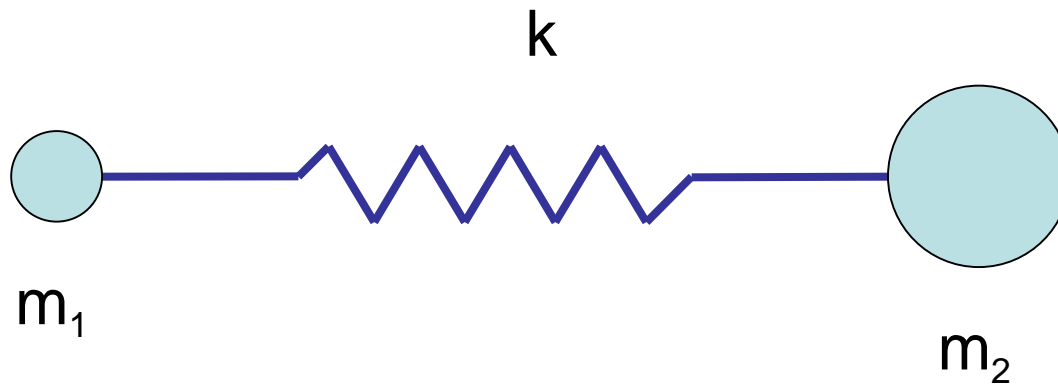


# Fizika I

Rezgések, hullámok

## *Molekula rezgés:*

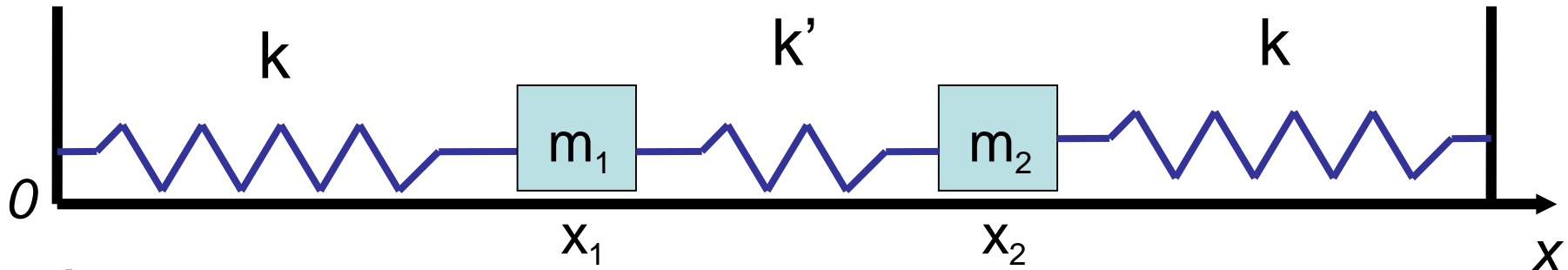


$$\omega = ?$$

$$k' = ?$$

$$m' = ?$$

## Csatolt rezgés:



Általában:  $k' \ll k$  (nullhosszúságú rugók)

$$I. \quad m_1 \ddot{x}_1 = -kx_1 + k'(x_2 - x_1)$$

$$II. \quad m_2 \ddot{x}_2 = k(\ell - x_2) - k'(x_2 - x_1)$$

$$m_2 = m_1 \quad \text{és} \quad \omega_o^2 = \frac{k}{m}$$

$$I. \quad \ddot{x}_1 = -\omega_o^2 x_1 + \frac{k'}{m}(x_2 - x_1)$$

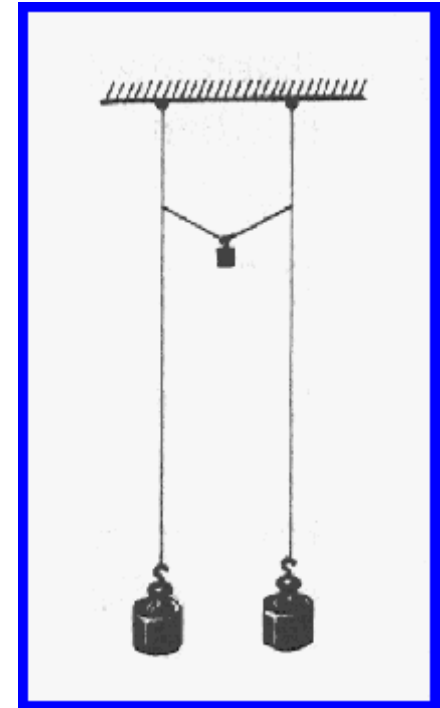
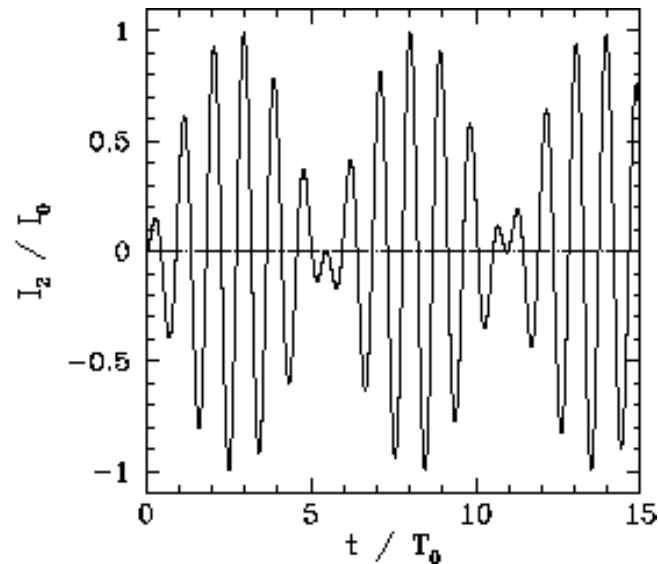
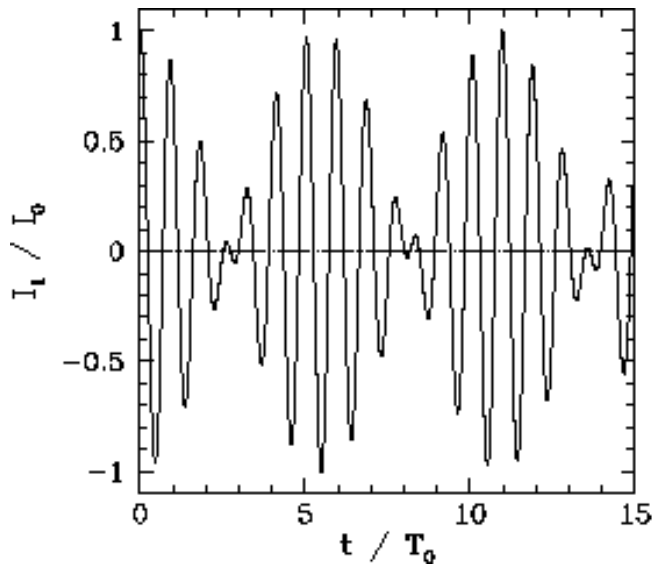
$$II. \quad \ddot{x}_2 = \omega_o^2(\ell - x_2) - \frac{k'}{m}(x_2 - x_1)$$

