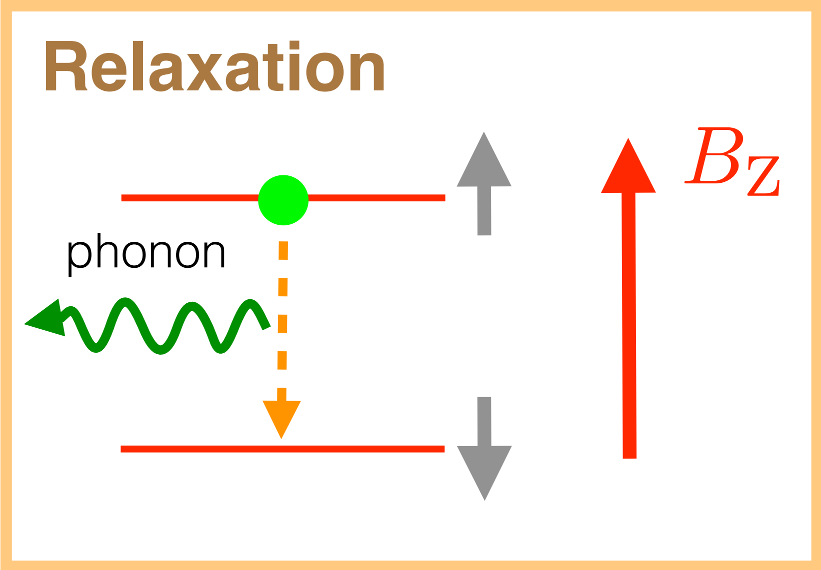
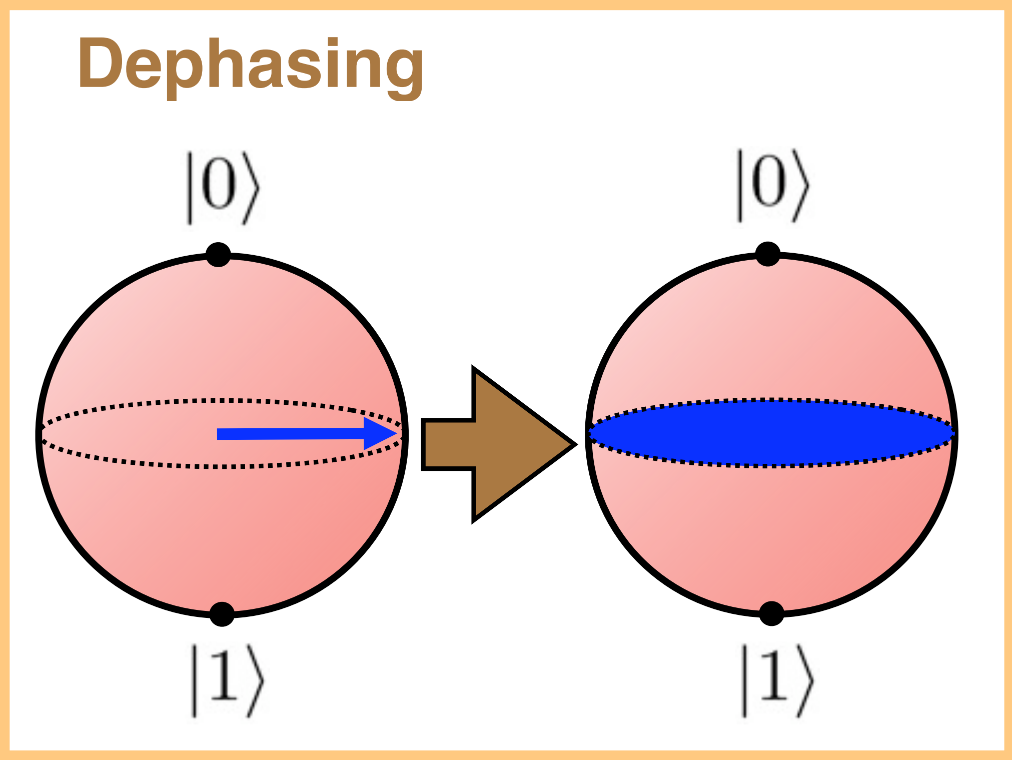
Quantum Computing Architectures

Budapest University of Technology and Economics
2018 Fall



Lecture 5

Information loss mechanisms
for electron spins



Schedule of this course

Szerda	
augusztus 29. - Regisztrációs hét -	
szeptember 5.	lecture 01
szeptember 12.	
szeptember 19.	lecture 02
szeptember 26. TTK Dékáni szünet	
október 3.	lecture 03
október 10.	
október 17.	lecture 04
október 24.	lecture 05 (today)
október 31.	lecture 06
november 7.	lecture 07
november 14. TDK konferencia	
november 21.	lecture 08
november 28.	lecture 09
december 5.	lecture 10

Introduction

Spin qubits
(electron spin)

Superconducting qubits
(transmon)

(Spin) Qubit Checklist

1. make a few qubits
2. initialize
3. control (1-qubit gate, 2-qubit gate)
4. read out
5. understand and reduce information loss

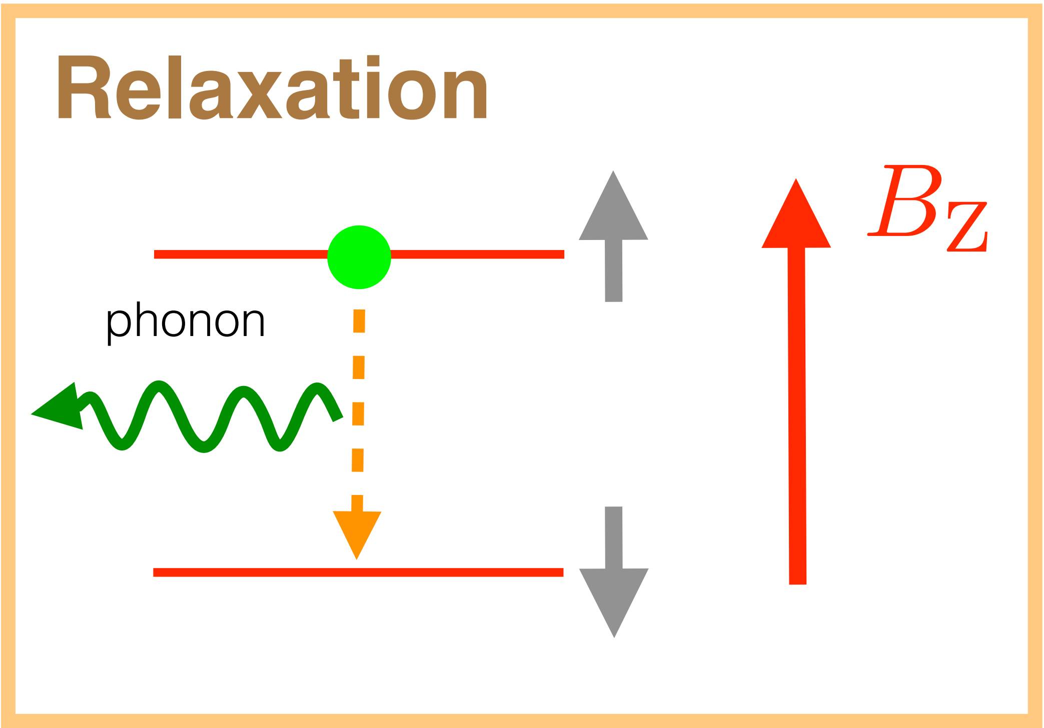
lecture 4

today

lecture 3

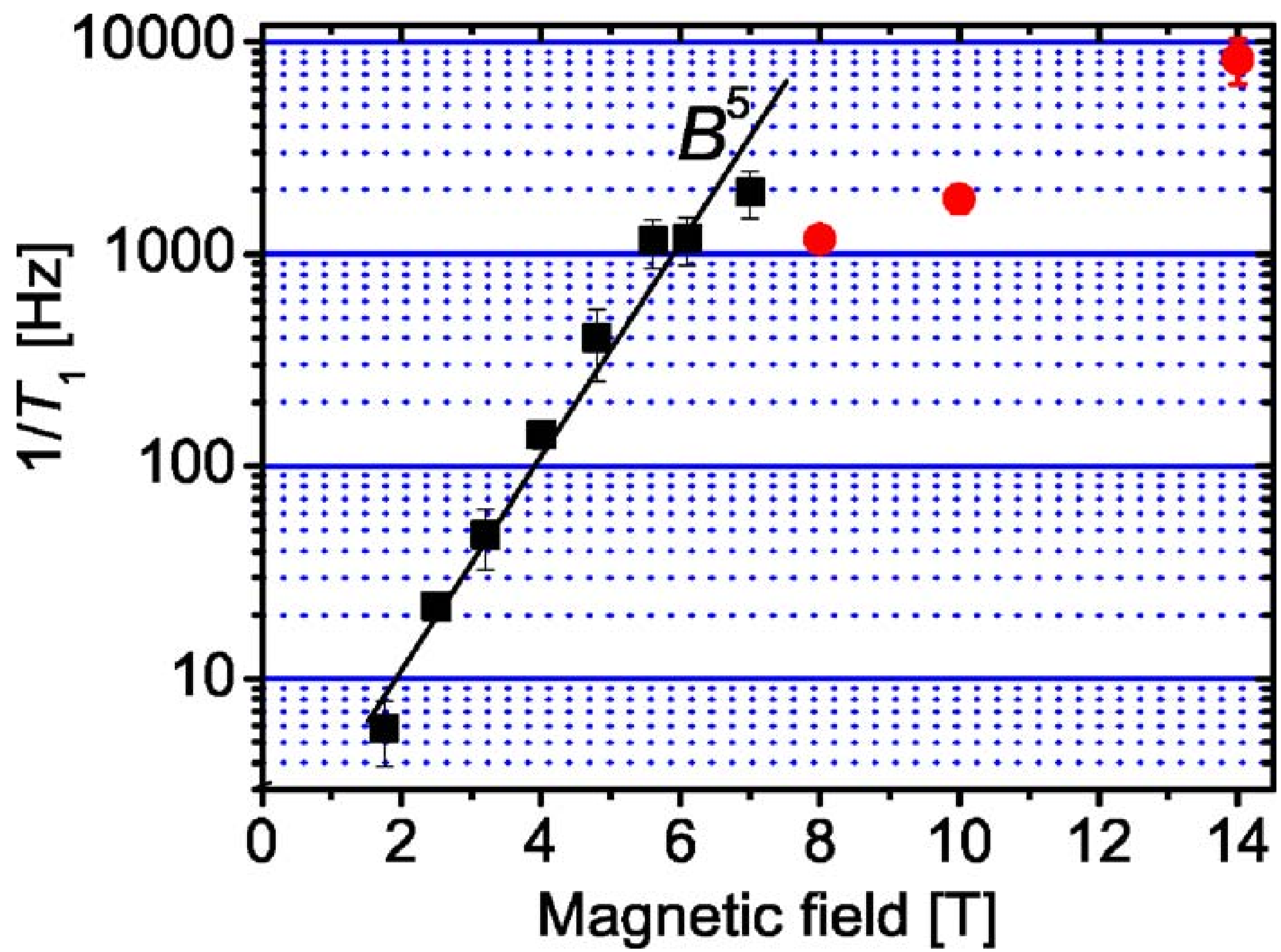
review papers: Hanson et al., Rev. Mod. Phys. (2007), Zwanenburg et al., Rev. Mod. Phys. (2013)

