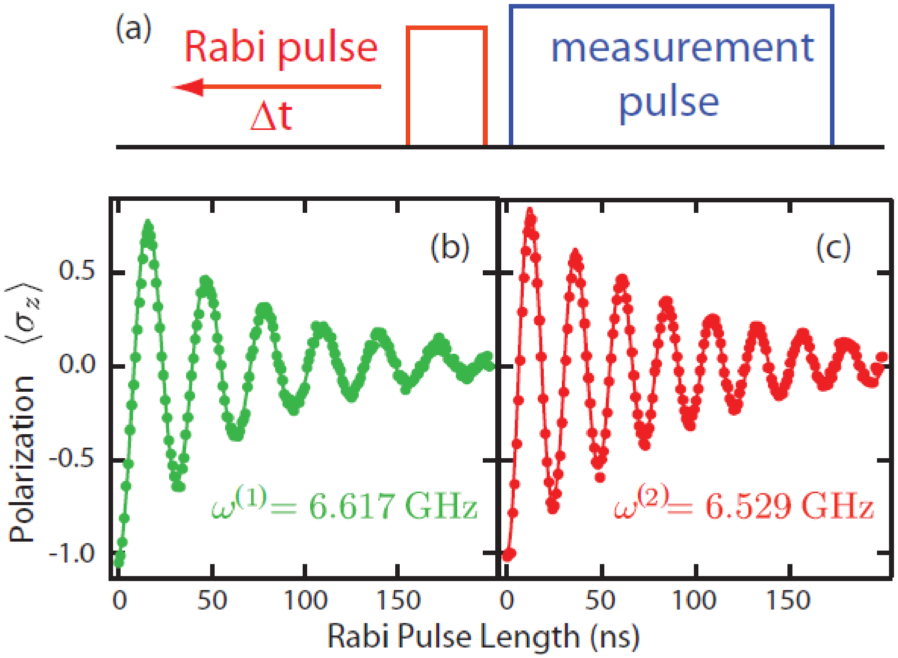
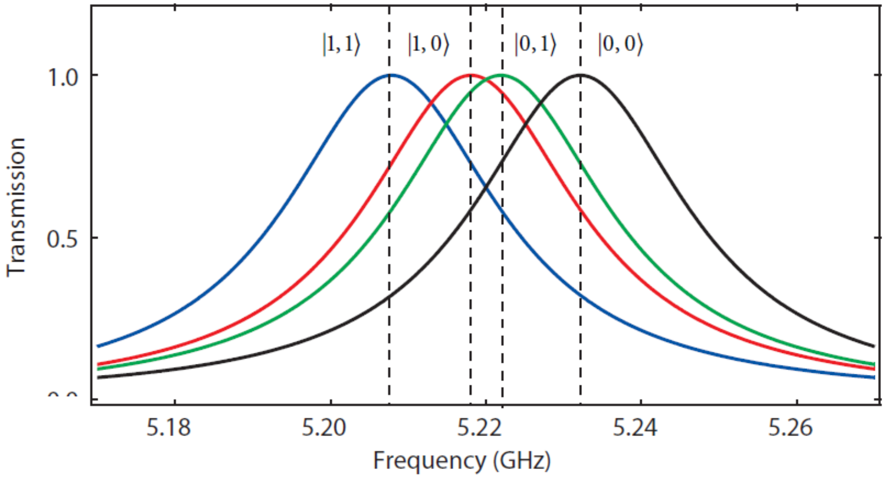


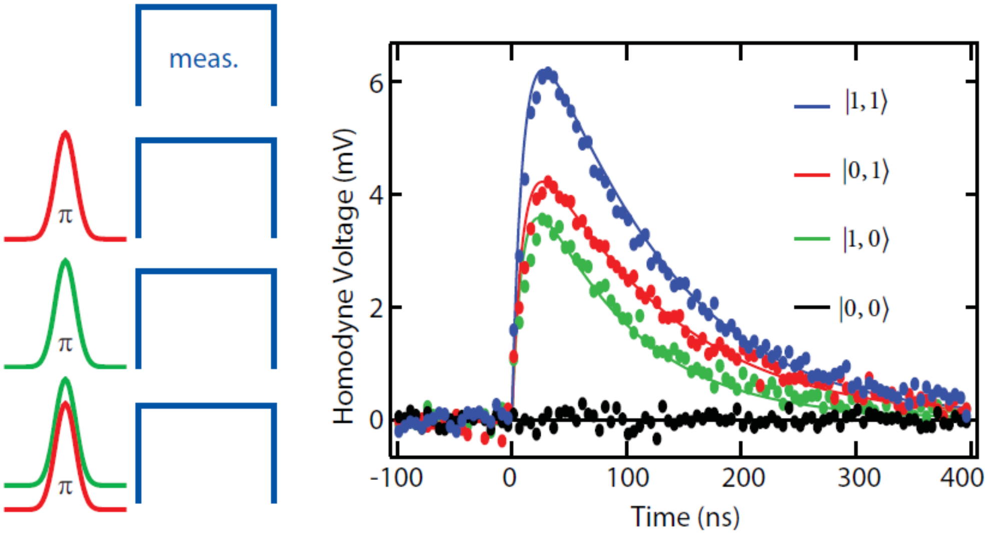
2 tone:

ω_S: qubit frequency (here continuous)

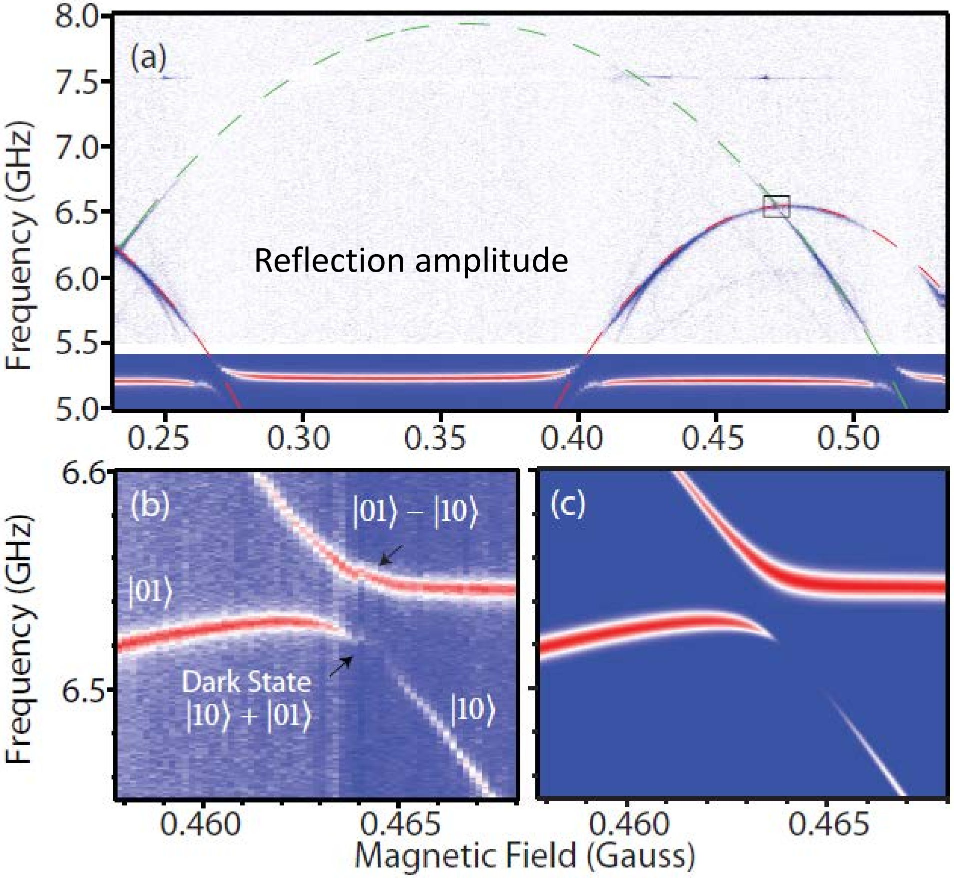
ω_{RF}: cavity frequency (here continuous)

The 2 qubits can be addressed separately if their frequncy is detuned
Separate characterization is possible





Dispersive readout: Due to the different parameters of the 2 qubit, all the states of the 2-qubit system can be read out with the cavity (different dispersive shift).
T1 and T2 can be measured.



2 tone:
ω_S: qubit frequency (here continuous)
ω_{RF}: cavity frequency (here continuous)

$$2J = 2g^{(1)}g^{(2)}/\Delta = 2\pi \cdot 26 \text{ MHz}$$

The qubit states hibridize with the cavity and also with each other if they are on resonance (dark state some interfence effect)

Global magnetic field – tunes both qubits.