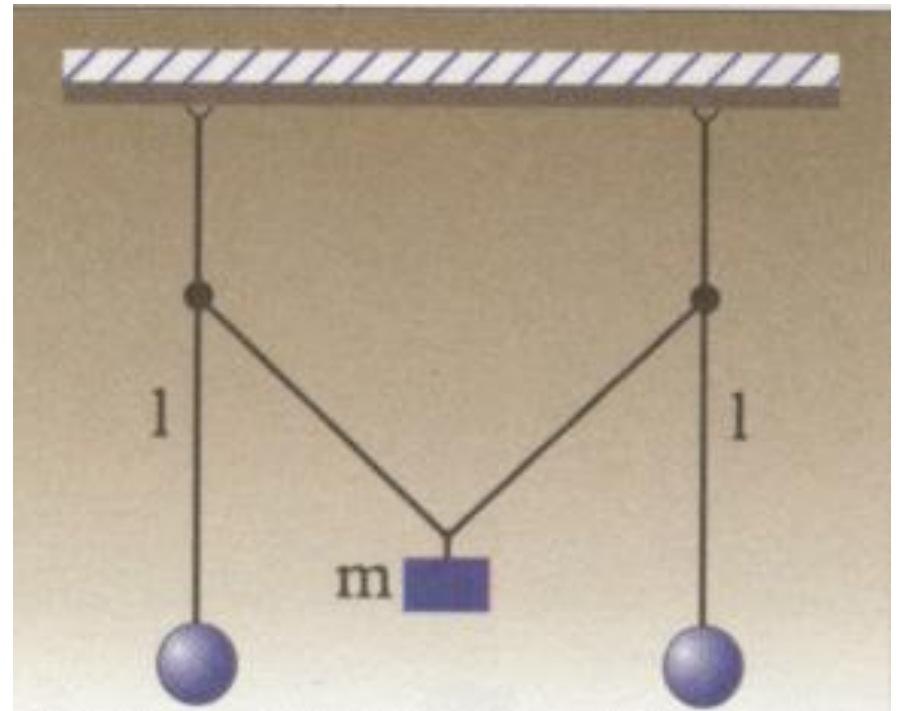
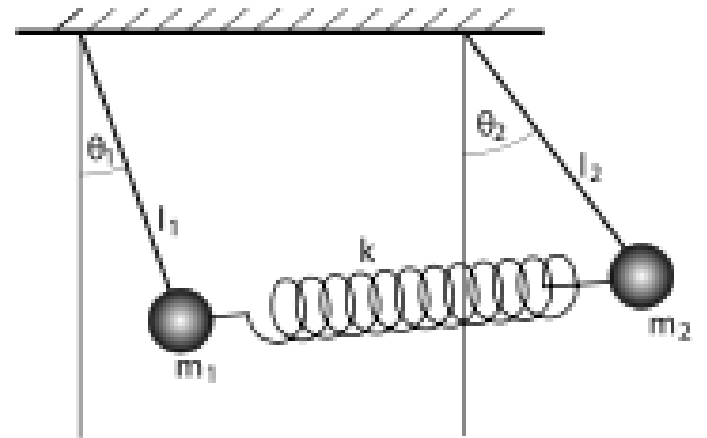
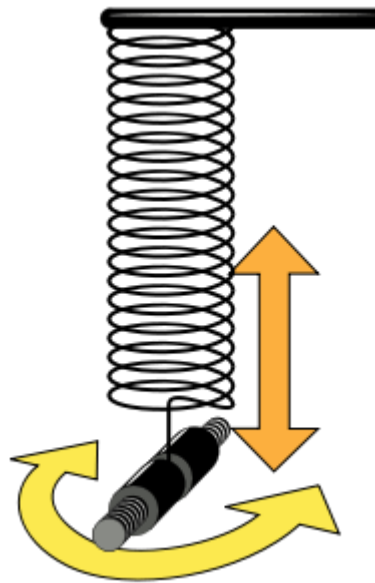
**megoldás:**

$$\omega = \sqrt{\omega_o^2 + 2\kappa} \quad \text{ahol} \quad \kappa = \frac{k'}{m}$$