Spin relaxation time measured in a GaAs quantum dot



Spin relaxation time: T_1

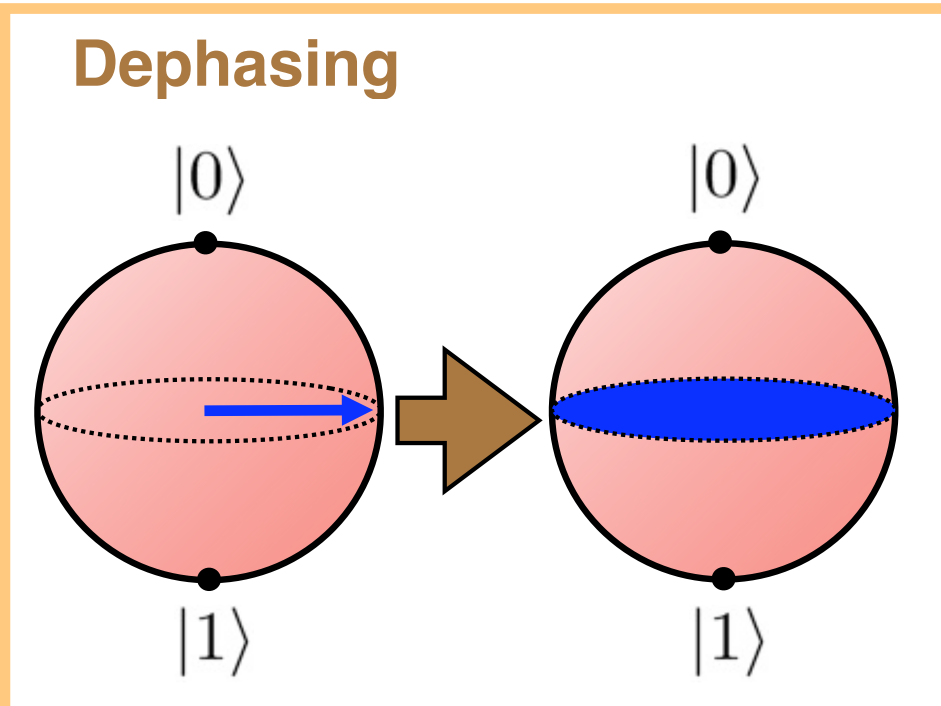
Spin relaxation rate: $\Gamma_1 = \frac{1}{T_1}$

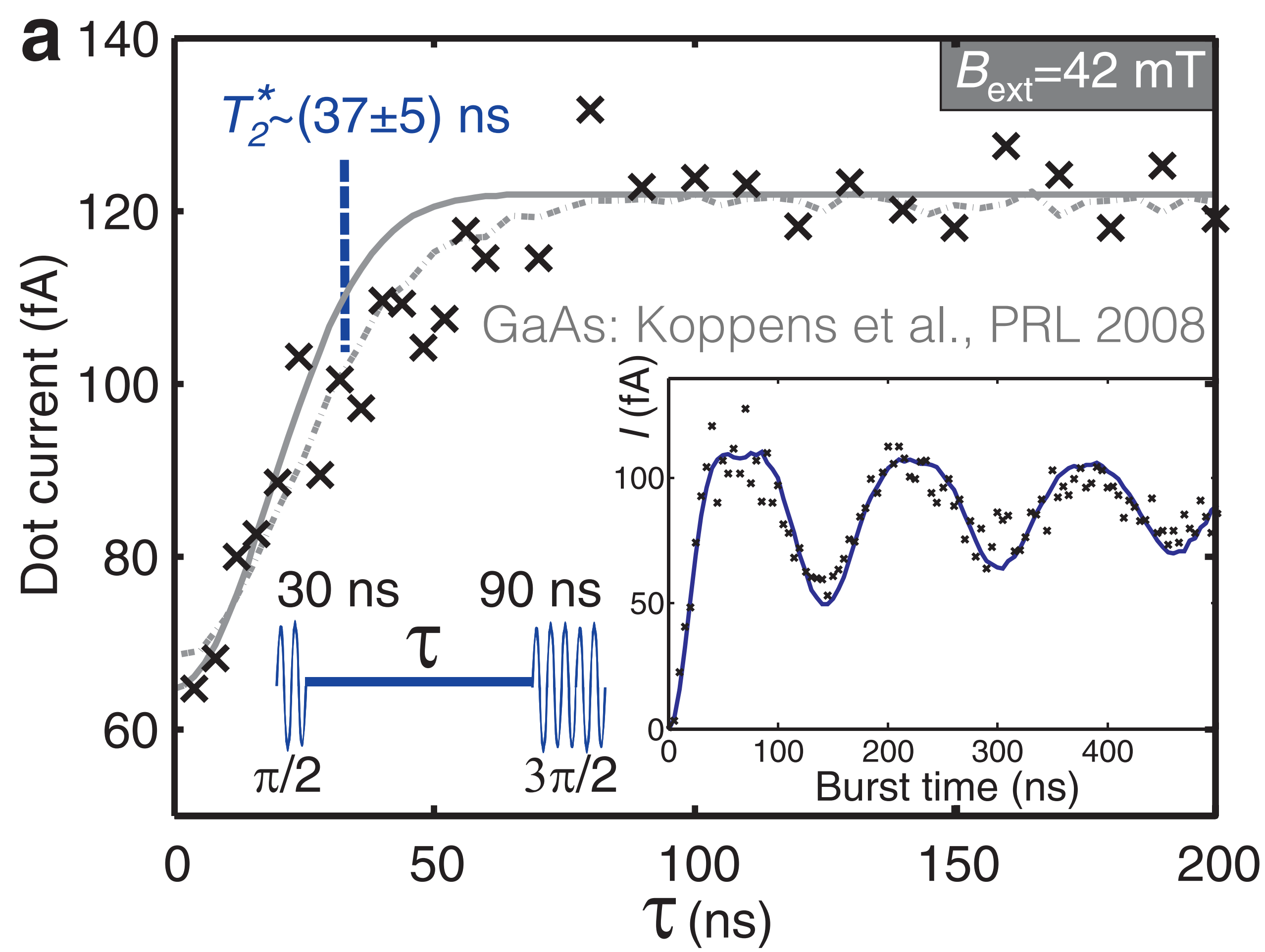


Simple power law: $\Gamma_1 \propto B^5$

Amasha et al., PRL 2008

Dephasing in GaAs due to nuclear spins



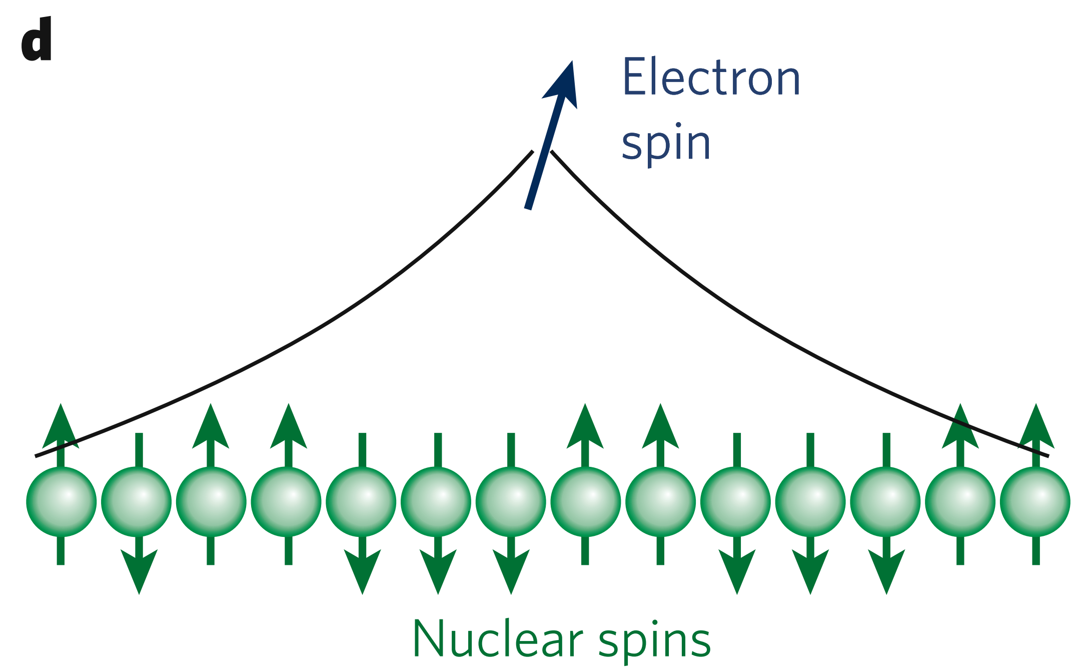


polarization vector in rotating frame:

$$\bar{p}_x(t) = e^{-(t/T_2^*)^2}$$

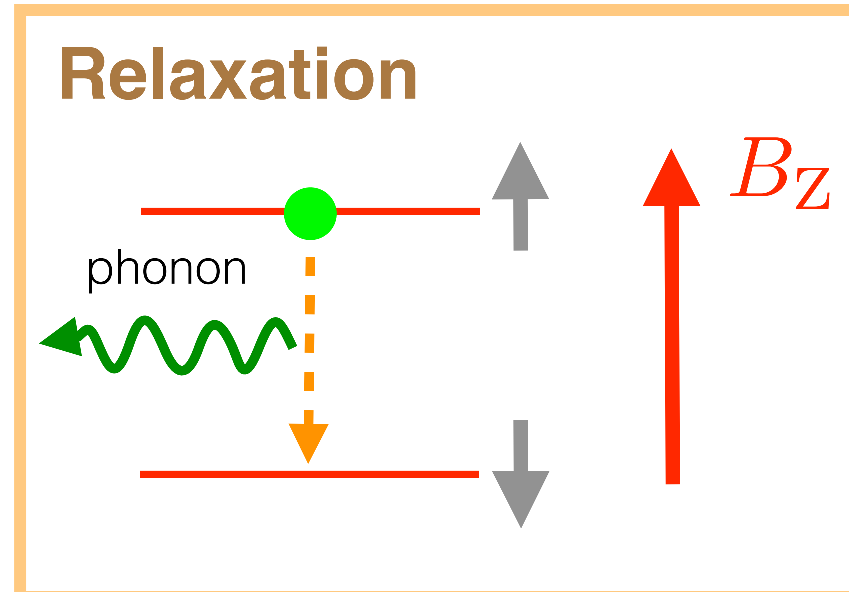
inhomogeneous dephasing time:

$$T_2^* = \frac{\sqrt{2}\hbar}{g\mu_B\sigma}$$



3000x improvement by using Si-28 instead of GaAs

A basic model for spin relaxation



- zero temperature
- a single phonon is emitted
- phonons are `bulk phonons'
- only acoustic phonons are considered
- **only longitudinal phonons are considered**
- acoustic phonon dispersion assumed to be isotropic
- dipole approximation: phonon wave length >> dot size
- **electron-phonon interaction: deformation-potential mechanism**
- mixing of spin and orbital: spin-orbit interaction
- relaxation rate from Fermi's Golden Rule

energy conservation

$$\Gamma_1 = \frac{2\pi}{\hbar} \sum_{\boldsymbol{q}_f} \left| \langle \overline{0_x 0_y \downarrow}, \boldsymbol{q}_f | H_{\mathrm{eph}} | \overline{0_x 0_y \uparrow}, \mathrm{vac} \rangle \right|^2 \delta(\hbar\omega_L - \hbar v_{\mathrm{LA}} q_f)$$

spin qubit states dressed by spin-orbit: see lecture 4

Do not go beyond linearly dispersing acoustic phonons

typical frequency of emitted phonon:

$$\nu = \frac{g^* \mu_B B}{h} \lesssim \frac{2 \cdot 60 \mu\mathrm{eV/T} \cdot 10\,\mathrm{T}}{4\,\mu\mathrm{eV/GHz}} = 300\,\mathrm{GHz}$$

