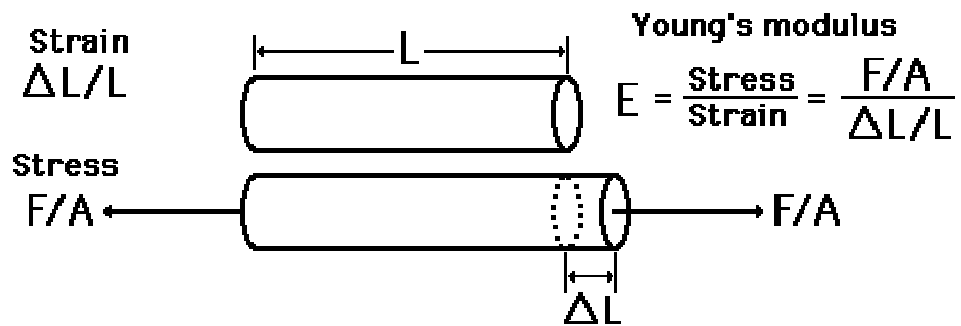
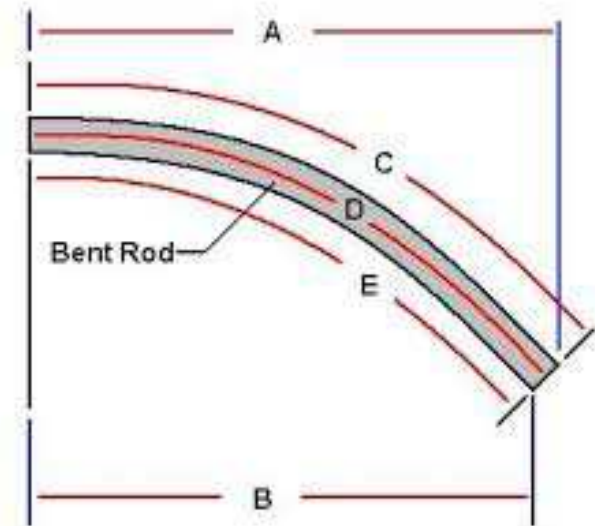
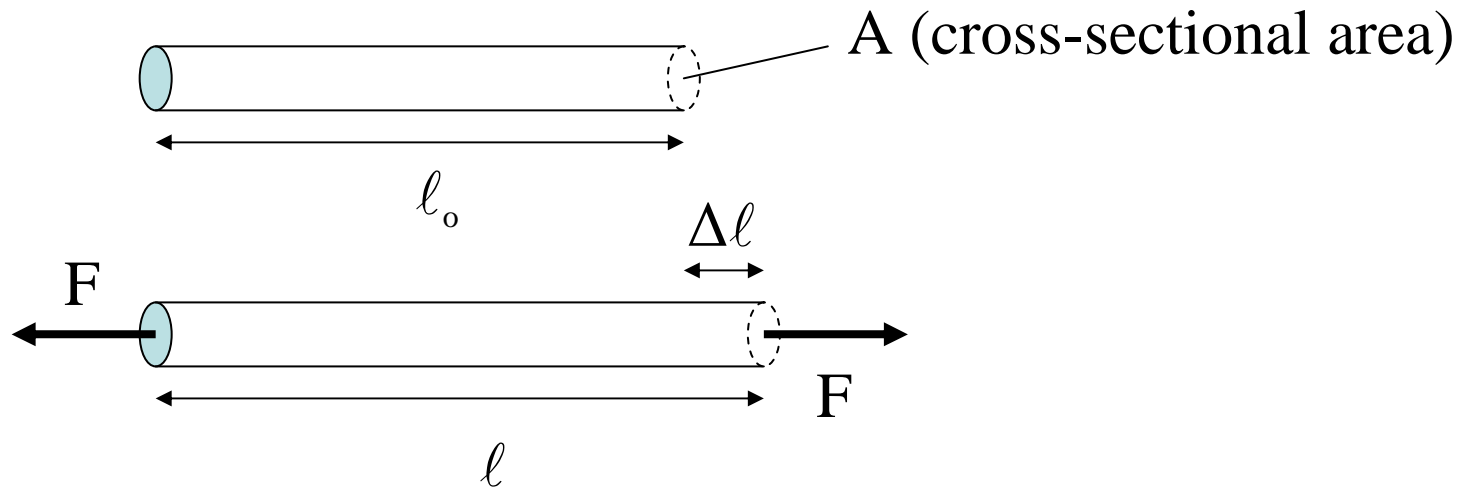


Elastic properties of material

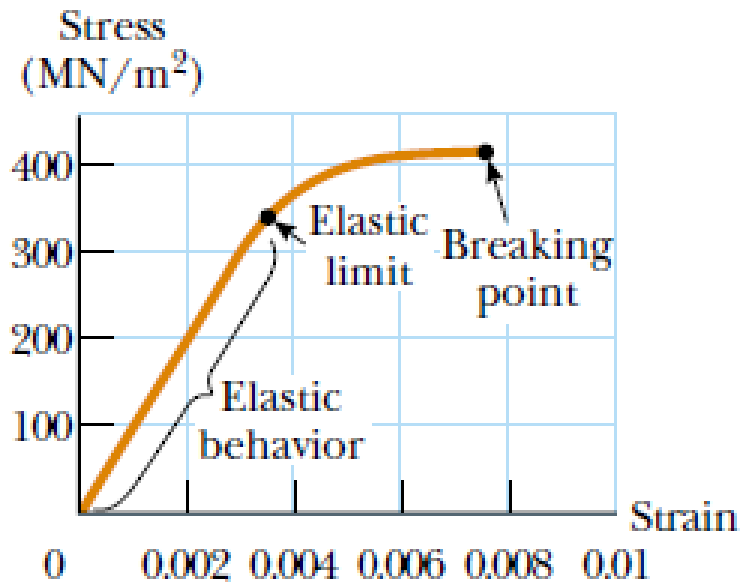


Elastic properties:

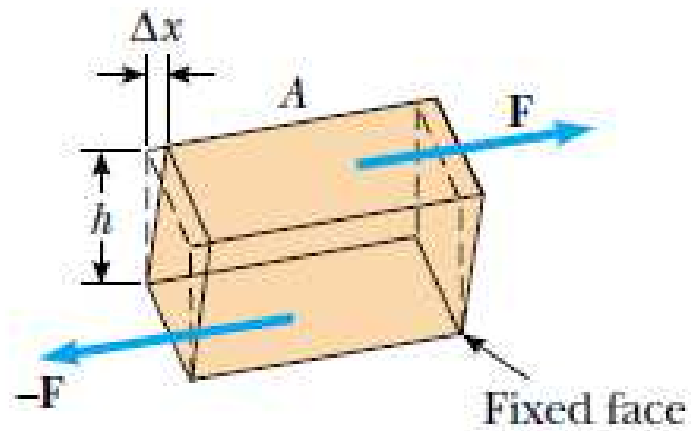


$$\left. \begin{aligned} \text{Tensile stress} &= \frac{F}{A} \\ \text{Tensile strain} &= \frac{l - l_0}{l_0} \end{aligned} \right\} \text{Young's modulus} = \frac{\text{tensile stress}}{\text{tensile strain}} = Y$$

$$\frac{F}{A} = Y \frac{\Delta l}{l_0}$$



Typical Values for Elastic Moduli			
Substance	Young's Modulus (N/m ²)	Shear Modulus (N/m ²)	Bulk Modulus (N/m ²)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5-7.8 \times 10^{10}$	$2.6-3.2 \times 10^{10}$	$5.0-5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}



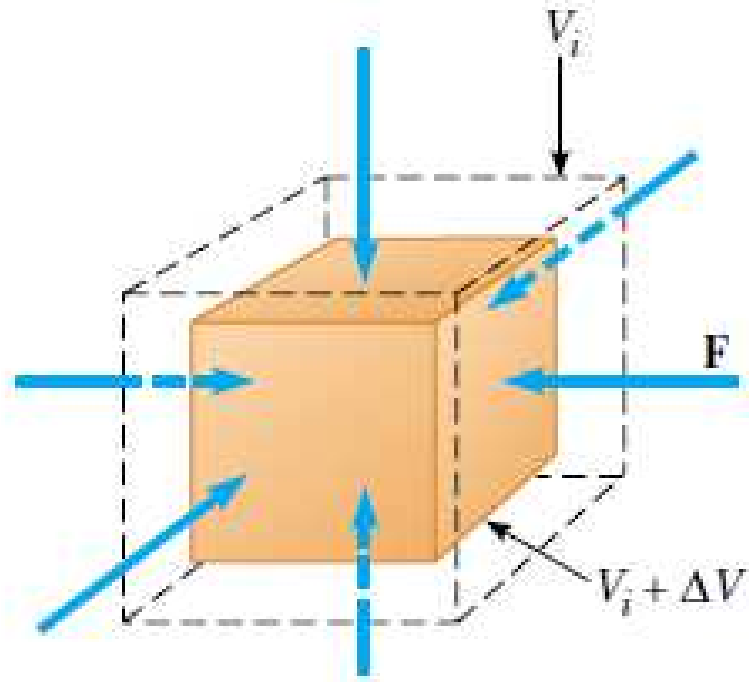
(a)



(b)