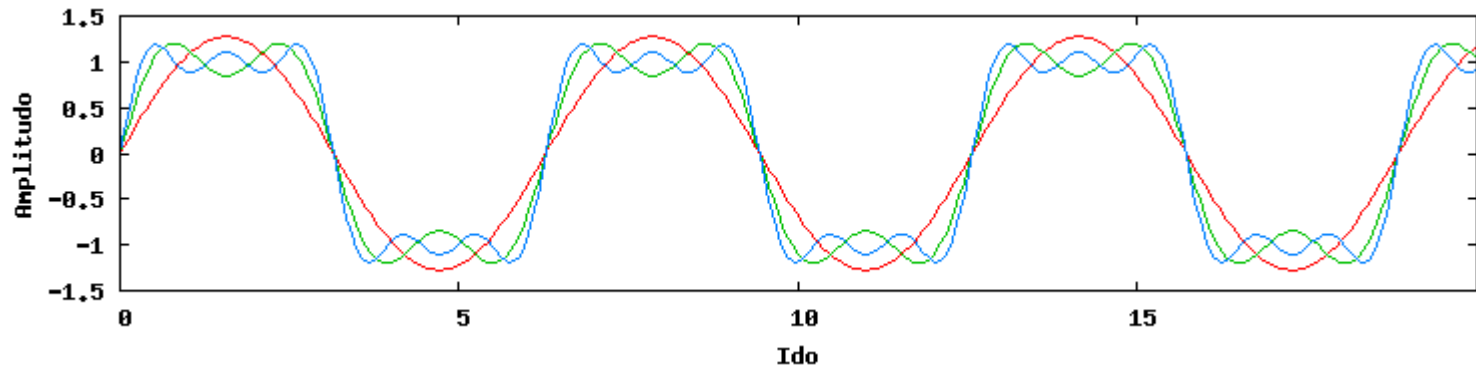
$$x_1 = C \cos\left(\frac{\omega - \omega_o}{2} t\right) \cos\left(\frac{\omega + \omega_o}{2} t\right)$$

$$x_2 = C \sin\left(\frac{\omega - \omega_o}{2} t\right) \sin\left(\frac{\omega + \omega_o}{2} t\right)$$





## Rezgések Fourier-felbontása:

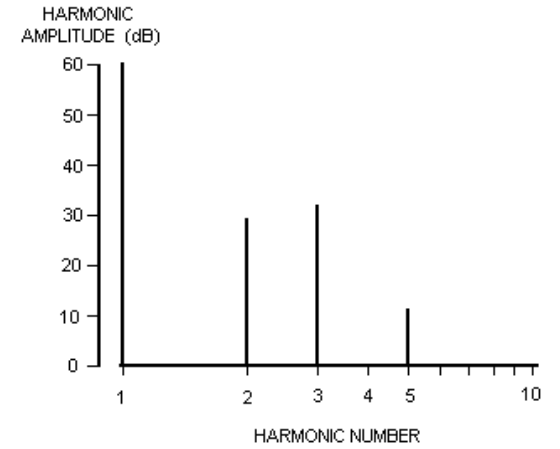
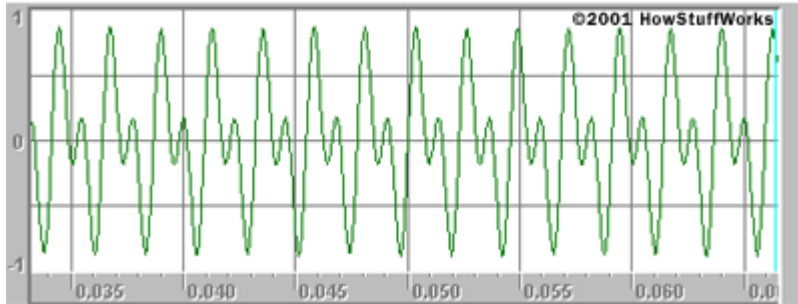
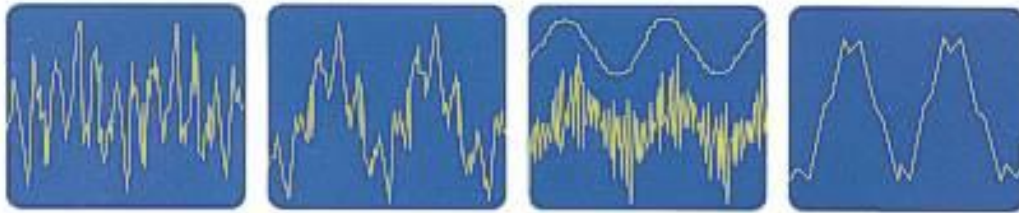


Tehát a Fourier transzformáció:

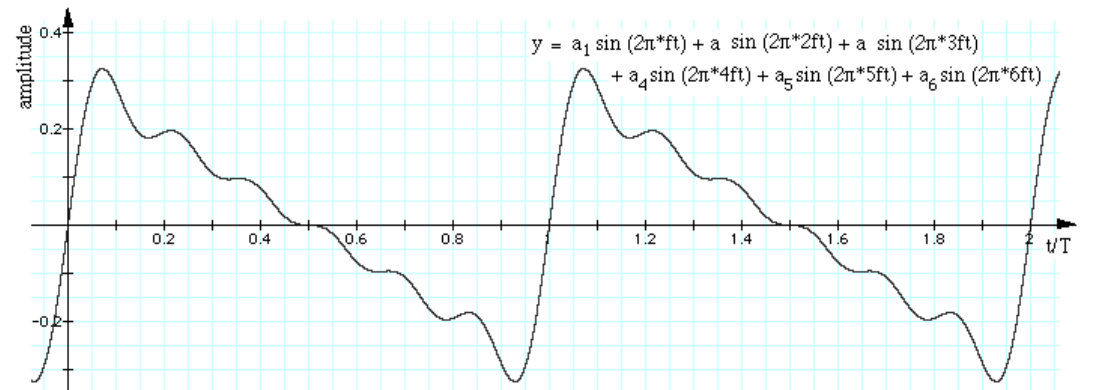
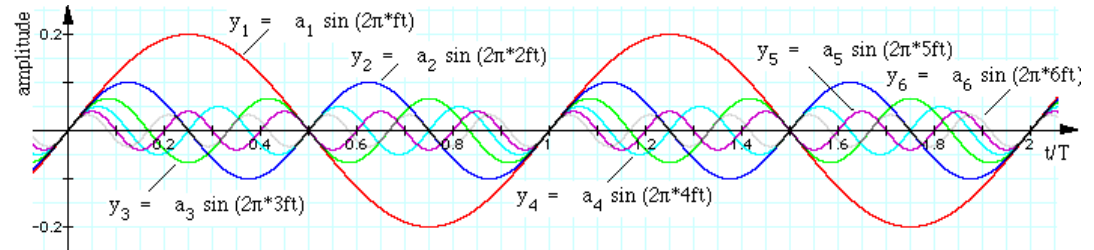
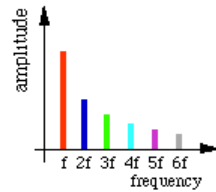
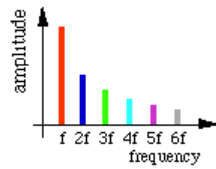
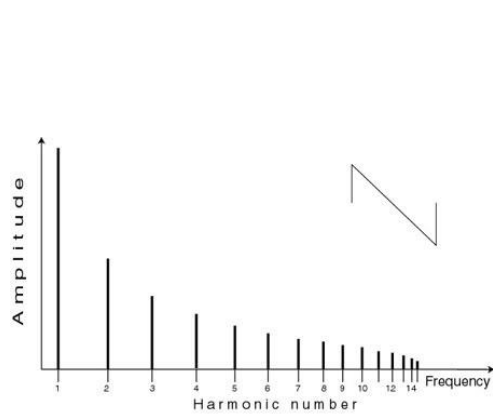
$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

$$F(j\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos\left(2\pi n \frac{t}{T}\right) dt \quad \longrightarrow \quad f(t) = c_0 + \sum_{n=1}^{\infty} \left[ a_n \cdot \cos\left(2\pi n \frac{t}{T}\right) + b_n \cdot \sin\left(2\pi n \frac{t}{T}\right) \right]$$