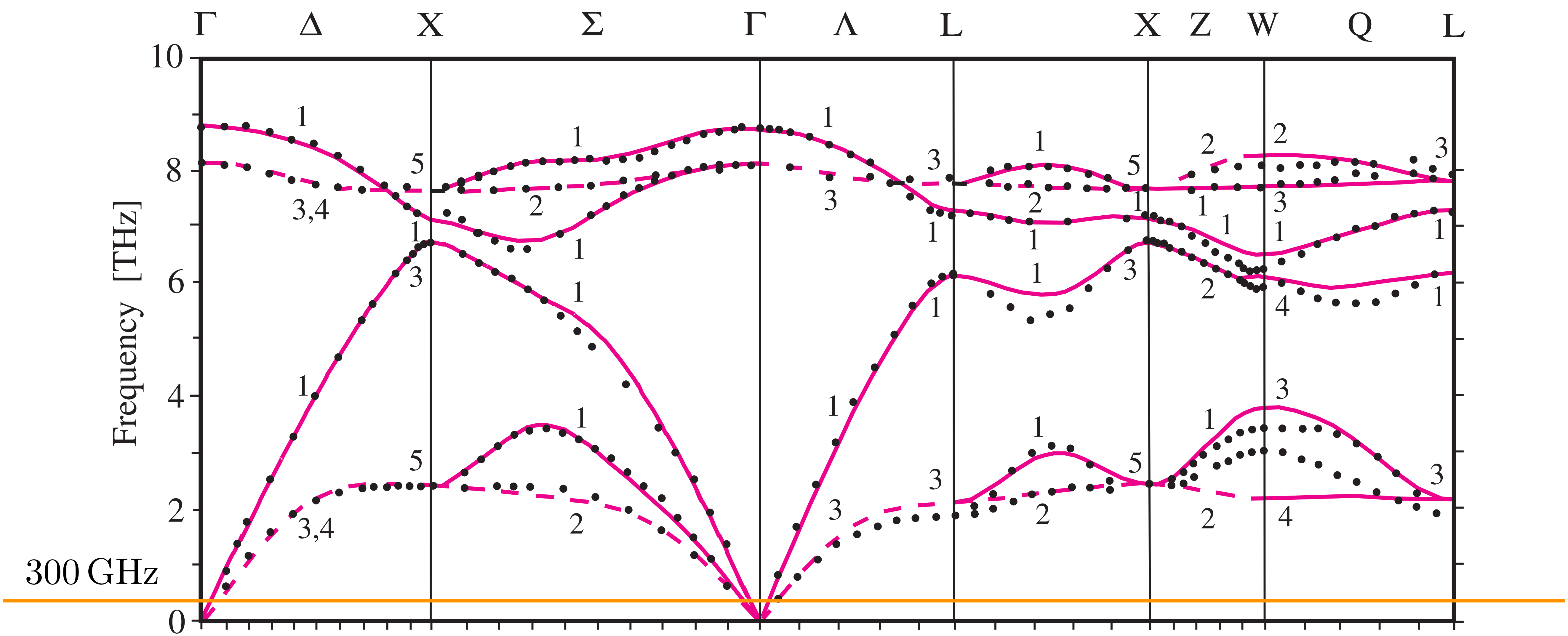
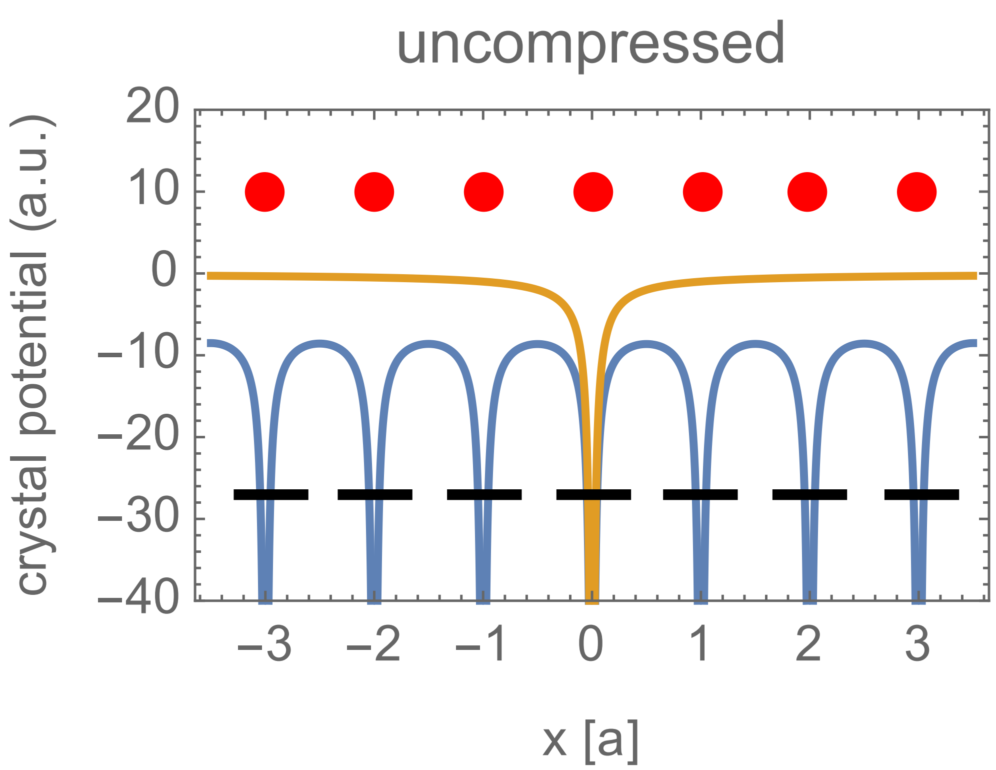
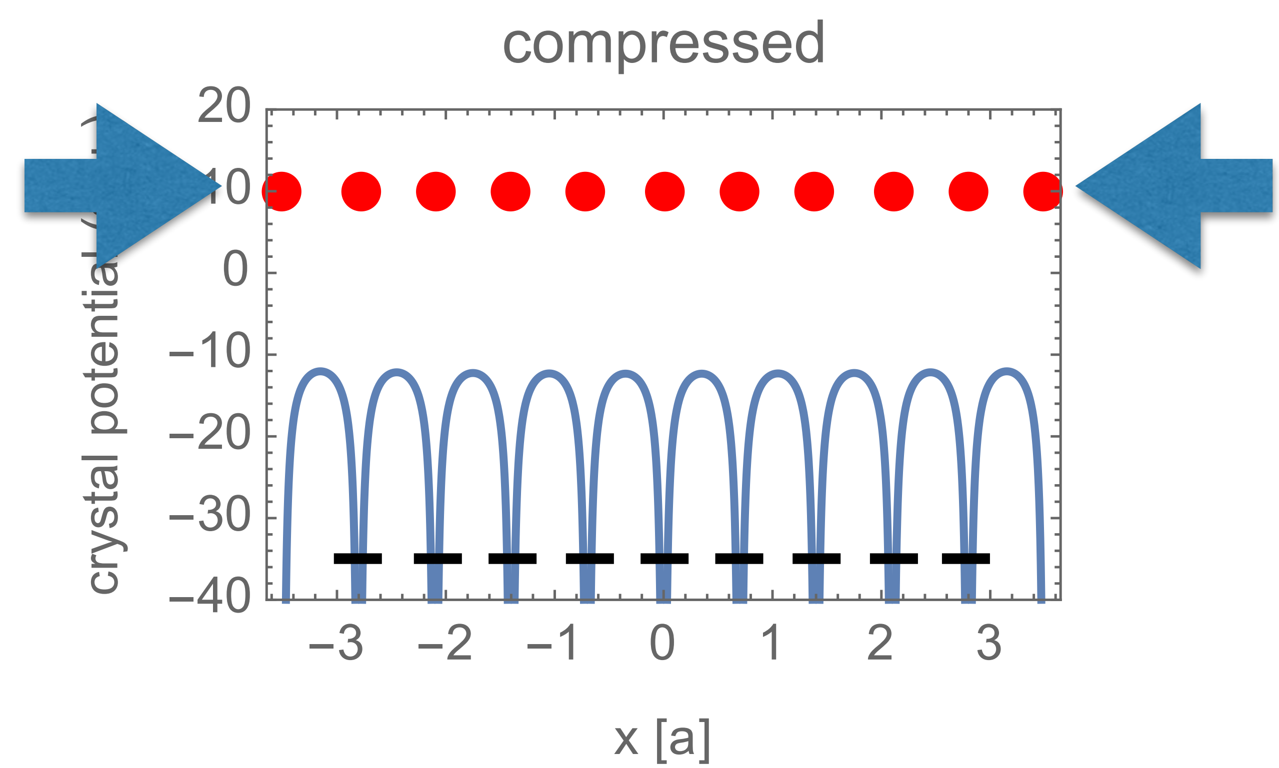


Fig. 3.2. Phonon dispersion curves in GaAs along high-symmetry axes [3.6]. The experimental data points were measured at 12 K. The *continuous lines* were calculated with an 11-parameter rigid-ion model. The numbers next to the phonon branches label the corresponding irreducible representations

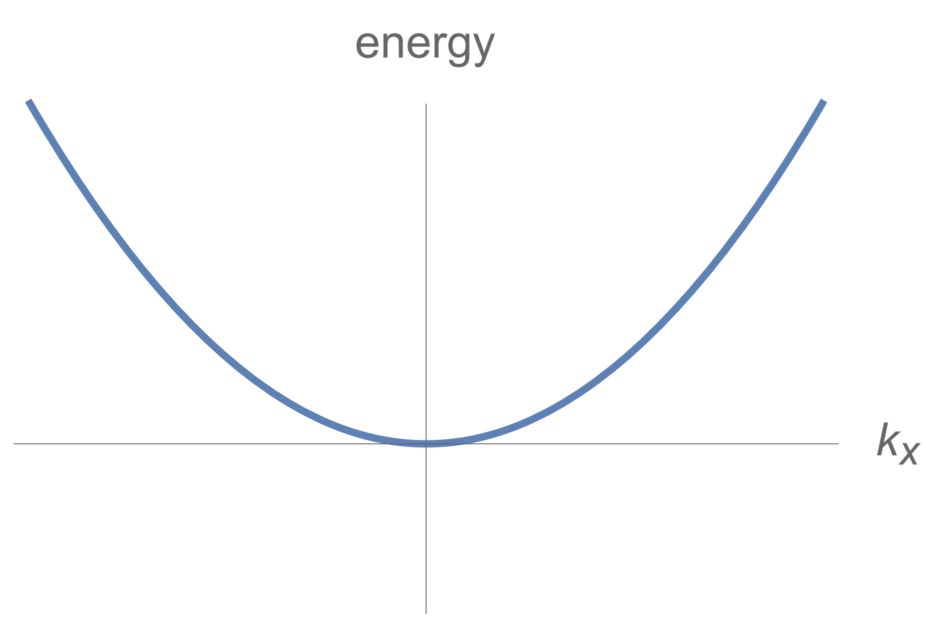
Yu & Cardona: *Fundamentals of Semiconductors*

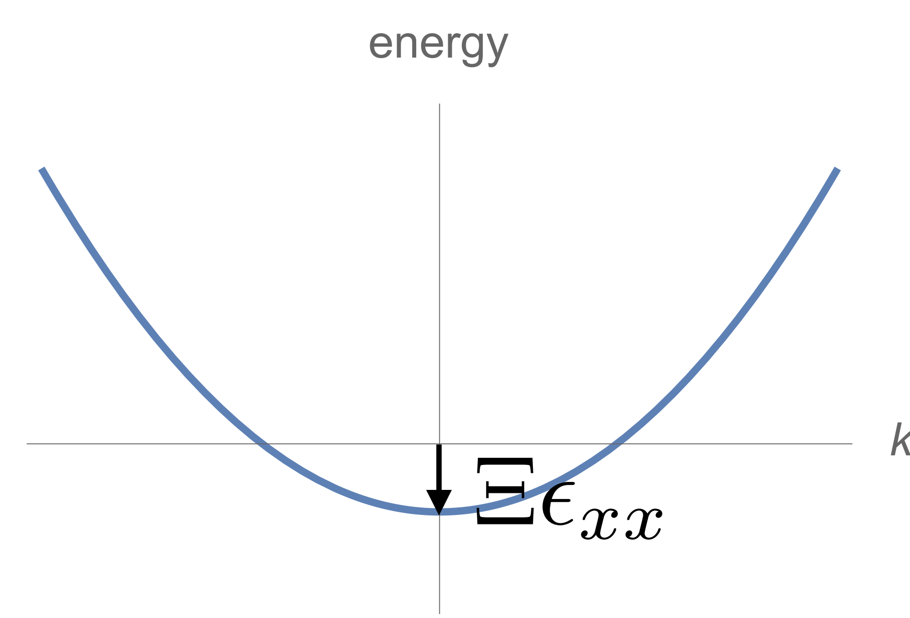
Electron-phonon interaction: deformation potential