Shear modulus:

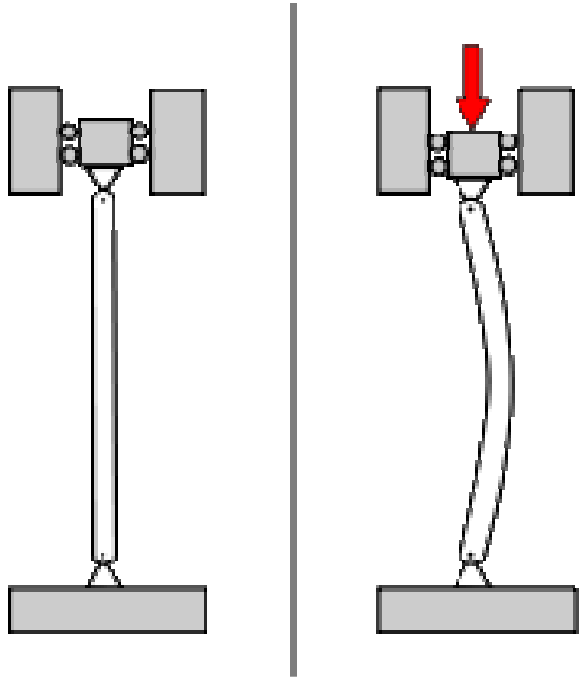
$$S \equiv \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$



Bulk modulus:

$$B \equiv \frac{\text{volume stress}}{\text{volume strain}} = - \frac{\Delta F/A}{\Delta V/V_i} = - \frac{\Delta P}{\Delta V/V_i}$$

Buckling:



Self buckling:

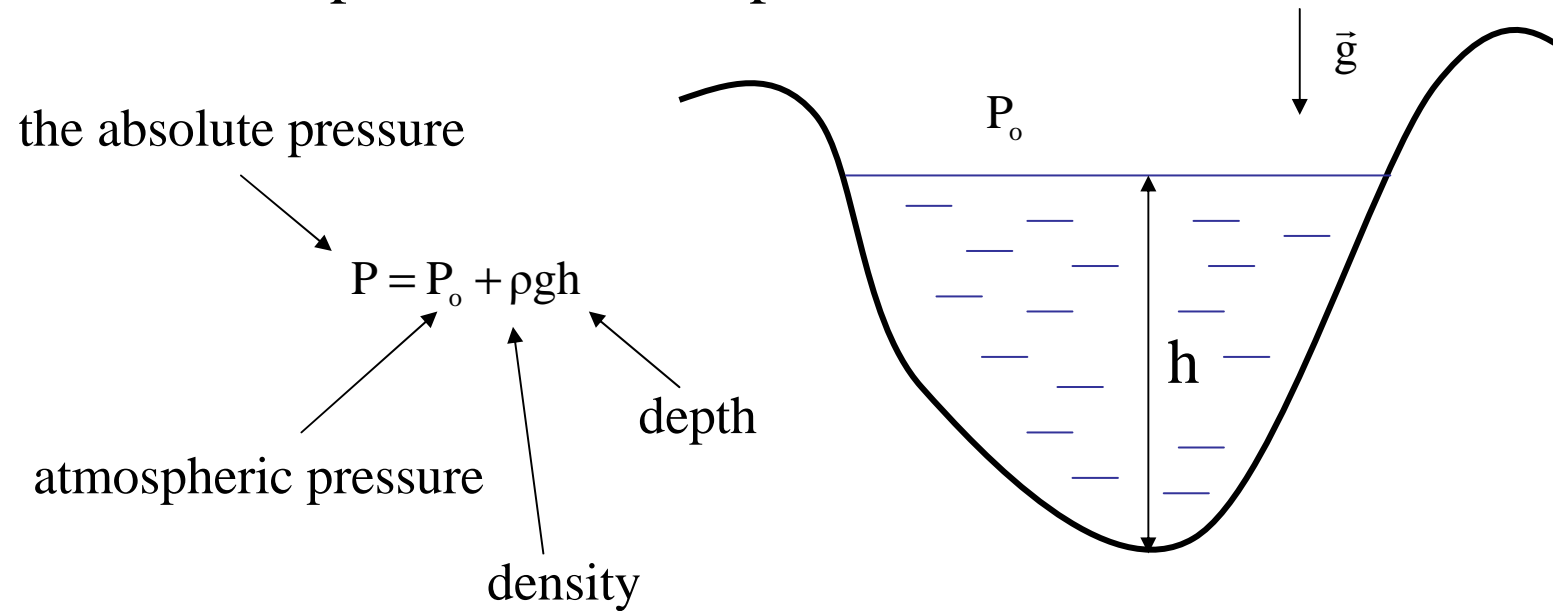
$$h_{crit} = \left(\frac{9B^2}{4} \frac{EI}{\rho g \pi r^2} \right)^{1/3}$$



Fluid mechanics

Def.: pressure $P = \frac{F}{A} \left[\frac{N}{m^2} \right]$

Variation of pressure with depth:

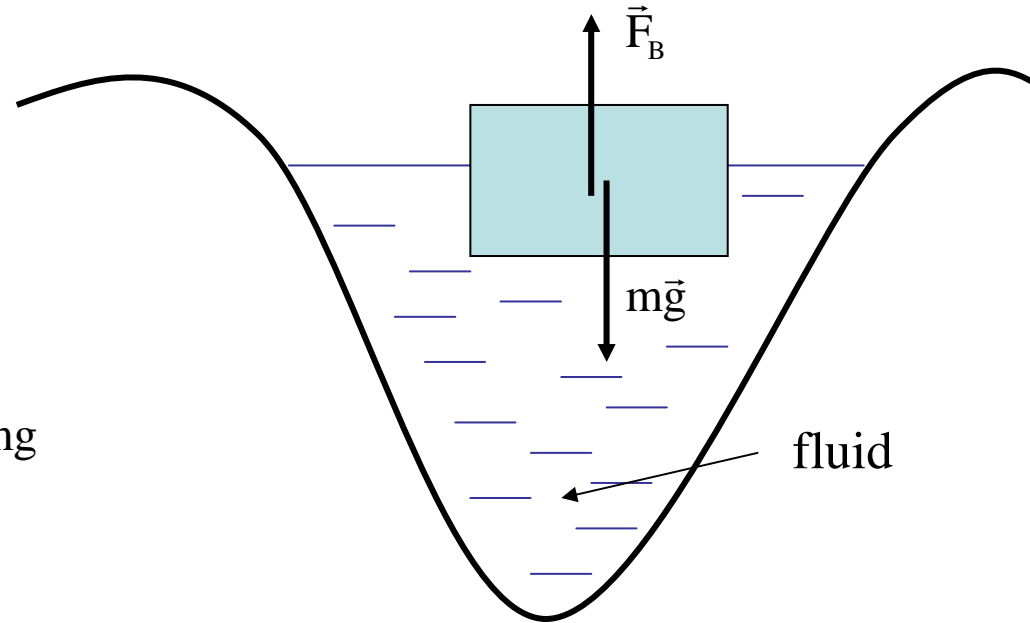


Buoyant force and Archimedes' principle:

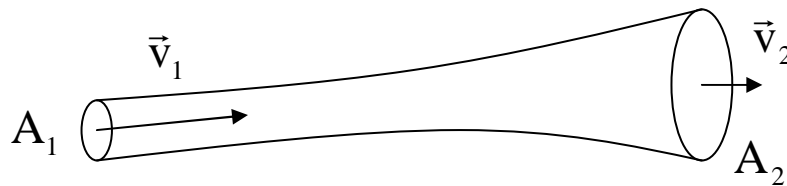
$$\vec{F}_B = \rho_{fl.} g V'$$

(floating object)

$$F_B = \rho_{fl.} g V' = \rho_o g V = mg$$



Fluid dynamics:



$$A_1 v_1 = A_2 v_2$$

Bernoulli's equation:

$$P + \frac{1}{2} \rho v^2 + \rho g y = \text{const}$$

Thermodynamics

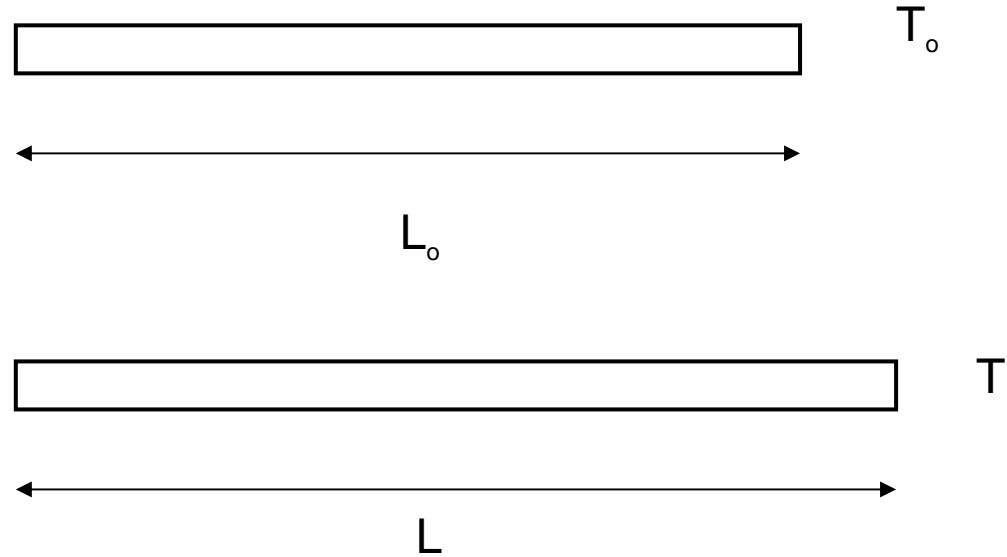
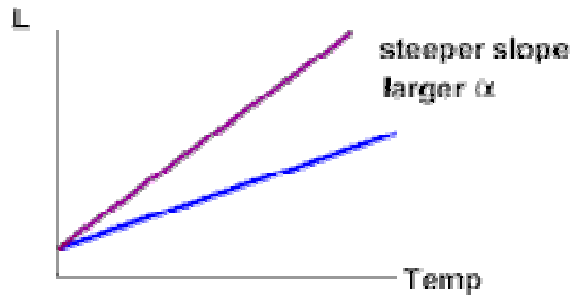
2. Heat expansion