**Figure 7** Harmonic Flute spectrum



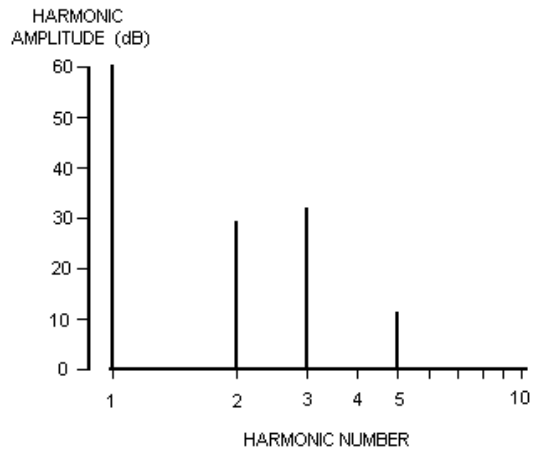
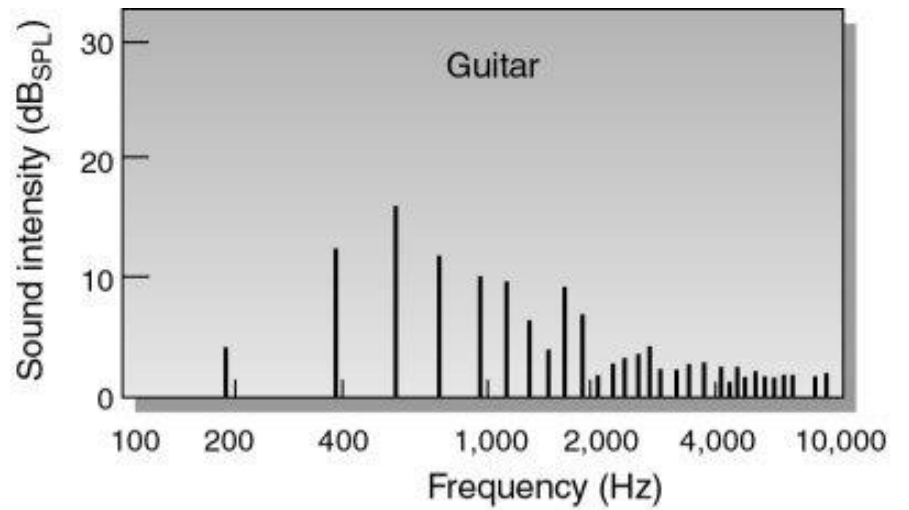
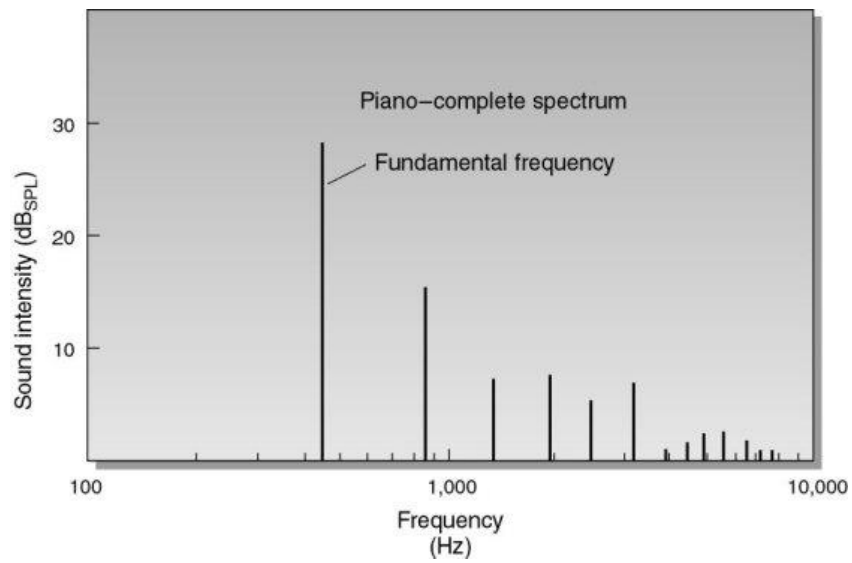
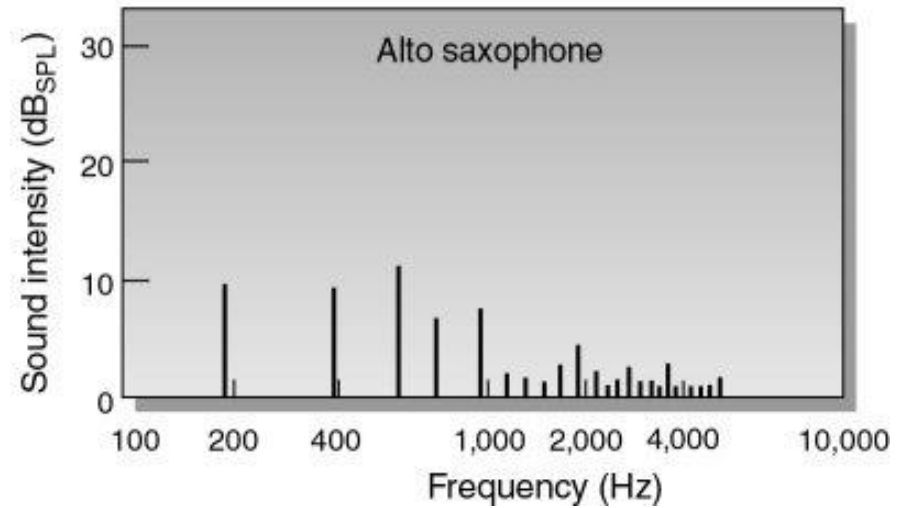