deeper confinement =>
conduction band lowered



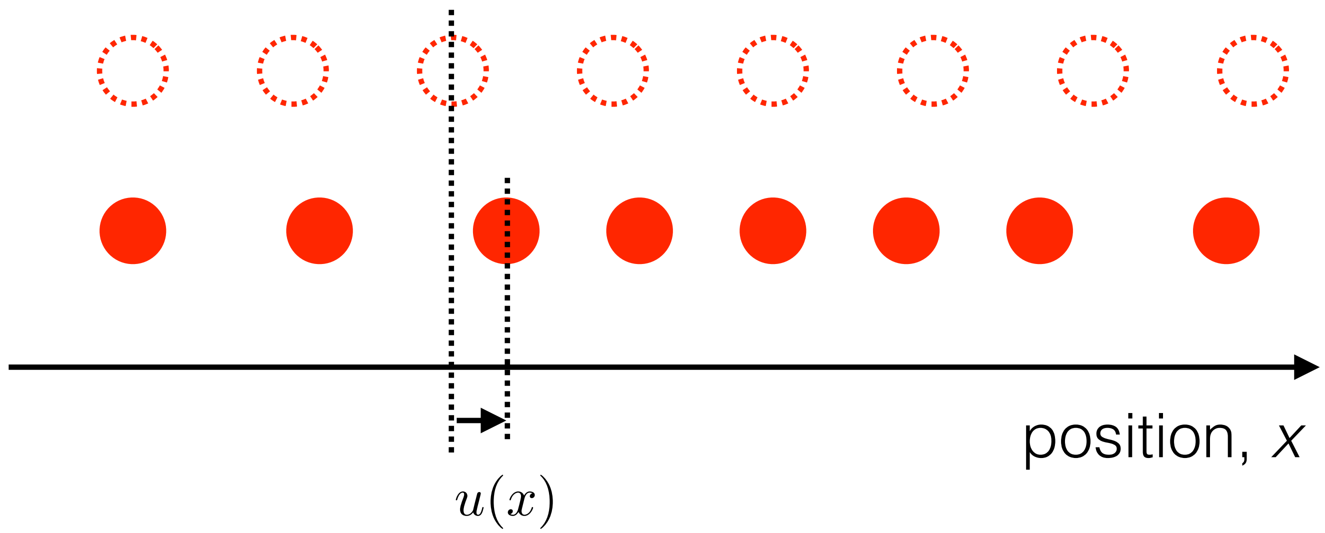


For example, $\Xi = 10$ eV and $\epsilon_{xx} = -1\%$ (compression) implies an energy shift of -0.1 eV.

Electron-phonon interaction: deformation potential

1D

displacement field in 1D: $u(x)$



strain in 1D: $\epsilon_{xx}(x) = \dfrac{du(x)}{dx}$

relative volume change in 1D = strain in 1D = $\epsilon_{xx}(x)$

3D

displacement field in 3D: $\boldsymbol{u}(\boldsymbol{r})$

strain in 3D: $\epsilon_{ij}(\boldsymbol{r}) = \dfrac{1}{2}\left(\partial_i u_j(\boldsymbol{r}) + \partial_j u_i(\boldsymbol{r})\right)$, where $i, j \in \{x, y, z\}$

relative volume change in 3D = $\mathrm{Tr}\left(\epsilon(\boldsymbol{r})\right) = \epsilon_{xx}(\boldsymbol{r}) + \epsilon_{yy}(\boldsymbol{r}) + \epsilon_{zz}(\boldsymbol{r})$

e-ph interaction: $H_{\mathrm{eph}} = \Xi\, \mathrm{Tr}\left(\epsilon(\boldsymbol{r})\right)$

Electron-phonon interaction: deformation potential

displacement of a harmonic oscillator:

$$x = \frac{\ell}{\sqrt{2}}(a + a^\dagger)$$

$$\ell = \sqrt{\frac{\hbar}{m\omega_0}}$$

displacement field due to many LA phonons:

$$\boldsymbol{u}(\boldsymbol{r}) = \sum_{\boldsymbol{q}} \frac{\ell_q}{\sqrt{2}} e^{i\boldsymbol{q}\boldsymbol{r}} \left(a_{\boldsymbol{q}} + a^\dagger_{-\boldsymbol{q}}\right) \hat{\boldsymbol{q}}$$

$$\ell_q = \sqrt{\frac{\hbar}{\rho V v_{\mathrm{LA}}\, q}},\ \hat{\boldsymbol{q}} = \boldsymbol{q}/q$$

relative volume change due to many LA phonons:

$$\mathrm{Tr}(\epsilon(\boldsymbol{r})) = i\sqrt{\frac{\hbar}{2\rho V v_{\mathrm{LA}}}} \sum_{\boldsymbol{q}} \sqrt{q} e^{i\boldsymbol{q}\boldsymbol{r}} \left(a_{\boldsymbol{q}} + a^\dagger_{-\boldsymbol{q}}\right)$$

dipole approximation:

if $\ell_{\mathrm{QD}} \ll 1/q$ then $e^{i\boldsymbol{q}\boldsymbol{r}} \mapsto 1 + i\boldsymbol{q}\boldsymbol{r}$ and

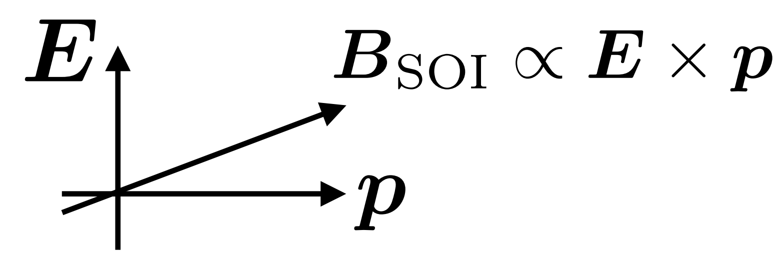
$$\mathrm{Tr}(\epsilon(\boldsymbol{r})) \mapsto -\sqrt{\frac{\hbar}{2\rho V v_{\mathrm{LA}}}} \sum_{\boldsymbol{q}} \sqrt{q}\, \boldsymbol{q}\boldsymbol{r} \left(a_{\boldsymbol{q}} + a^\dagger_{-\boldsymbol{q}}\right)$$

Khaetskii & Nazarov et al., PRB 2001

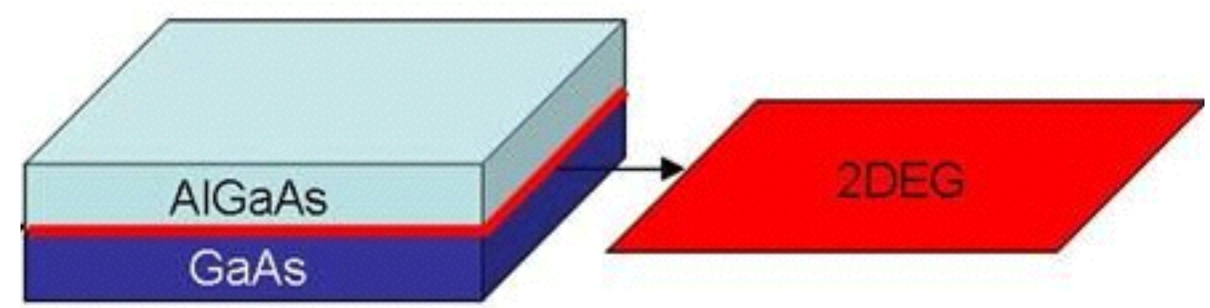
Spin-orbit-induced spin relaxation

$$H = H_{\text{osc}} + H_{\text{hom}} + H_{\text{SOI}} + H_{\text{eph}}$$

$$H_{\text{osc}} = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2} m \omega_0^2 (x^2 + y^2)$$



$$H_{\text{hom}} = \frac{1}{2} g^* \mu_B B_0 \sigma_z$$



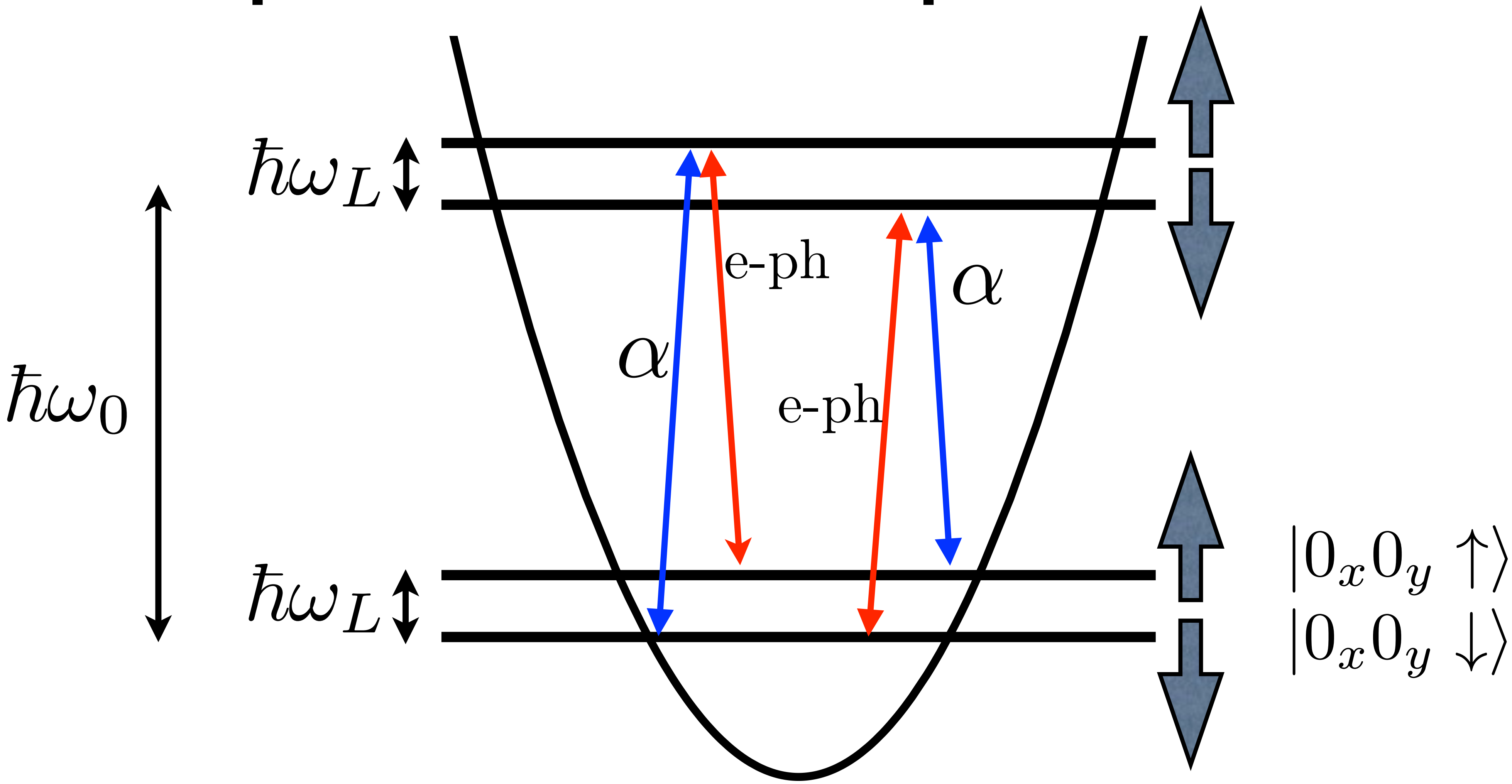
$$H_{\text{SOI}} = \alpha(\sigma_x p_y - \sigma_y p_x)$$

Rashba spin-orbit interaction

$$H_{\text{eph}} = -\Xi \sqrt{\frac{\hbar}{2\rho V v_{\text{LA}}}} \sum_{\boldsymbol{q}} \sqrt{q}\, \boldsymbol{q}\boldsymbol{r} \left(a_{\boldsymbol{q}} + a_{-\boldsymbol{q}}^\dagger \right)$$

Golovach et al., PRB 2006

Spin-orbit-induced spin relaxation



$$H_0 = H_{\rm osc} + H_{\rm hom}$$

$$H_1 = H_{\rm SOI}$$

qubit basis states dressed by SOI:

$$|\overline{0_x 0_y \uparrow}\rangle \approx |0_x 0_y \uparrow\rangle + \sum \ldots$$

$$|\overline{0_x 0_y \downarrow}\rangle \approx |0_x 0_y \downarrow\rangle + \sum \ldots$$

$$\Gamma_1 = \frac{2\pi}{\hbar} \sum_{\boldsymbol{q}_f} \left| \langle \overline{0_x 0_y \downarrow}, \boldsymbol{q}_f | H_{\rm eph} | \overline{0_x 0_y \uparrow}, {\rm vac} \rangle \right|^2 \delta(\hbar\omega_L - \hbar v_{\rm LA} q_f)$$

Spin-orbit-induced spin relaxation

further tools for the calculation:

$$\frac{1}{V}\sum_{\boldsymbol{q}} \cdots = \int \frac{d^3q}{(2\pi)^3} \cdots$$

$$a_{\boldsymbol{q}} \left|\mathrm{vac}\right\rangle = 0$$

$$a_{\boldsymbol{q}}^\dagger \left|\mathrm{vac}\right\rangle = \left|\boldsymbol{q}\right\rangle$$

convert 3D cartesian integral to spherical coordinates

The result:

$$\Gamma_1 = \frac{1}{6\pi} \frac{\Xi^2\alpha^2\omega_L^7}{\rho v_{\mathrm{LA}}^7\omega_0^2} \propto B^7$$

Exercises: (1) Do the calculation.
(2) Assume 1D structure with 1D LA phonons, 1 subband. Redo the calculation.
(3) Replace SOI with inhomogeneous field as in lecture 4. Redo the calculation.