J. Majer et al., Nature 449, 443 (2007)
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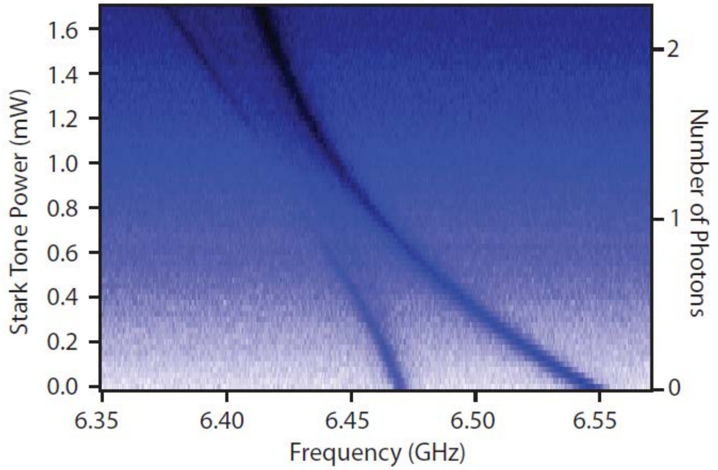
$$\hat{H} = \frac{1}{2}\hbar\left(\omega_q^{(1)} + \chi^{(1)}\hat{a}^\dagger\hat{a}\right)\sigma_Z^1 + \frac{1}{2}\hbar\left(\omega_q^{(2)} + \chi^{(2)}\hat{a}^\dagger\hat{a}\right)\sigma_Z^2 + \hbar\omega_r\hat{a}^\dagger\hat{a} + \hbar J\left(\sigma_-^{(1)}\sigma_+^{(2)} + \sigma_-^{(2)}\sigma_+^{(1)}\right)$$

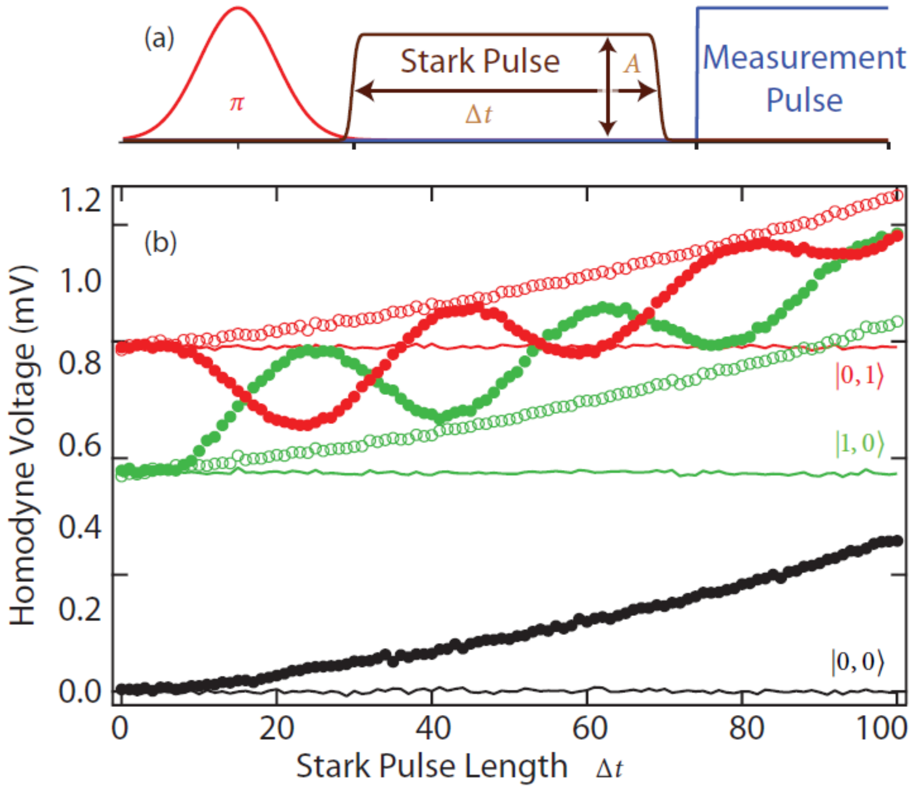
Flux biasing to avoided crossing
not fast enough
Use stark effect
2 qubits at 6.47 and 6.55 GHz -
close to resonance
Drive at 6.675 GHz

Avoided crossing using Stark effect



Size of the Stark shift depends on photon number and detuning

$$\chi = \frac{g^2}{\Delta}$$



Start at state 1,0 → pulse to the avoided crossing where the bonding and the antibonding states are the eigenstates.

The state evolves between 1,0 and 0,1. After Δt waiting time measurement of the state at the cavity frequency

SWAP operation

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