Thermal expansion:

$$T_C = T - 273,15$$

↑ ↑
[°C] [K]

$$T_F = \frac{2}{5} T_C + 32^\circ F$$

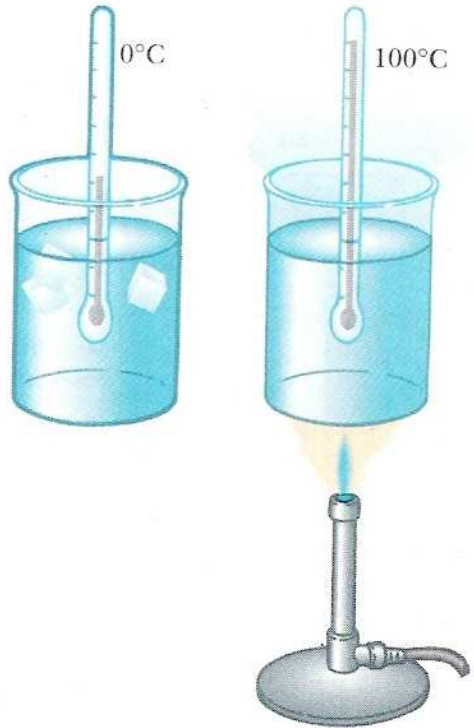
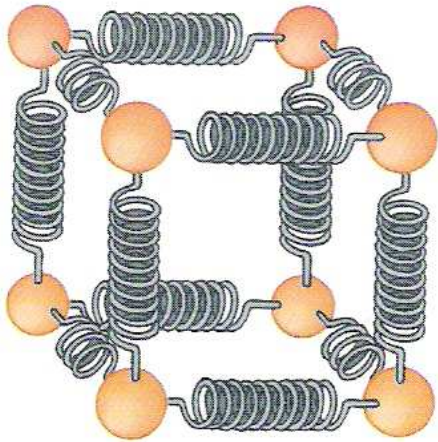
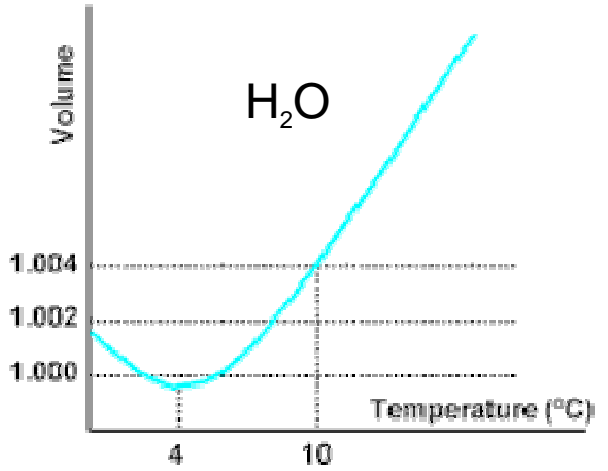
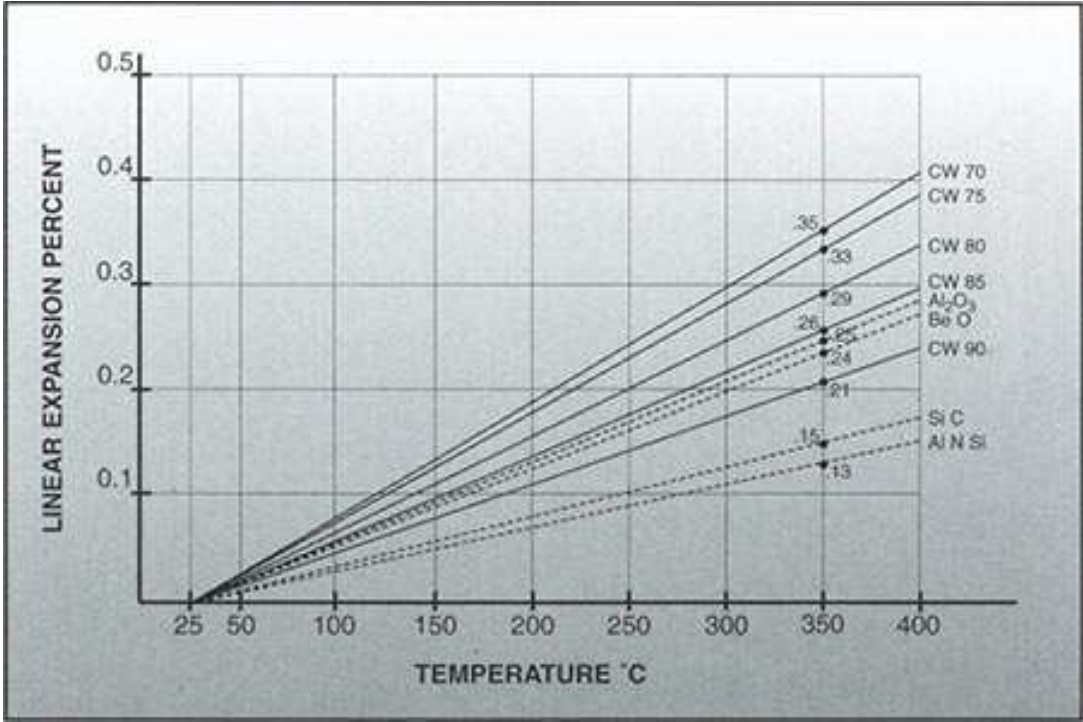


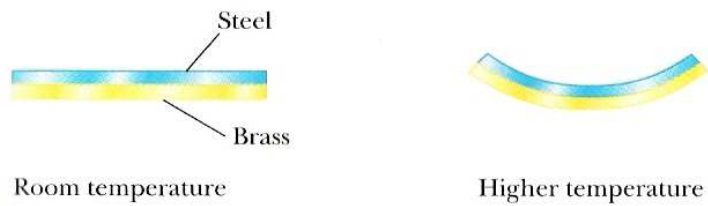
$$\Delta L = \alpha L_0 \Delta T$$

$$L - L_0 = \alpha L_0 (T - T_0)$$

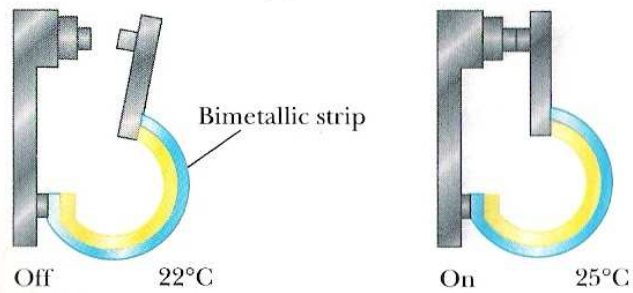
α : average coefficient of linear expansion

[1 / °C]





(a)



(b)



(c)