Discussion: power counting is powerful

$$\Gamma_1 \propto B^7 = B^{2\cdot(\frac{1}{2}+1+1)+2}$$

FGR has matrix element squared

$\sqrt{q} = q^{1/2}$ in eph Hamiltonian

q^1 in eph Hamiltonian
due to dipole approximation

'van Vleck cancellation'
time reversal symmetry
$\langle \overline{0_x 0_y \downarrow} | x | \overline{0_x 0_y \uparrow} \rangle \propto B^1$

3D acoustic phonons
DOS $\propto \omega^2$

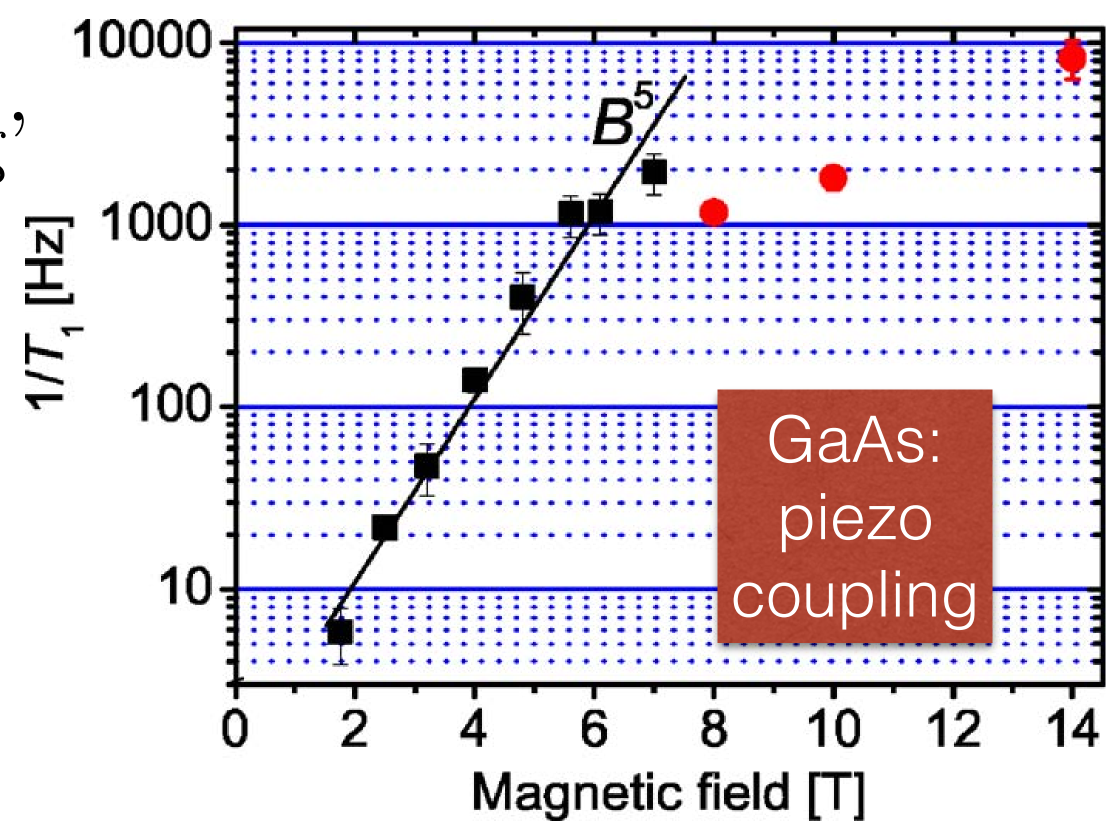
replace 'deformation potential e-ph coupling'
with 'piezoelectric e-ph coupling'

$$\Downarrow$$

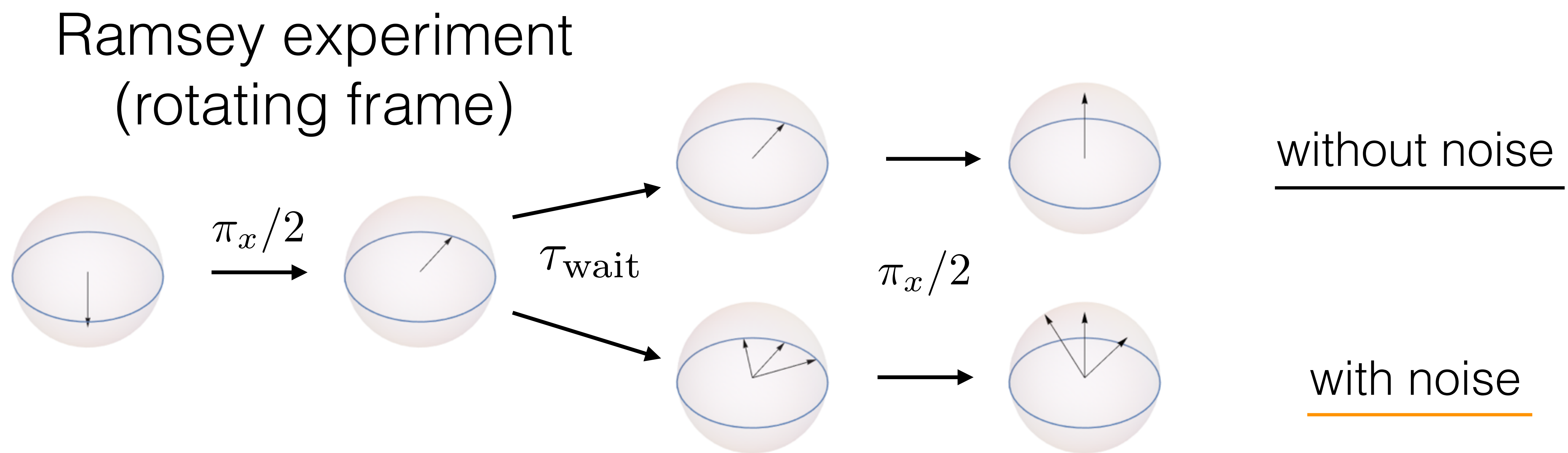
$$1/\sqrt{q} = q^{-1/2} \text{ in the e-ph Hamiltonian}$$

