Quantum Information Processing, BME 2019 Spring
Lecture 10, Feb 20, 2019
Classical error correction example: the repetition code
Exercises

Akos Budai, Andras Palyi, Zoltan Zimboras
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I. EXERCISES

1. Take a classical channel, with bitflip error with probability $p = 0.1$. Consider the 3-bit repetition code. Calculate the maximum error and the average error for the majority vote decoder. Specify the maximum-likelihood decoder. Is it the same as the majority vote decoder, or is it different?

2. Generalize the previous calculation to the N-bit case. In particular, plot the maximum error as a function of code size N, with N going for odd integers starting from 3 to 13, on a lin-lin plot as well as on a lin-log (x: lin, y: log) plot.

3. Characterize a qubit on a chosen QX machine: how well does it function as a classical channel? Prepare state 0 and measure this in the computational basis, with 8192 shots. What is the probability of finding 1 as the readout result? Prepare state 1 with an X gate, and measure this in the computational basis, again with 8192 shots. What is the probability of finding 0 as the readout result? What is the average error and maximum error of this qubit (see lecture slides for definition of average error and maximum error)?

4. Implement a classical 3-bit error correction code on QX. Prepare 000 and measure the output bitstring, with 8192 shots. Prepare 111 using three X gates, measure the output bitsring, again with 8192 shots. Use the data set you obtained to empirically determine the 'empirical maximum-likelihood decoder'. Is it the same as the majority vote decoder? Is it the same as the maximum-likelihood decoder for the relaxation channel described in the lecture? What is the maximum error of this empirical decoder? What is its average error? Is the 3-bit repetition code better than a single bit?

5. Repeat the previous exercise, with the modification that the preparation of the 111 is done using a single X gate and two CNOTs: preparation = $\mathrm{CNOT}_{1\to 3}\mathrm{CNOT}_{1\to 2}X_1$. Compare the decoder errors you obtain with those in the previous exercise.