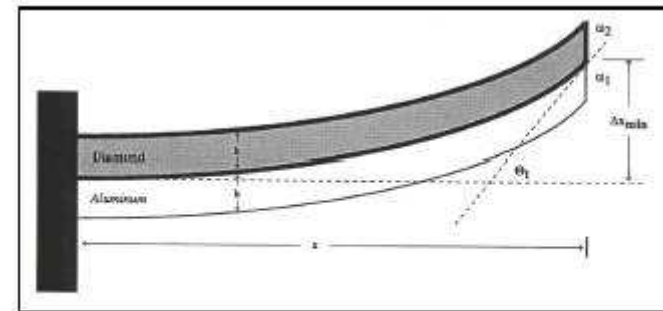
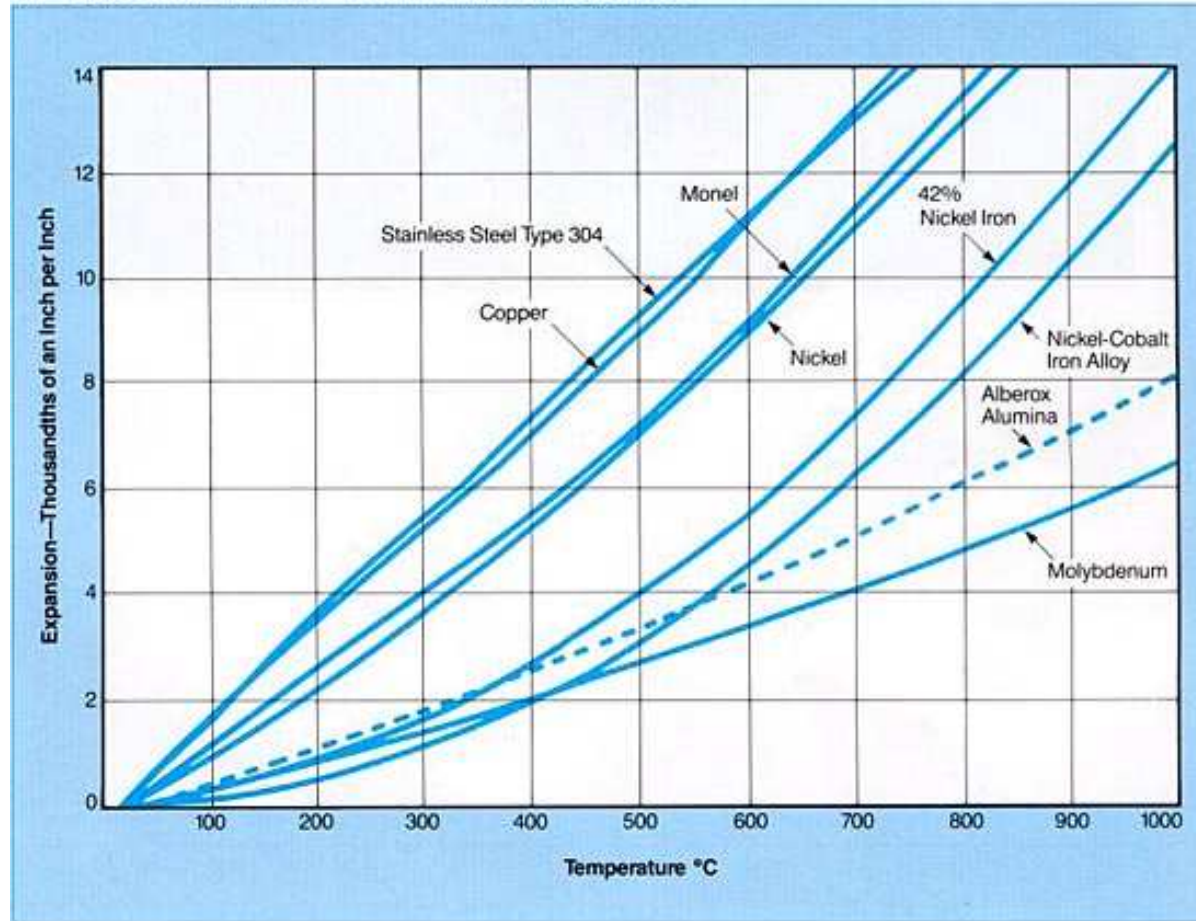


Fig. 4.8. Thermal expansion temperature sensor (schematic representation only).

Material	Linear expansion Coeff. α ($1/^\circ\text{C}$)
Aluminum	$24 \cdot 10^{-6}$
Brass & bronze	$19 \cdot 10^{-6}$
Copper	$17 \cdot 10^{-6}$
Glass	$9 \cdot 10^{-6}$
Glass (Pyrex)	$3,2 \cdot 10^{-6}$
Steel	$11 \cdot 10^{-6}$
Concrete	$12 \cdot 10^{-6}$

APPROXIMATE THERMAL EXPANSIONS



Extreme high temp.



Thermal expansion joints

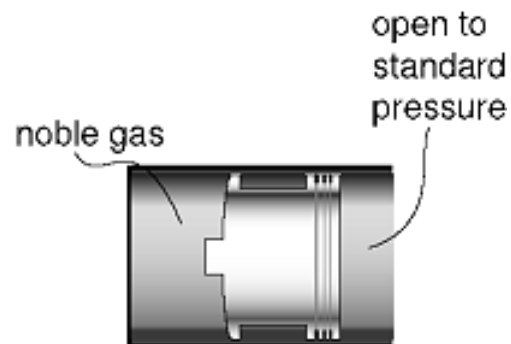
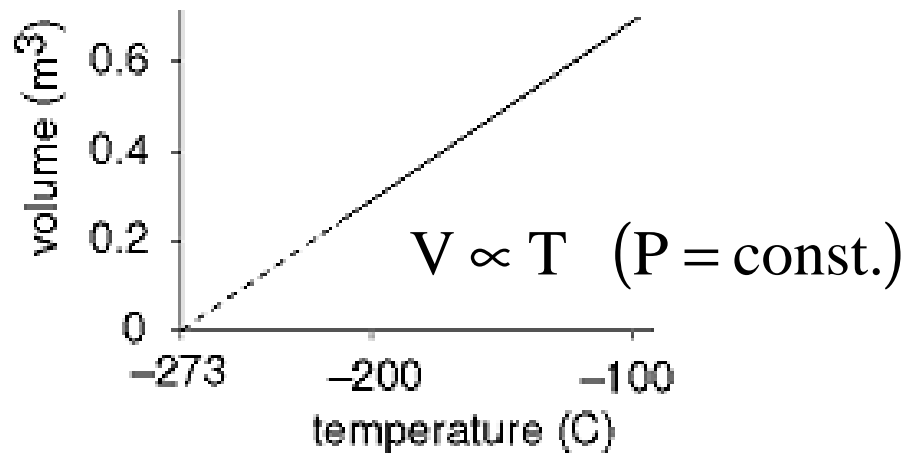
Heat expansion in 2D:

$$\Delta A = 2\alpha A \Delta T$$

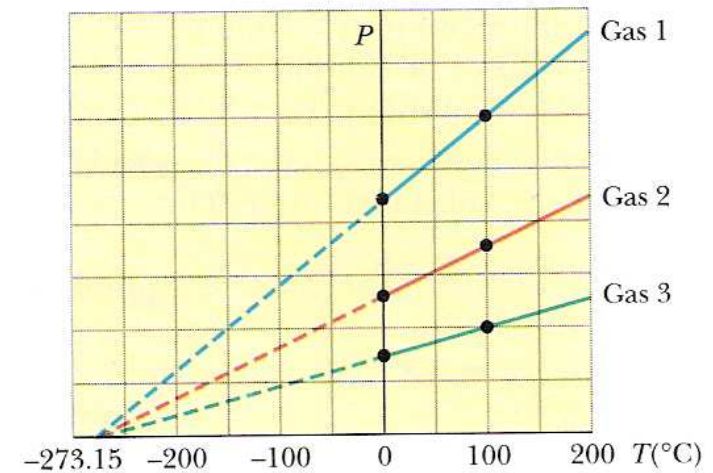
Heat expansion in 3D:

$$\Delta V = \beta V \Delta T \rightarrow \beta = 3\alpha$$

Gas thermometer (experiments)



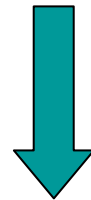
$$P \propto T \quad (V = \text{const.})$$



Ideal gas:

$$\left. \begin{aligned} P &\propto T \quad (V = \text{const.}) \\ V &\propto T \quad (P = \text{const.}) \end{aligned} \right\}$$

$$PV \propto T \quad (m = \text{const.})$$



Equation of state for an ideal gas

$$P \text{ [Pa]}$$

$$V \text{ [m}^3\text{]}$$

$$T \text{ [K]}$$

$$PV = nRT$$

Ideal gas law

Number of moles

$$n = \frac{m}{M}$$

Mass of the Ideal gas

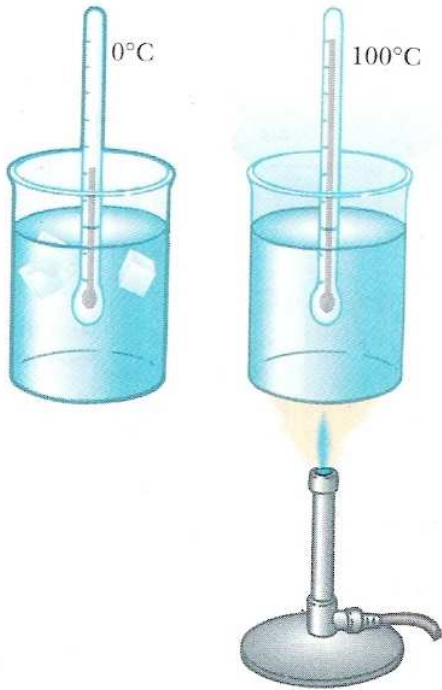
Molar mass

Universal gas constant: $R = 8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}$