Figure 7 Harmonic Flute spectrum

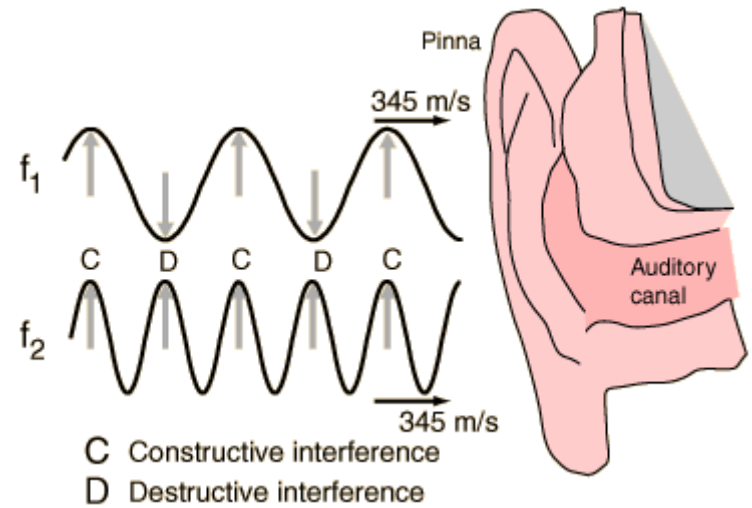




# A lebegés jelensége:

$$y_1(t) = A \cos(\omega_1 t)$$

$$y_2(t) = A \cos(\omega_2 t)$$



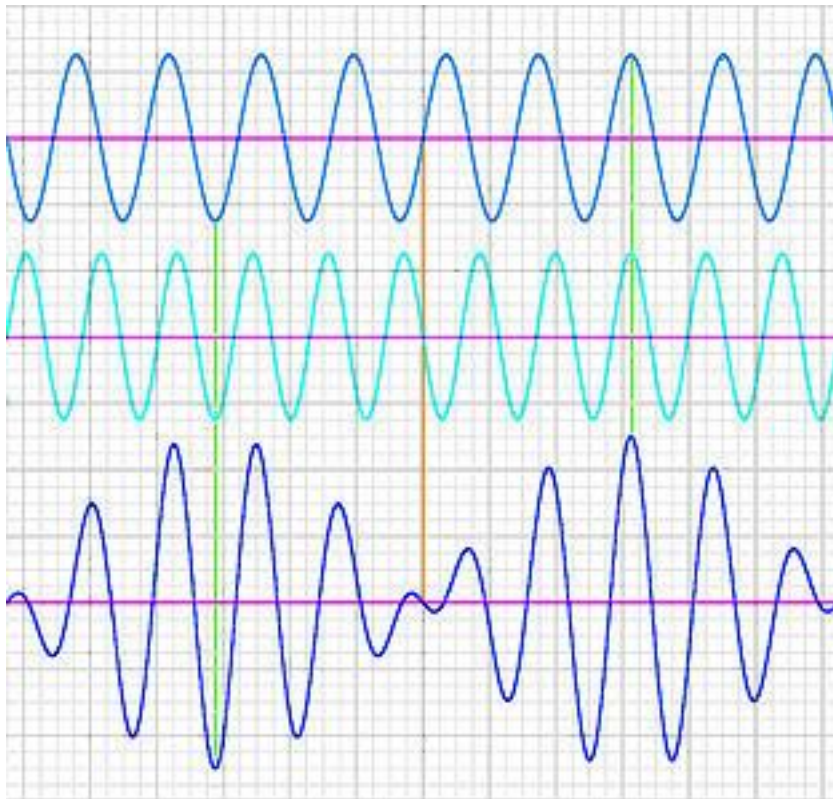
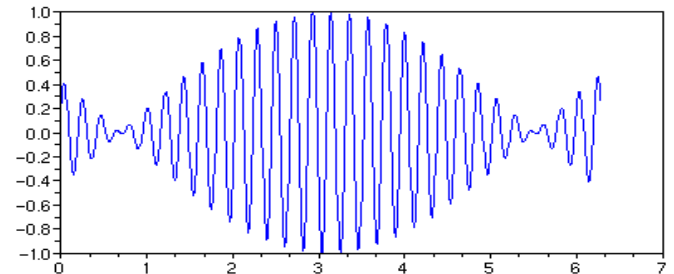
$$y(t) = y_1(t) + y_2(t)$$

$$y(t) = A \cos(\omega_1 t) + A \cos(\omega_2 t)$$

$$y(t) = 2A \cos\left(\frac{\omega_1 + \omega_2}{2} t\right) \cos\left(\frac{\omega_2 - \omega_1}{2} t\right)$$

$$y(t) = 2A \cos\left(2\pi \frac{f_1 + f_2}{2} t\right) \cos\left(2\pi \frac{f_2 - f_1}{2} t\right)$$

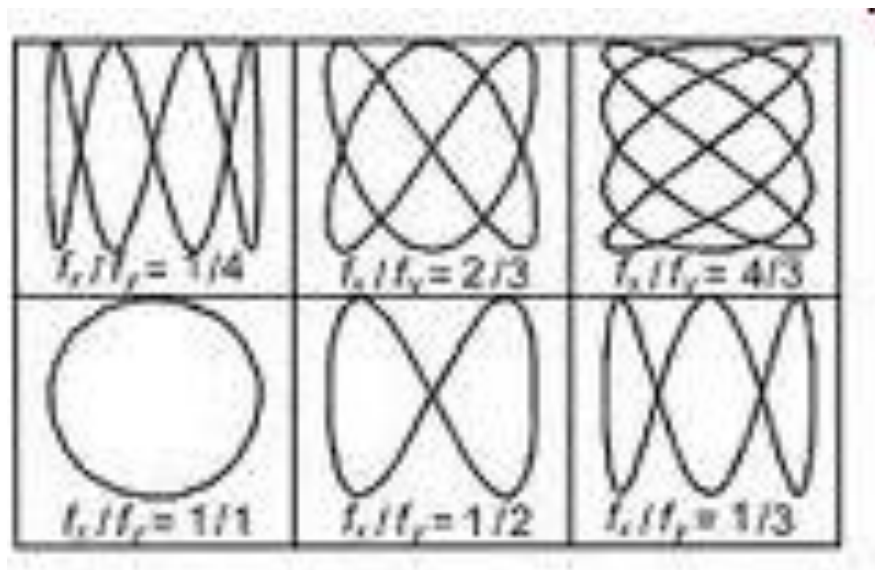
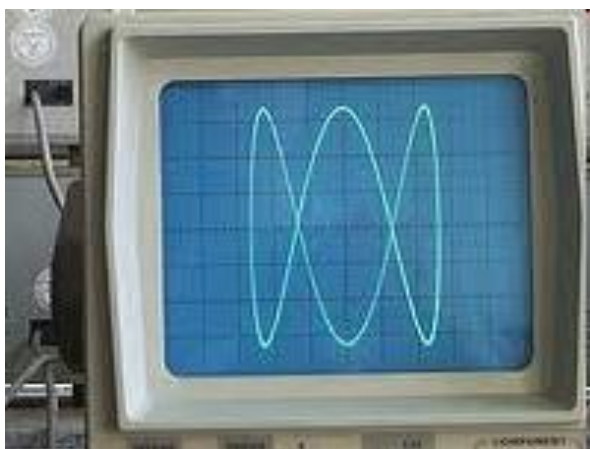
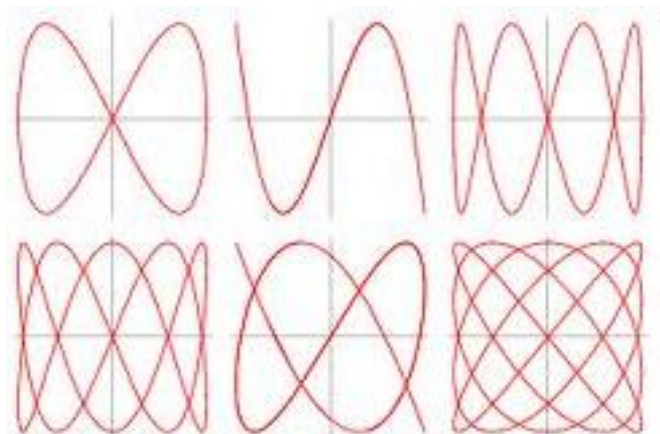
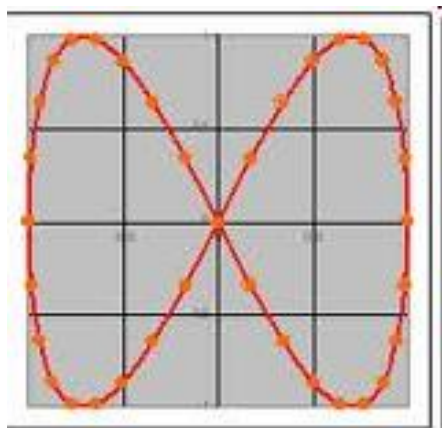
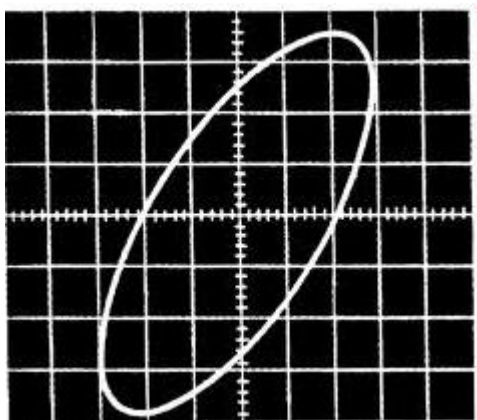
$$f_{\text{lebegés}} = \Delta f$$