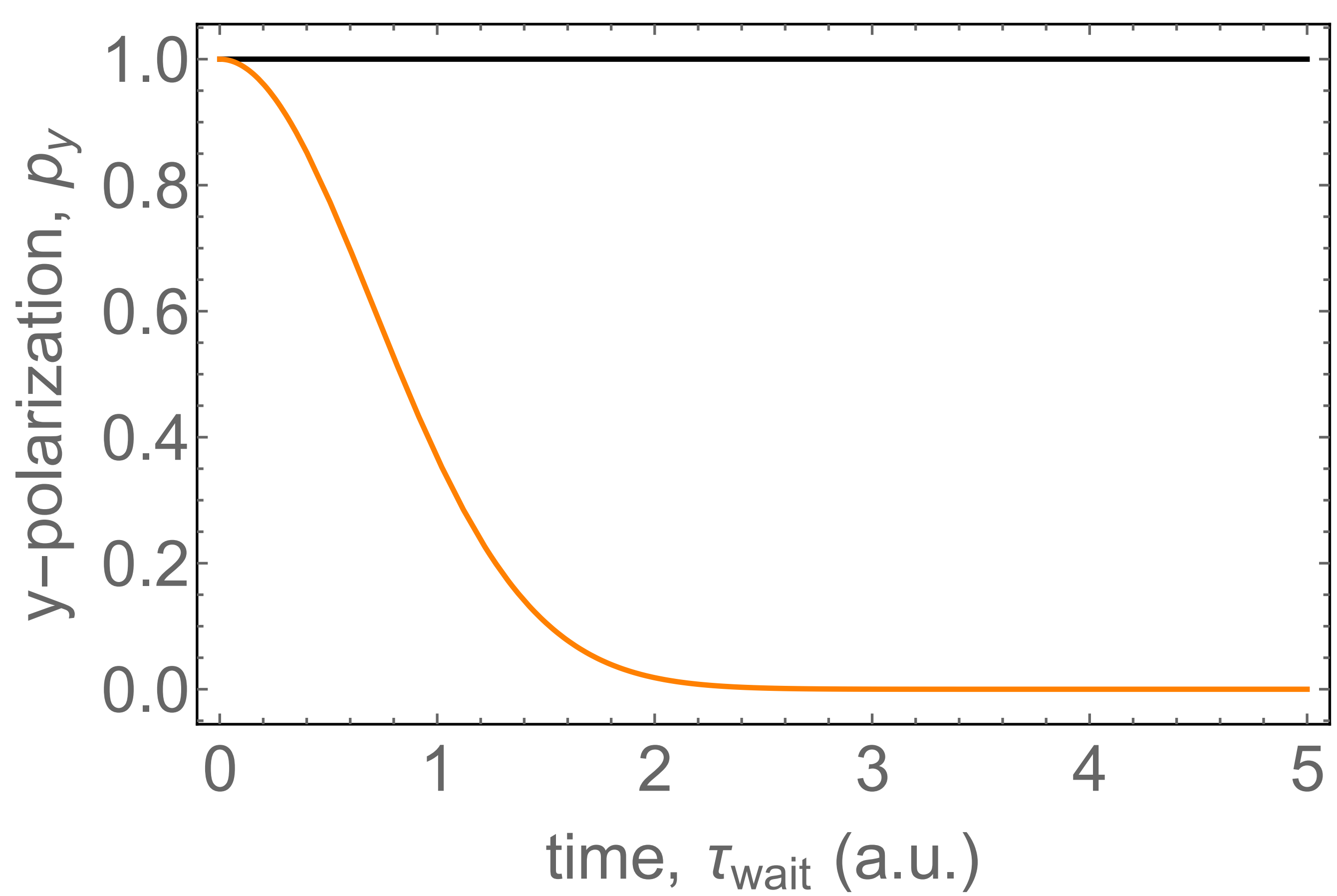
$$\Downarrow$$

$$\Gamma_1 \propto B^5$$



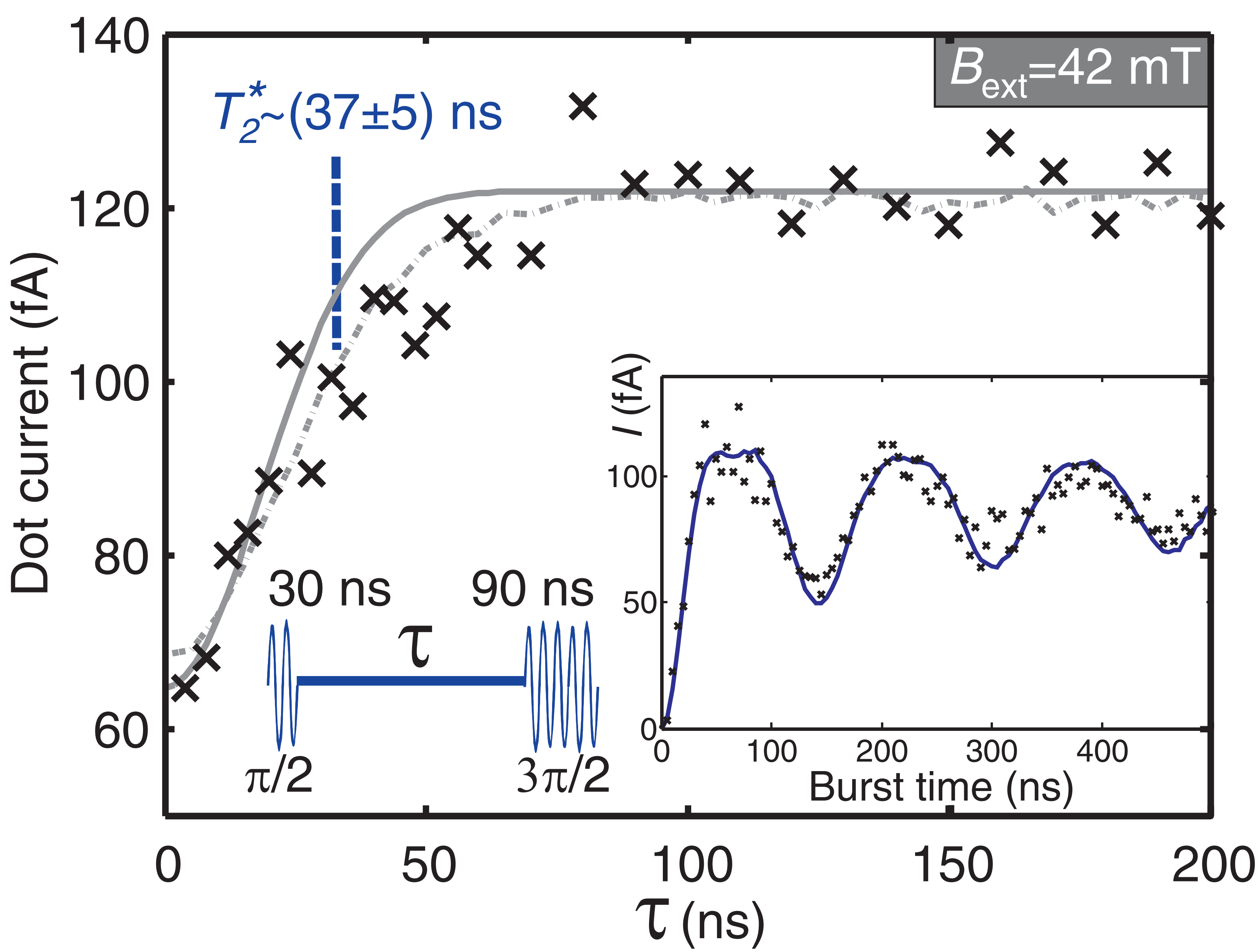
Measuring dephasing via the Ramsey experiment





Koppens et al., PRL 2008

A Ramsey experiment in GaAs

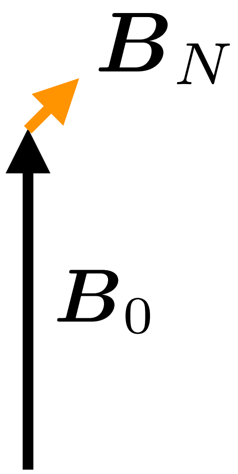


crosses: experimental data

solid line: fit, $I(\tau) = I_0 + \Delta I e^{-(\tau/T_2^*)^2}$

Claim: data is consistent with `quasistatic nuclear field' model

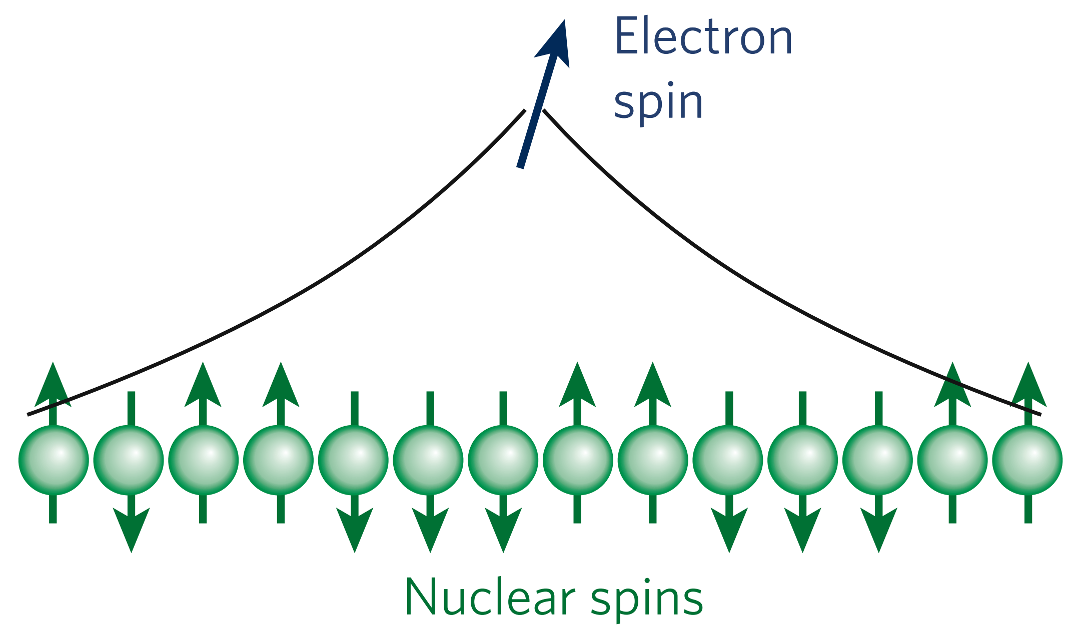
`Quasistatic nuclear field' model



external B-field

$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_N = \begin{pmatrix} 0 \\ 0 \\ B_0 \end{pmatrix} + \begin{pmatrix} B_{Nx} \\ B_{Ny} \\ B_{Nz} \end{pmatrix}$$

field created by nuclear spins
`nuclear field' = `Overhauser field'



Quasistatic approximation:
(1) $\boldsymbol{B}_N$ is constant for each run of the experiment
(2) $\boldsymbol{B}_N$ changes randomly between subsequent runs

Weak nuclear field approximation:
$\boldsymbol{B}_N << \boldsymbol{B}_0$

Gaussian nuclear field approximation:
Each component of $\boldsymbol{B}_N$ has Gaussian distribution with stdev σ

Noise-averaged dynamics of the polarization vector

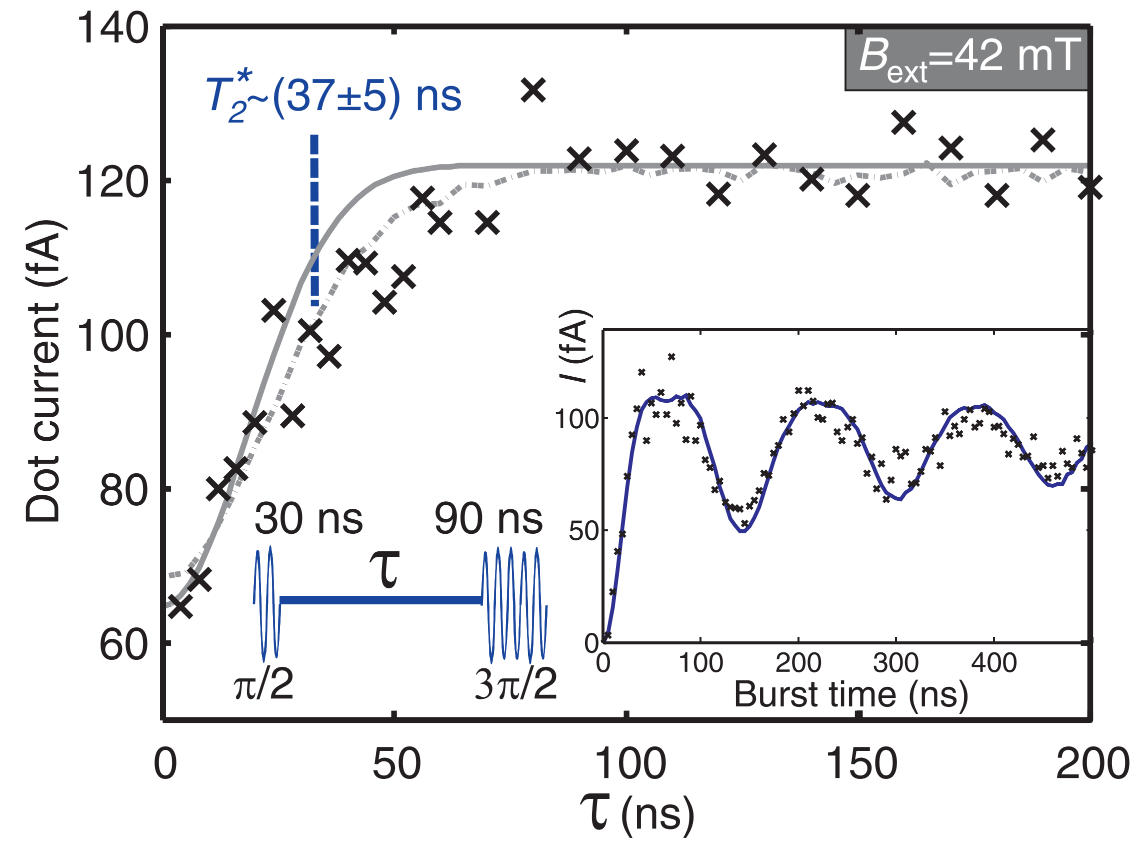
$$\overline{\boldsymbol{p}}(t) = \int_{-\infty}^{\infty} d(\delta\omega) P(\delta\omega) \begin{pmatrix} -\sin(\delta\omega\, t) \\ \cos(\delta\omega\, t) \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{-\frac{1}{2}\Sigma^2 t^2}$$

$$P(\delta\omega) = \frac{1}{\sqrt{2\pi}\Sigma} e^{-\delta\omega^2/\Sigma^2}, \text{ with } \Sigma = \frac{g^* \mu_B \sigma}{\hbar}$$