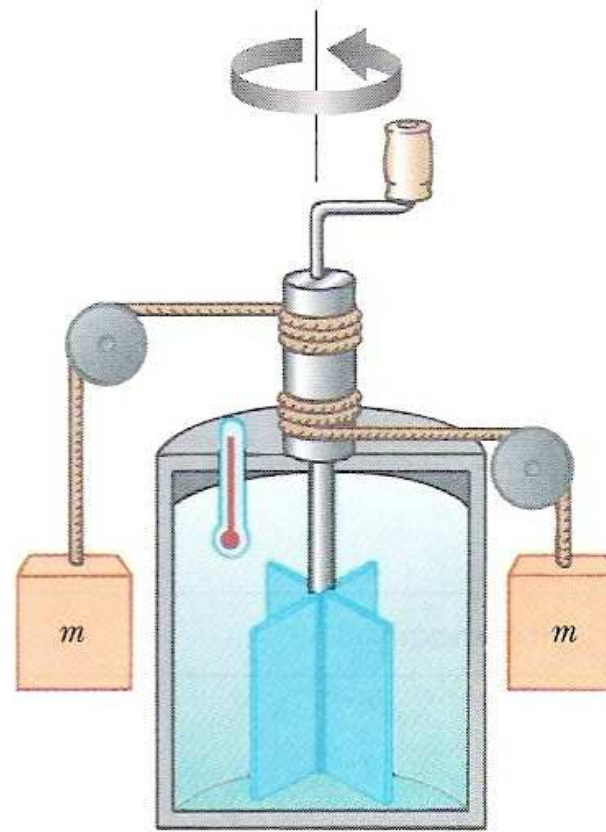
$$1 \text{ mole} : N_A = 6,022 \cdot 10^{23}$$

Work, heat, latent heat & 1st law of thermodynamics

To increase the temperature



Heat transfer



Work
(Joule's experiment)

$$\left. \begin{array}{l} Q \propto \Delta T \\ Q \propto m \end{array} \right\}$$

$$Q = cm\Delta T$$

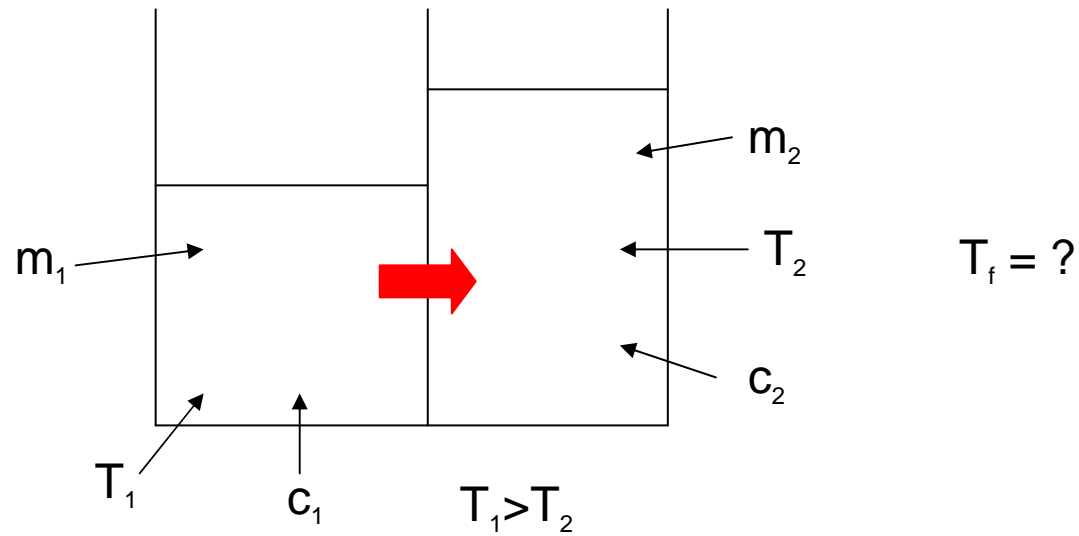
C: specific heat

TABLE 10.3

Specific heat for some common materials at 25°C*

Substance	Specific heat (J/kg·°C)
Water (0°C to 100°C)	4190
Ice (-10°C to 0°C)	2090
Steam (100°C)	2010
Wood	1700
Aluminum	900
Marble	860
Glass	840
Iron	448
Copper	390
Zinc	386
Silver	236
Lead	128

*The specific heat of most materials varies slightly with temperature, but is often taken to be constant.



Heat rejected : $Q_1 = c_1 m_1 (T_1 - T_f)$

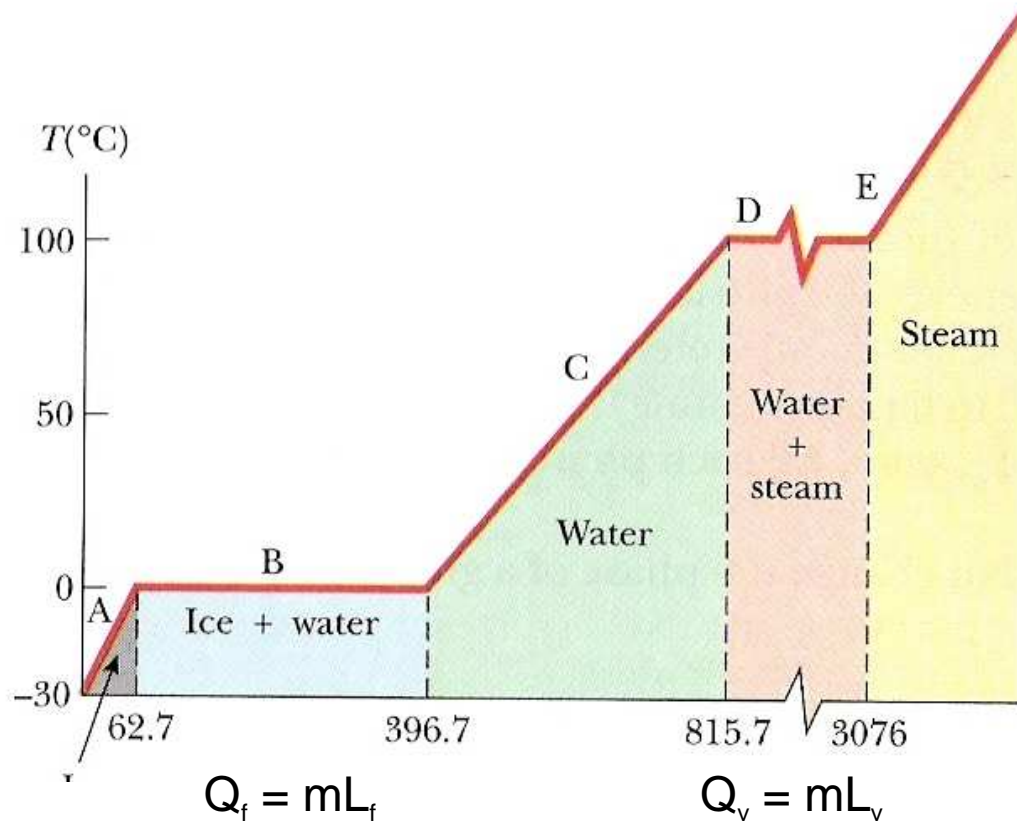
Heat accepted : $Q_2 = c_2 m_2 (T_f - T_2)$

$$Q_1 = Q_2$$

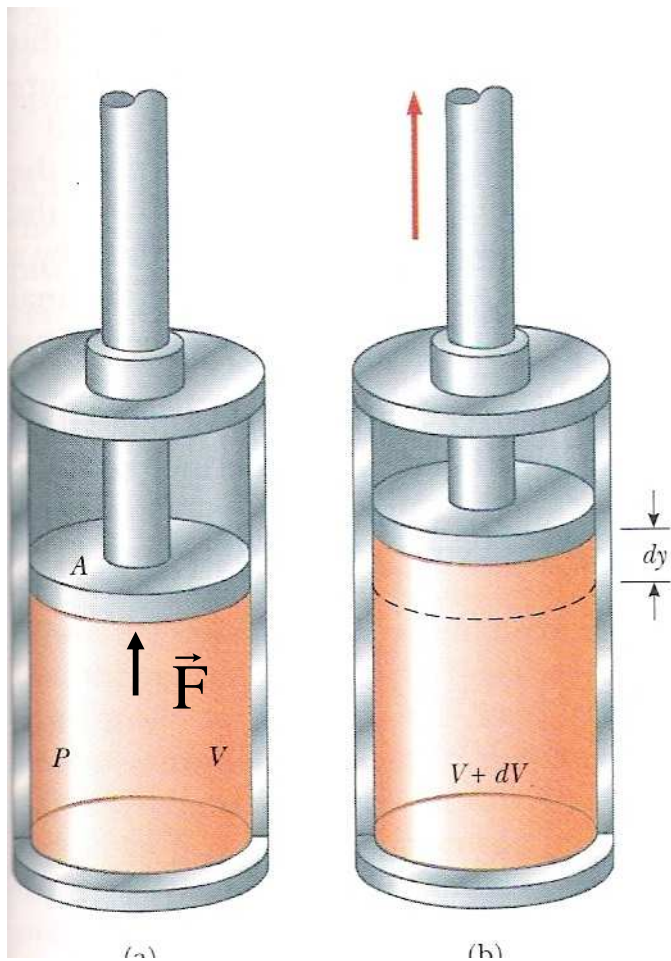
$$T_f = \frac{c_1 m_1 T_1 + c_2 m_2 T_2}{c_1 m_1 + c_2 m_2}$$