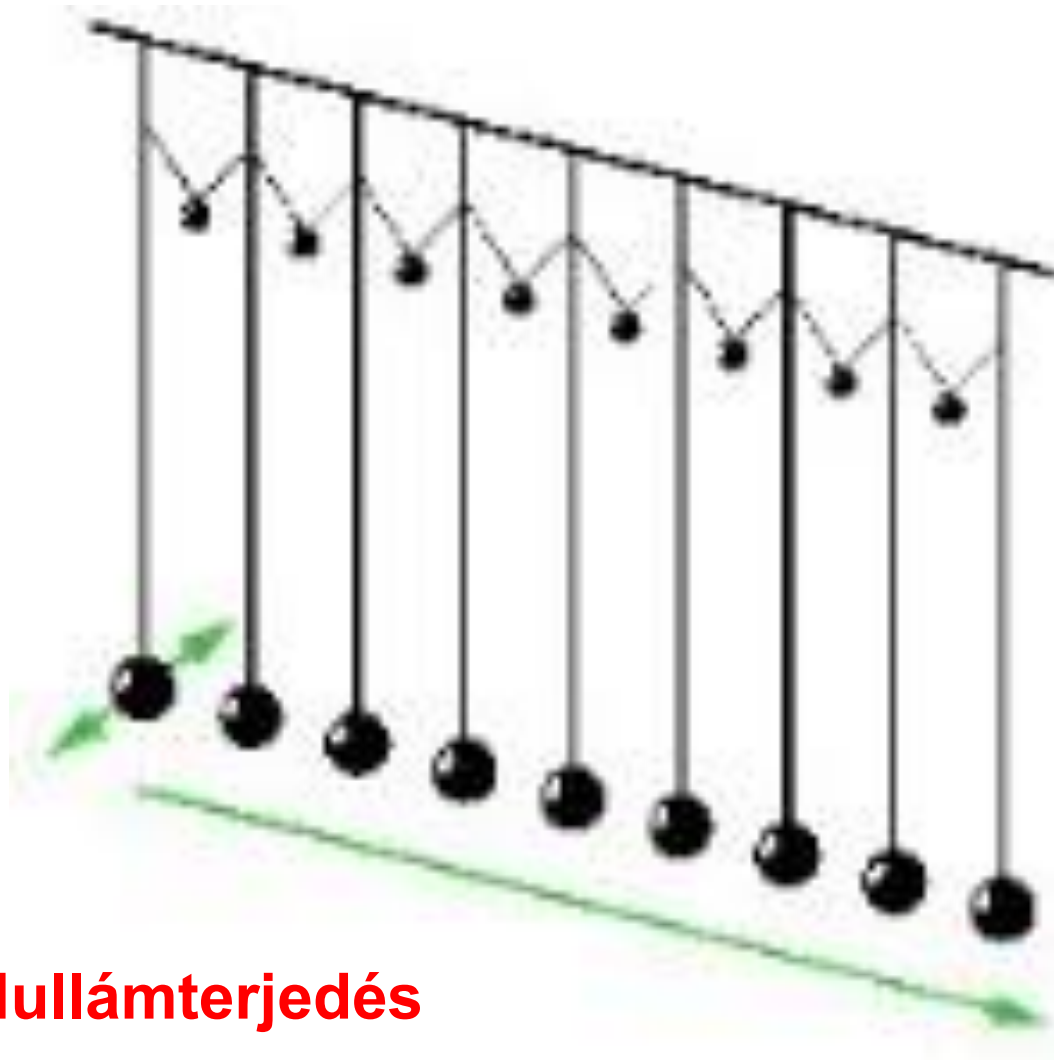
# Egymásra merőleges rezgések összetétele – Lissajou görbék