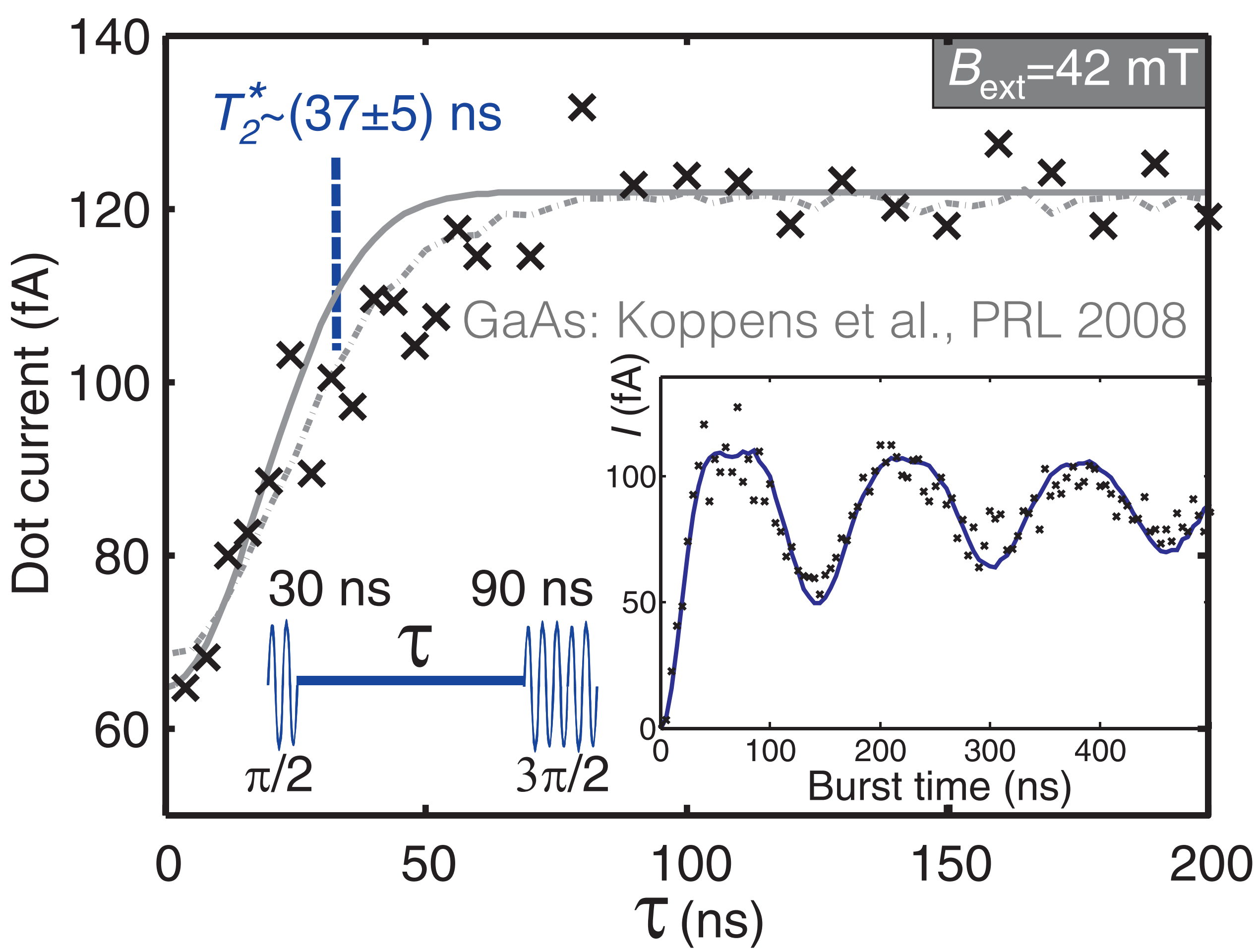
Conclusion:

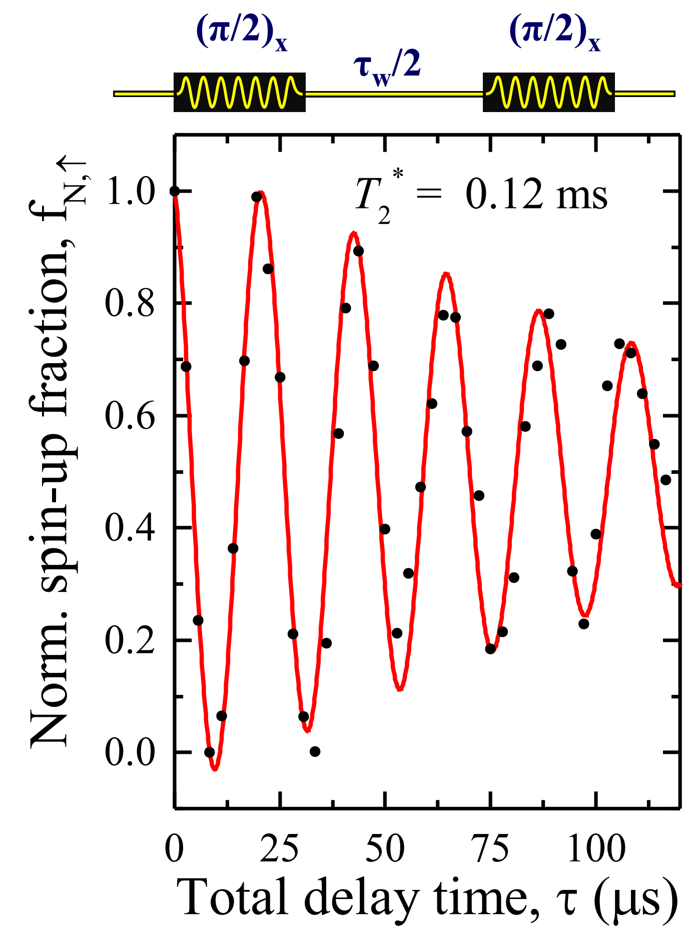
$$\bar{p}_y(t) = e^{-(t/T_2^*)^2}, \text{ with } T_2^* = \frac{\sqrt{2}\hbar}{g^* \mu_B \sigma}$$

$$\text{e.g., } T_2^* = \frac{\sqrt{2}\hbar}{0.4 \times \mu_B \times 1\,\text{mT}} \approx 40\,\text{ns}$$

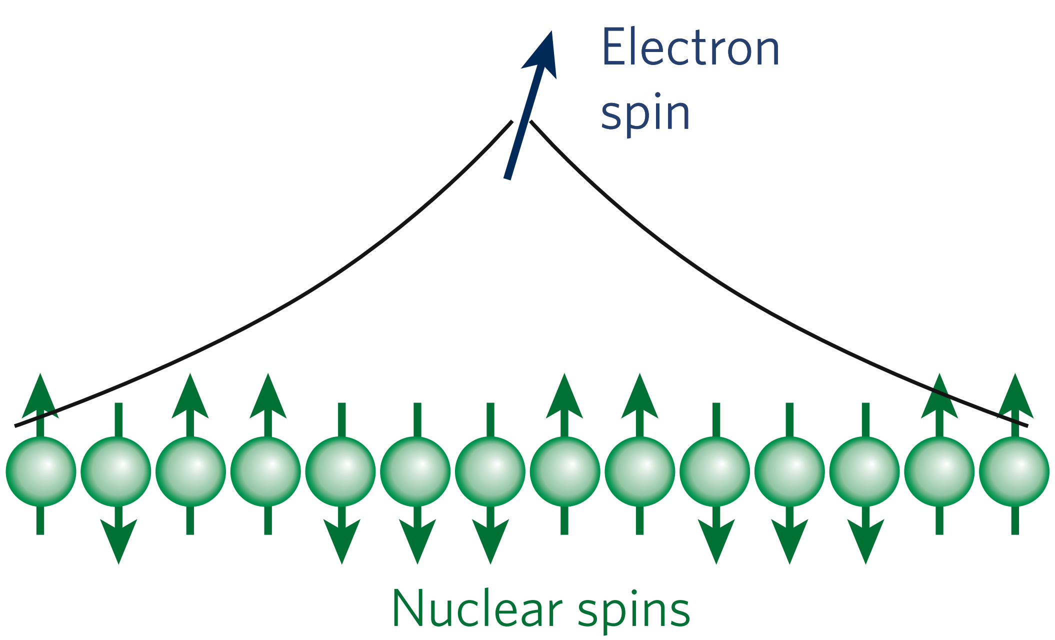


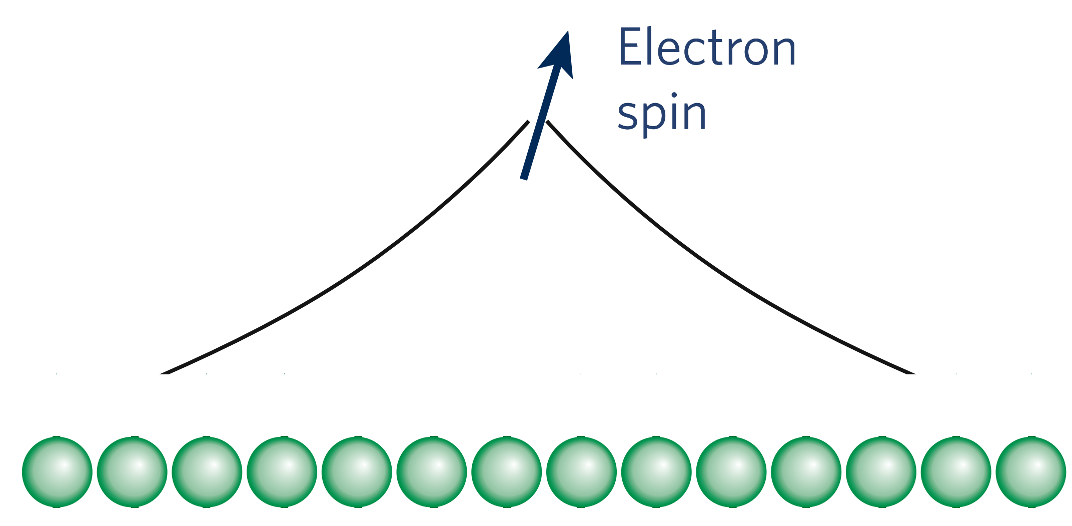
3000x improvement by using Si-28 instead of GaAs





Si-28
Veldhorst et al.,
Nat. Nanotech
2014



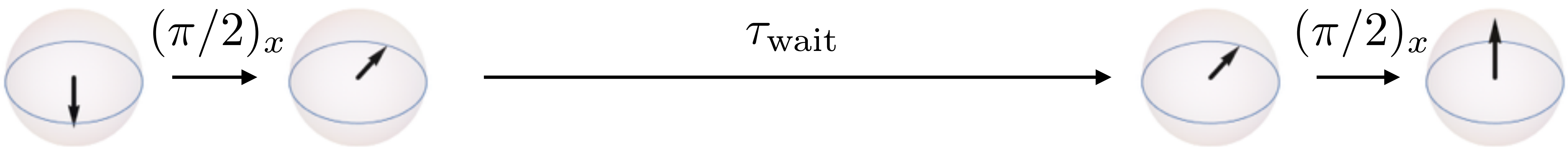


Si-28: no nuclear spins; `semiconductor vacuum'

How accurate is the quasistatic noise model?