Latent heat

The thermal energy required to change the phase : $Q = mL$



Work done by the gas

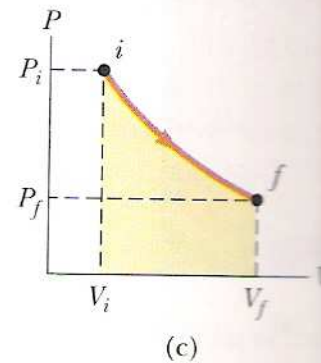
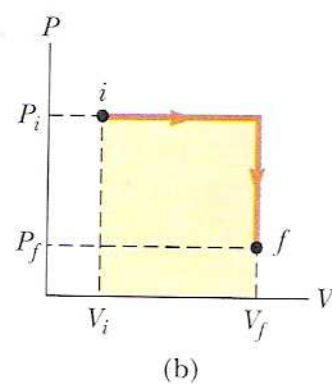
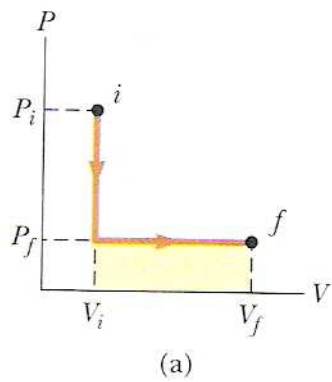
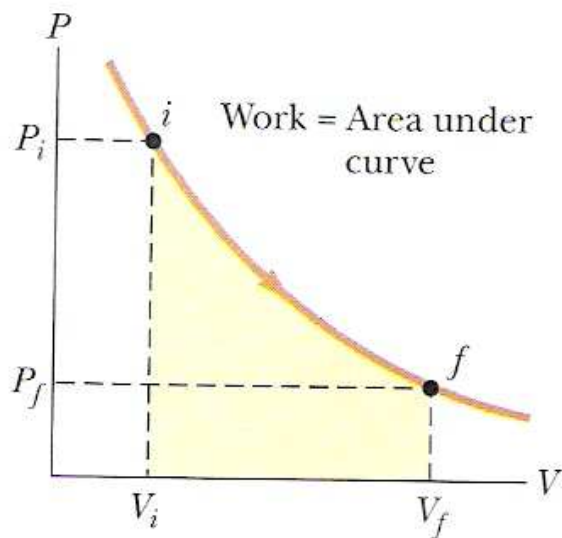


$$\Delta V = As$$

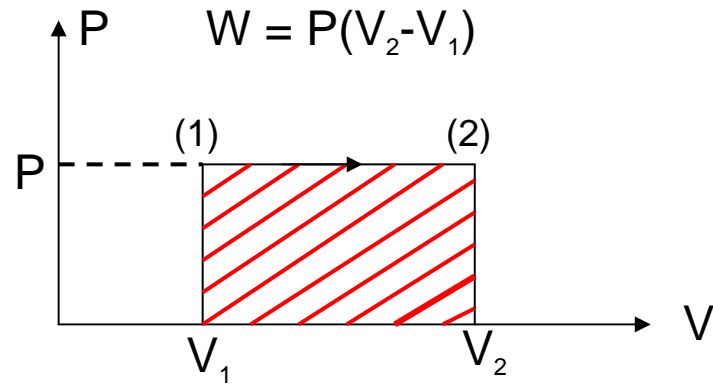
F
a
c
t

$$dy = s$$

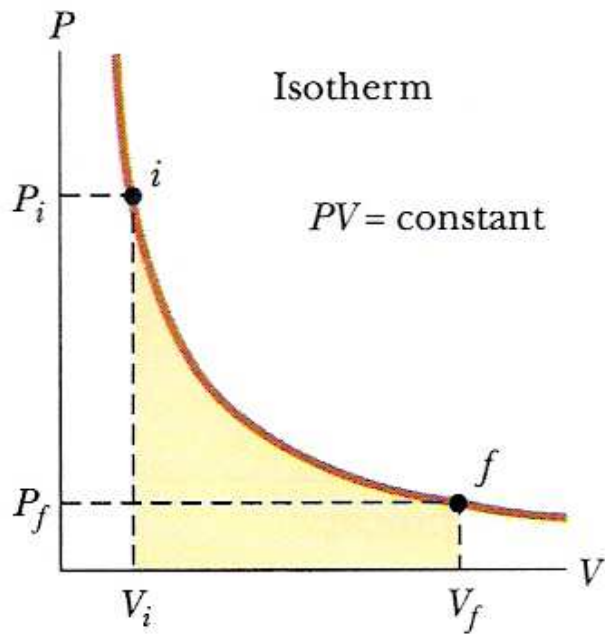
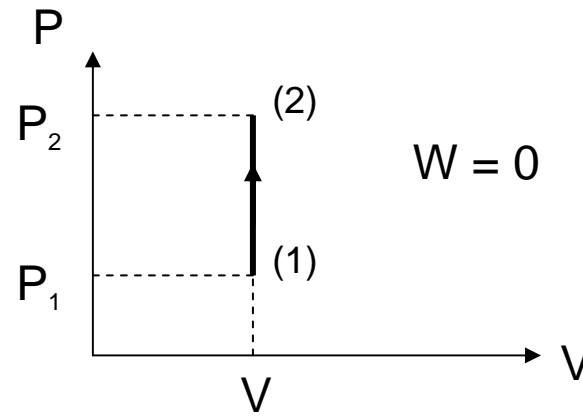
$$W = Fs = PAs = P\Delta V$$



If $P = \text{const.}$ isobar



If $V = \text{const.}$ Isochor (isovolumic)



If $T = \text{const.}$

$$W = nRT \ln \left(\frac{V_f}{V_i} \right)$$

1st law of thermodynamics :

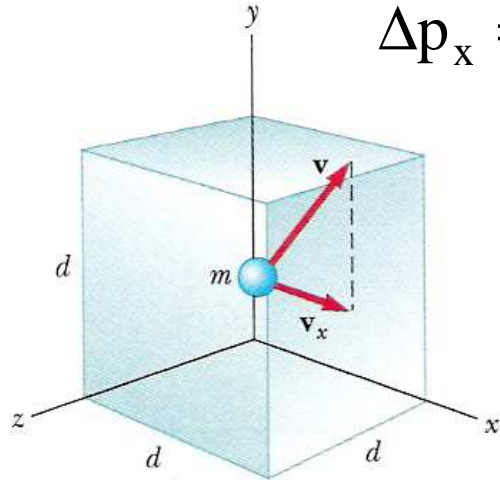
$$\Delta U = \Delta Q - W_g$$

U : internal energy

$$\Delta U = \Delta Q + W_s$$

$$W_g = -W_s$$

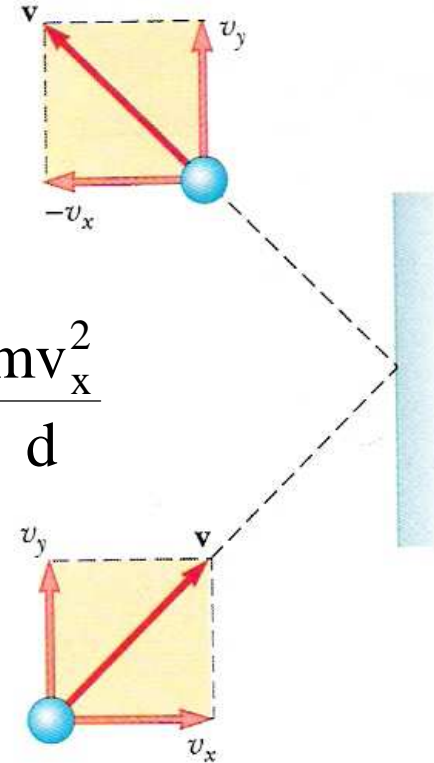
Kinetic theory of gases



$$\Delta p_x = -mv_x - (mv_x) = -2mv_x$$

$$F_x \Delta t = \Delta p_x = 2mv_x$$

$$F_x = \frac{2mv_x}{\Delta t} = \frac{2mv_x}{2d/v_x} = \frac{mv_x^2}{d}$$



$$F_x = \frac{m}{d} (v_{1x}^2 + v_{2x}^2 + \dots)$$

$$\overline{v_x^2} = \frac{v_{1x}^2 + \dots + v_{Nx}^2}{N}$$

$$\overline{v^2} = 3\overline{v_x^2}$$

$$F_x = \frac{Nm}{d} \overline{v_x^2}$$

$$F_x = \frac{N}{3d} m \overline{v^2}$$

$$P = \frac{F}{A} = \frac{F}{d^2}$$

$$P = \frac{2}{3} \frac{N}{V} \left(\frac{1}{2} m \overline{v^2} \right)$$

$$PV = \frac{2}{3} N \left(\frac{1}{2} m \overline{v^2} \right) = nRT = \frac{N}{N_A} RT = N \left(\frac{R}{N_A} \right) T = N k_B T$$

$$\left(\frac{1}{2} m \overline{v^2} \right) = 3 \cdot \frac{1}{2} k_B T \quad f = 3$$