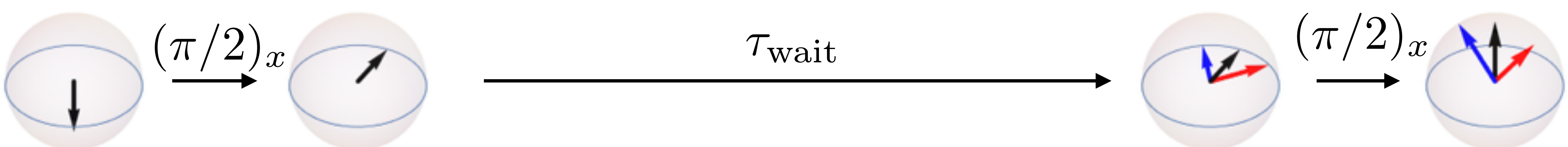
perfect memory

(1) Ramsey, no noise



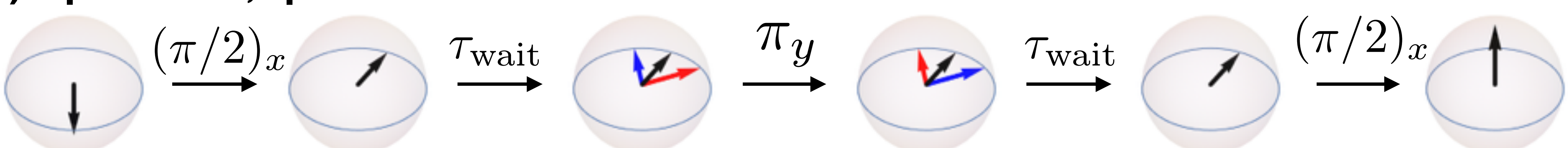
lossy memory

(2) Ramsey, quasistatic noise



perfect memory

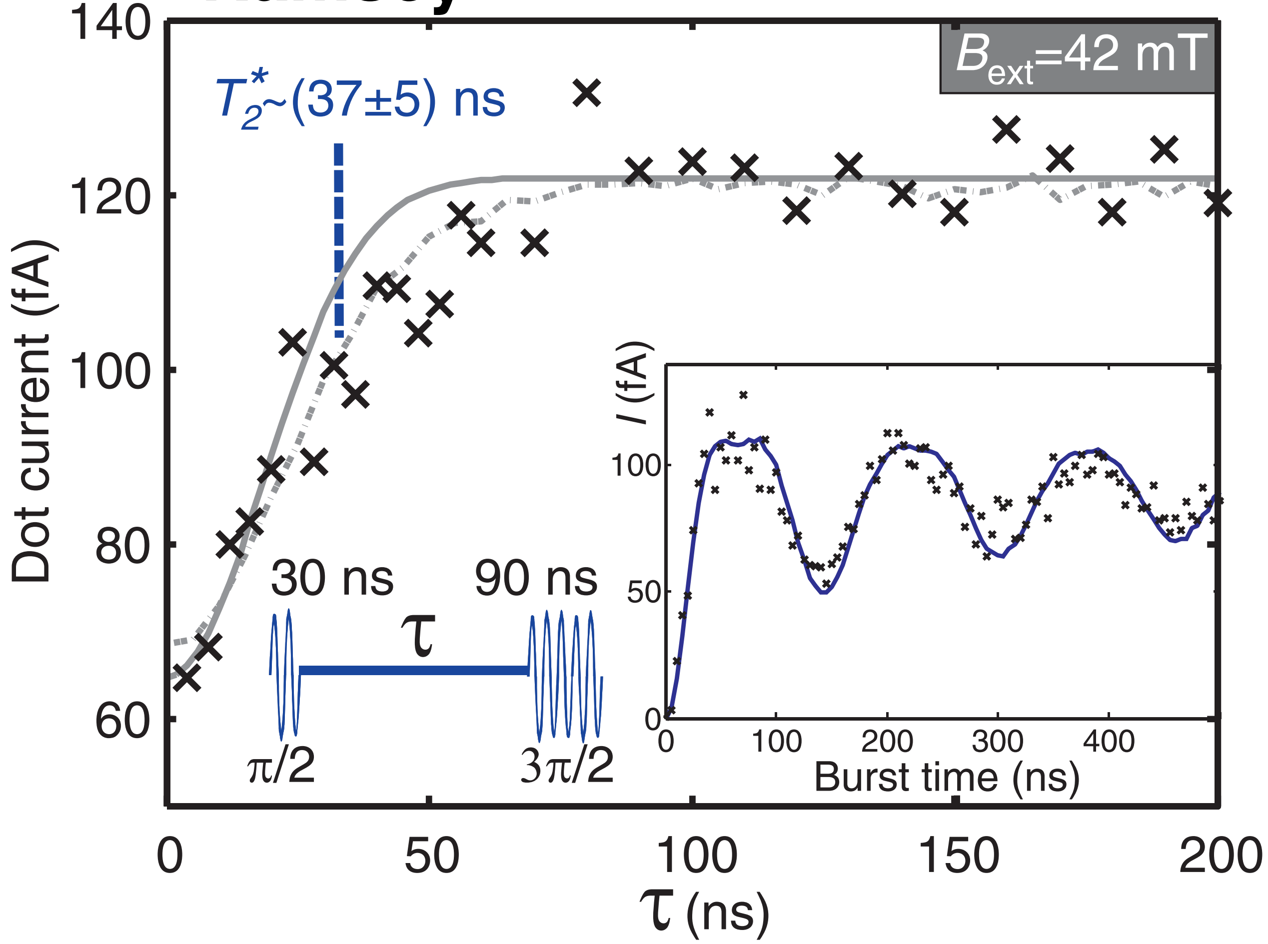
(3) Spin Echo, quasistatic noise



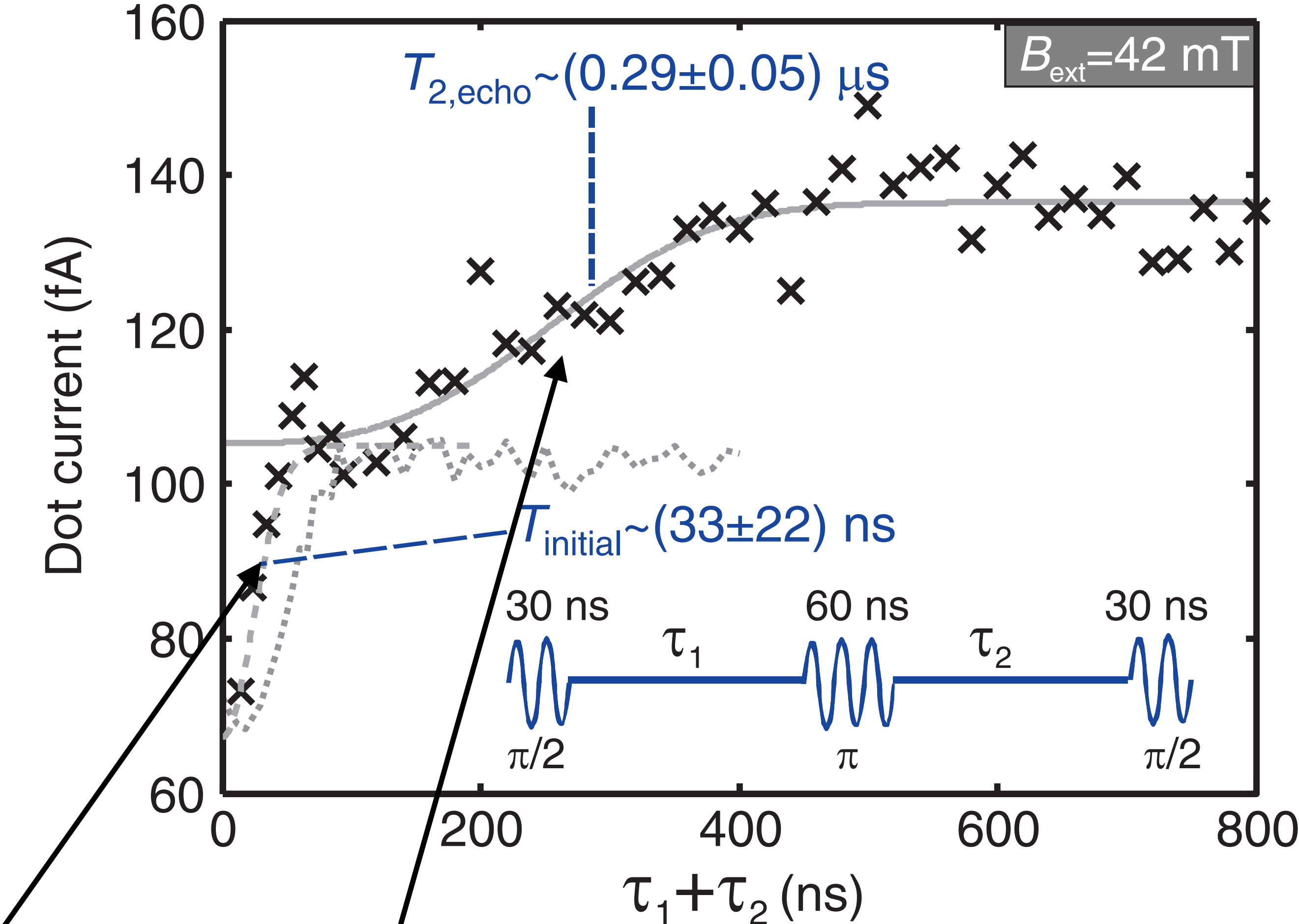
IF Spin Echo experiment yields perfect memory, THEN noise is quasistatic.

How accurate is the quasistatic noise model?

Ramsey



Spin Echo



Initial decay:
time scale as in Ramsey
caused by imperfect

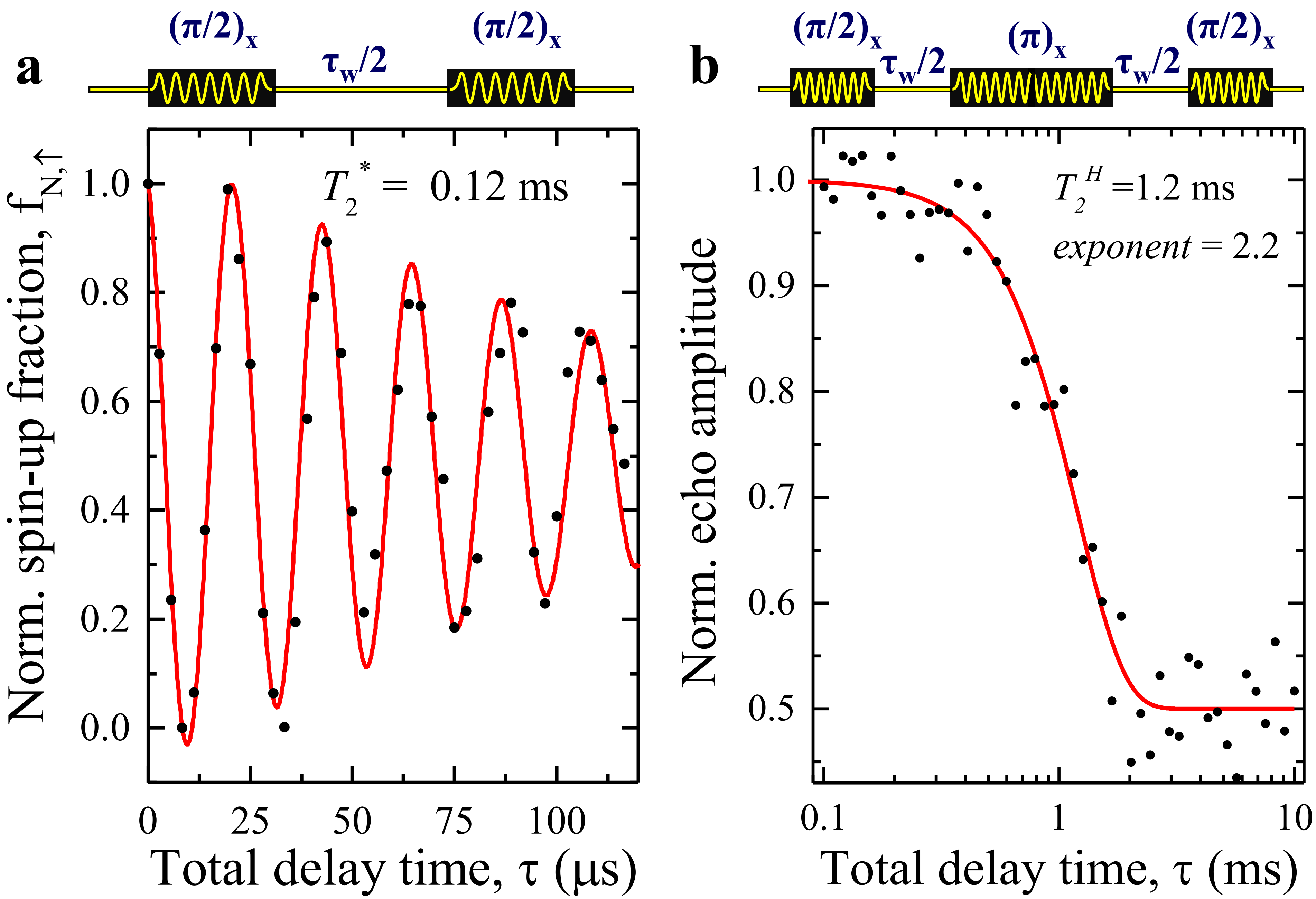
$$a + be^{-[(\tau_1+\tau_2)/T_{2,\mathrm{echo}}]^d}$$

$$d = 3$$

Spin Echo is not perfect, but works: decay 7x slower.

Spin Echo in Si-28

Veldhorst et al., Nat. Nanotech 2014



Spin Echo works also in Si-28

Summary

1. Spin relaxation: spin-orbit + phonon emission
2. GaAs: inhomogeneous dephasing due to nuclear spins
3. Spin Echo prolongs the quantum memory lifetime
4. Change GaAs to Si-28: 3000x improvement
5. Si-28: bottleneck is $T2^*$ ~ 0.12 ms (T1 much longer)

Potential extensions

1. Role of temperature in spin relaxation
2. Geometric spin dephasing
3. Decay in Spin Echo: nuclear-spin dynamics (GaAs), charge noise (Si)
4. Reducing the Overhauser field via increasing the dot size
5. Anisotropic hyperfine interaction of holes
6. Is spin echo useful in quantum computing?
7. Low-frequency (1/f) charge noise
8. Beyond lifetimes: quality of quantum gates (randomized benchmarking)