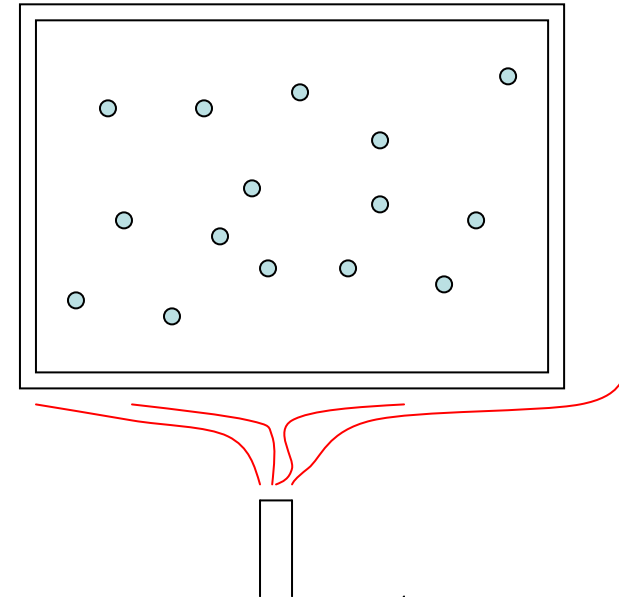
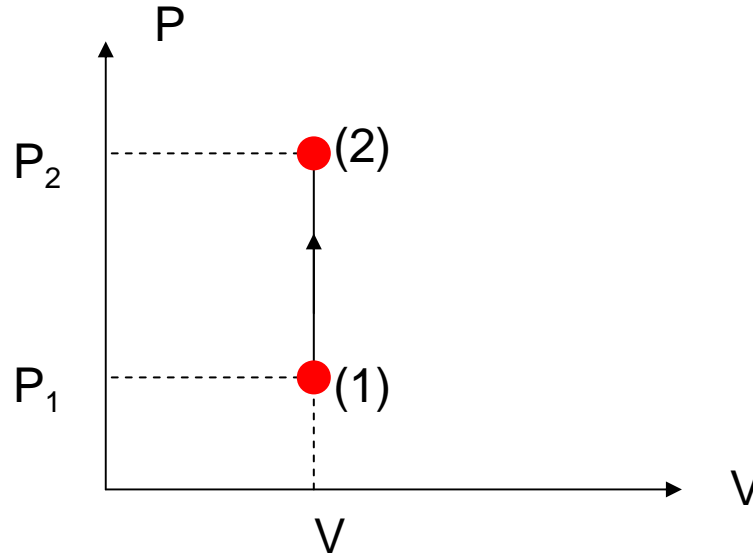
$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}}$$

Internal energy: $E_k = N \left(\frac{1}{2} m \overline{v^2} \right) = N \frac{3}{2} k_B T = n \frac{3}{2} RT$

Isochoric process (isovolumetric pr.)

$V = \text{const.}$

$W=0$



$$\left. \begin{aligned} P_1 V_1 &= nRT_1 \\ P_2 V_2 &= nRT_2 \end{aligned} \right\}$$

$V\Delta P = nR\Delta T$

$f=3$

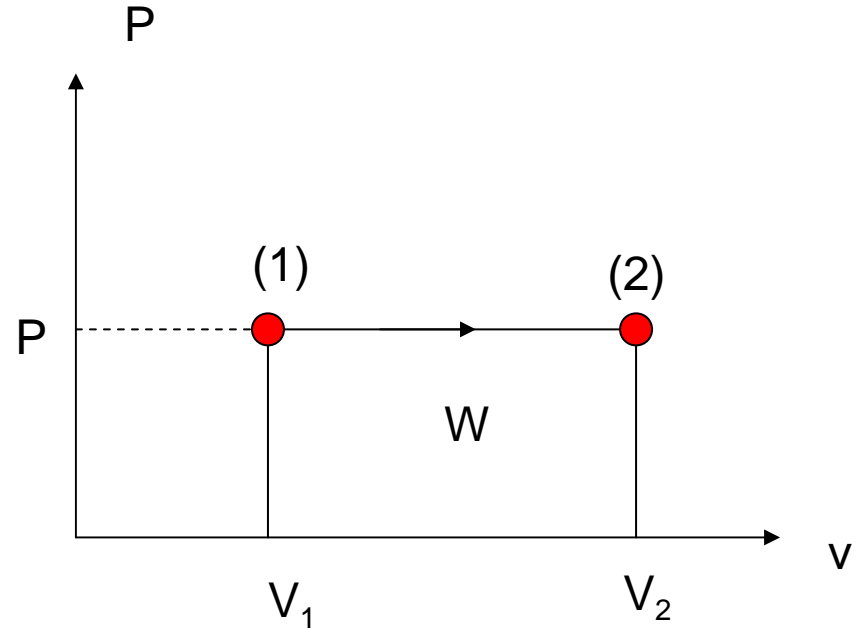
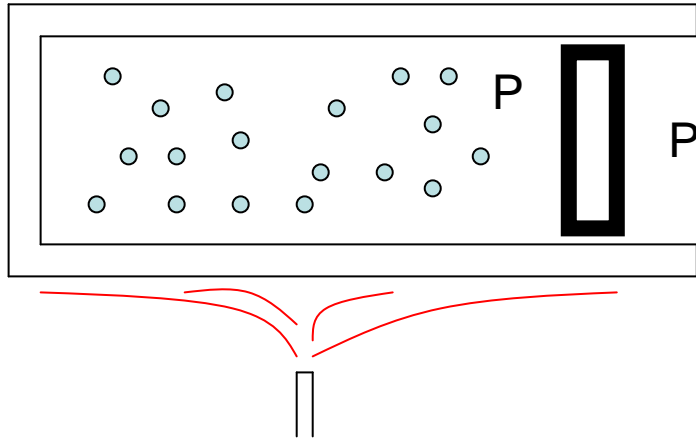
$$\Delta U = \Delta Q + \cancel{W_s} = 0$$

$$U = n \frac{3}{2} RT \rightarrow \Delta Q = \Delta U = n \left(\frac{3}{2} R \right) \Delta T = n C_V \Delta T$$

$$C_V = \frac{f}{2} RT$$

$$U = n C_V T = n \frac{f}{2} RT$$

Isobaric process



$$\left. \begin{array}{l} PV_1 = nRT_1 \\ PV_2 = nRT_2 \end{array} \right\} P(V_2 - V_1) = nR(T_2 - T_1)$$

$$\downarrow$$

$$W = P\Delta V = nR\Delta T$$

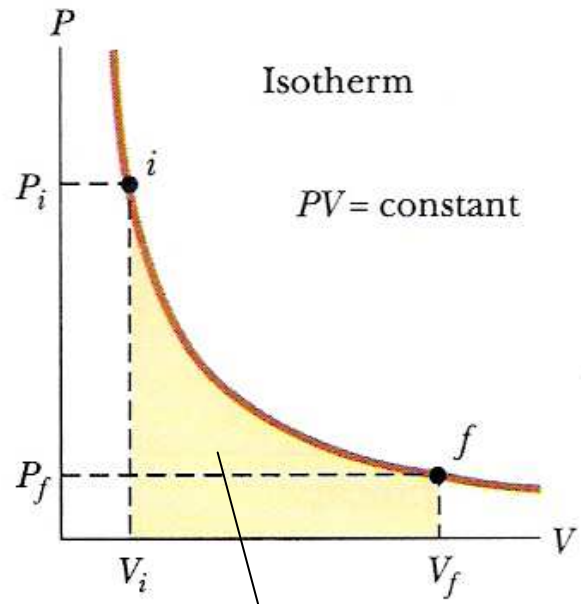
$$\Delta U = \Delta Q - W_g$$

$$\Delta Q = \Delta U + W_g = n \frac{f}{2} R \Delta T + P\Delta V = n \frac{f}{2} R \Delta T + nR\Delta T = n \left(\frac{f+2}{2} R \right) \Delta T$$

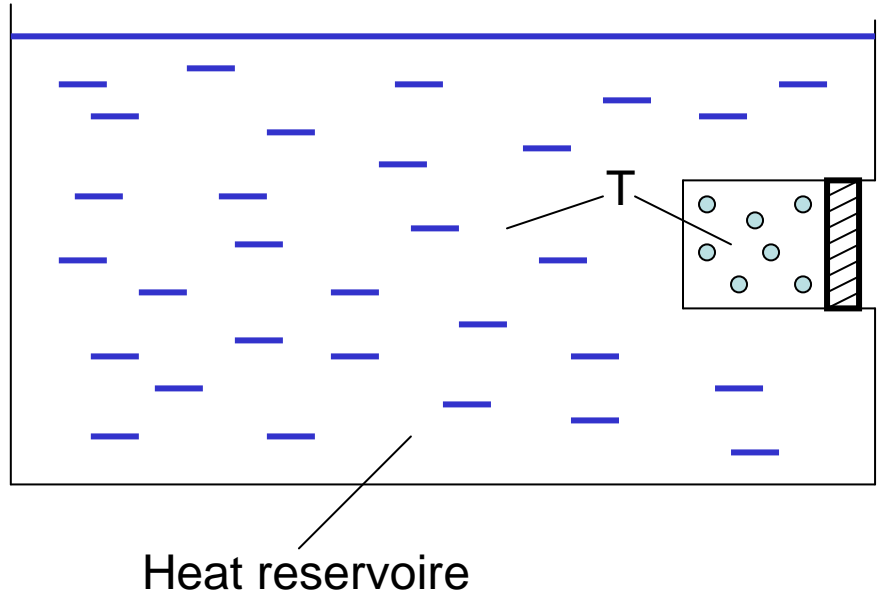
$$\Delta Q = nC_p \Delta T$$

$$C_p = \frac{f+2}{2} R$$

Isothermal process $\Delta T=0$



$$W_{\text{gr}} = nRT \ln\left(\frac{V_2}{V_1}\right)$$



$$\Delta U = \Delta Q - W_{\text{gr}} \quad \Delta U = nC_V \Delta T = 0$$

$$W_{\text{gr}} = \Delta Q$$

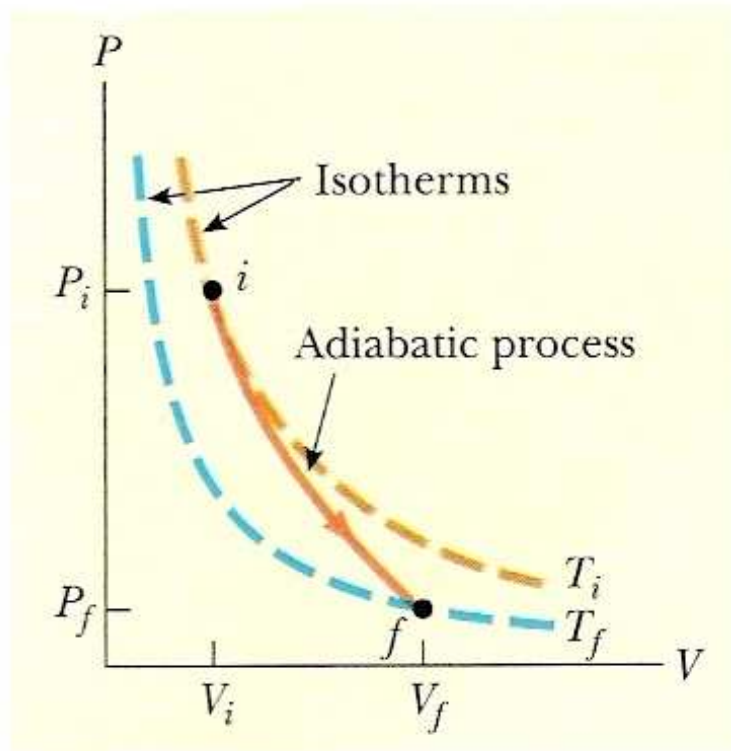
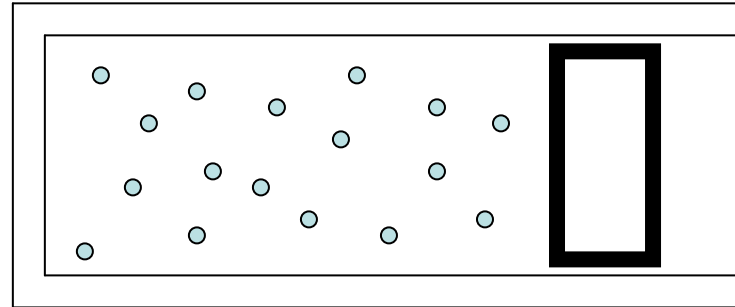
Adiabatic process

$$\Delta Q = 0$$

(no heat transfer)

$$\Delta U = \Delta Q - W_g$$

$$\Delta U = -W_g$$

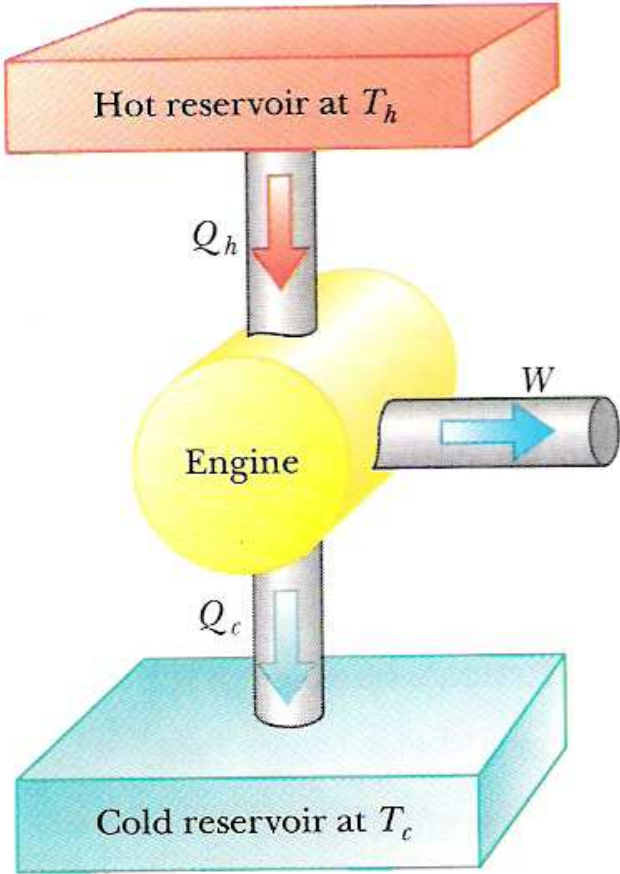


$$PV^\kappa = \text{const.}$$

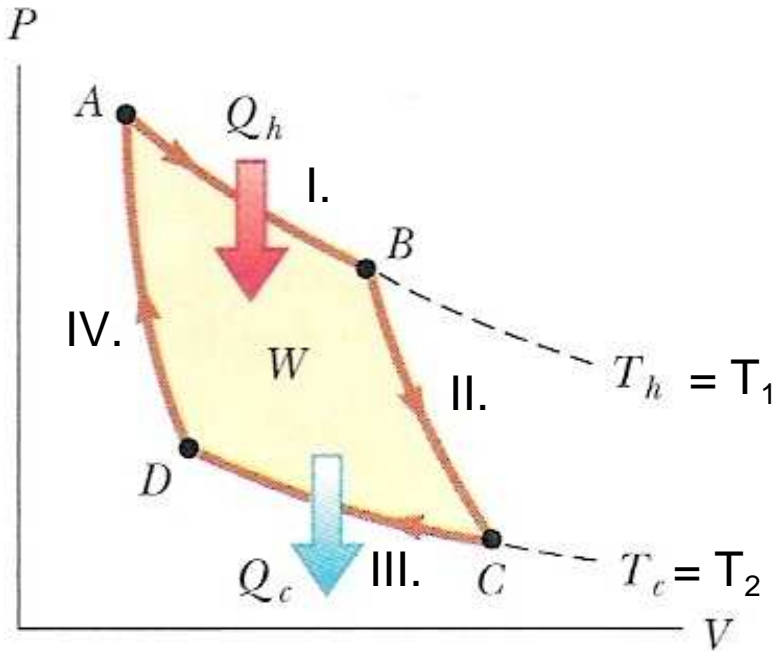
$$\kappa = \frac{C_P}{C_V} = \frac{f + 2}{f}$$

$$P_1 V_1^\kappa = P_2 V_2^\kappa$$

Heat engine



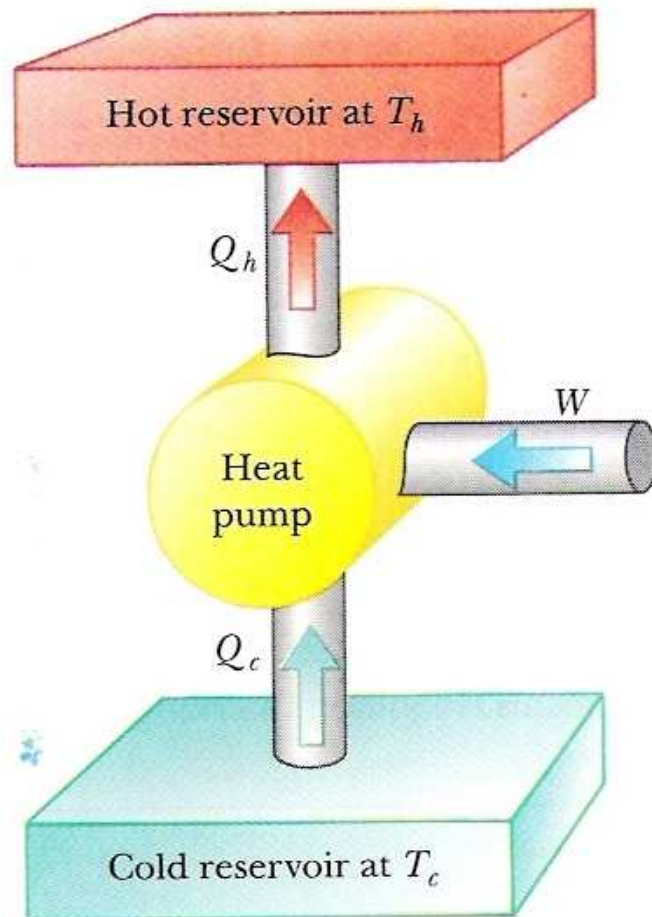
Carnot-cycle



in

Efficiency:
$$\varepsilon = \frac{W}{Q_I - Q_{II}} = 1 - \frac{T_2}{T_1}$$

Heat pump and refrigerator



Schematic diagram for a refrigerator and a heat pump

Coefficient of performance

$$\text{COP (heat pump)} = \frac{Q_h}{W} = \frac{Q_1}{W} = \frac{T_1}{T_1 - T_2}$$

$$\text{COP (refrigerator)} = \frac{Q_c}{W} = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$