$$x(t) = A \sin(\omega t)$$

$$y(t) = A \sin(\omega t + \varphi)$$



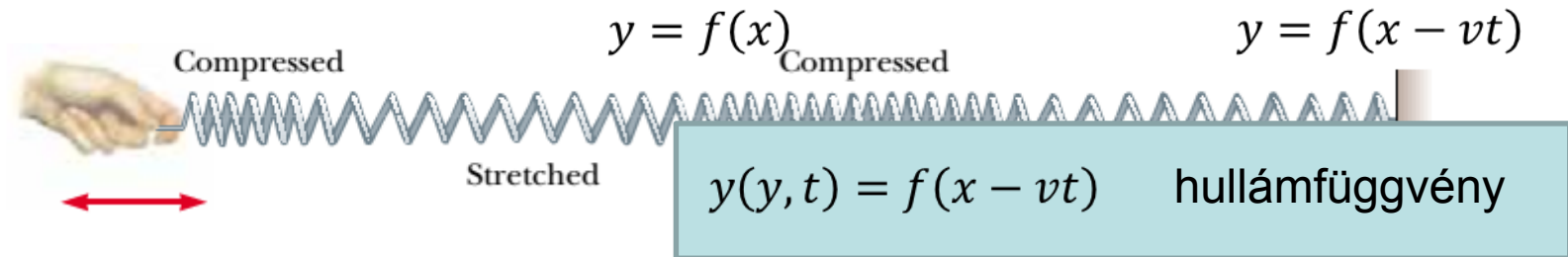
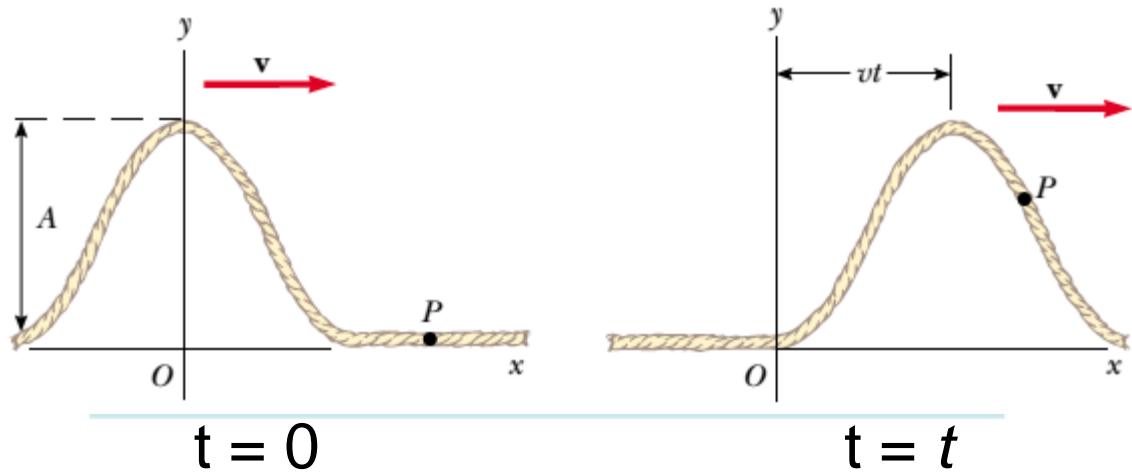
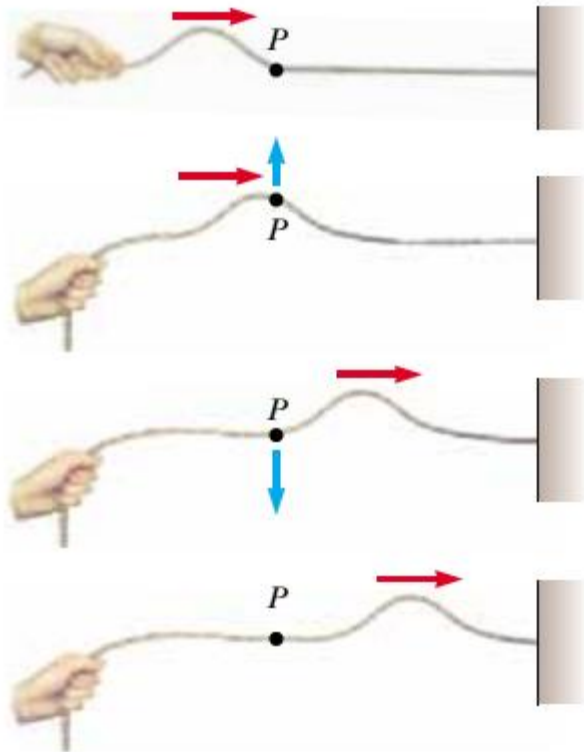
## *Csatolt rezgés még egyszer*



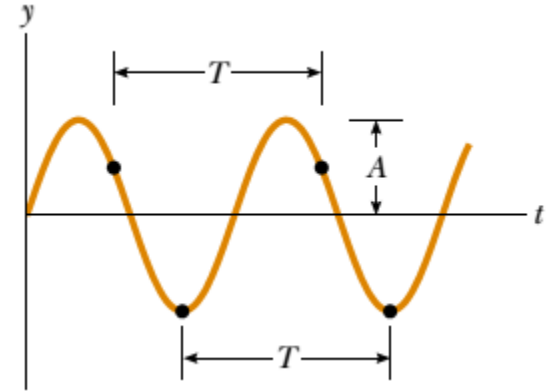
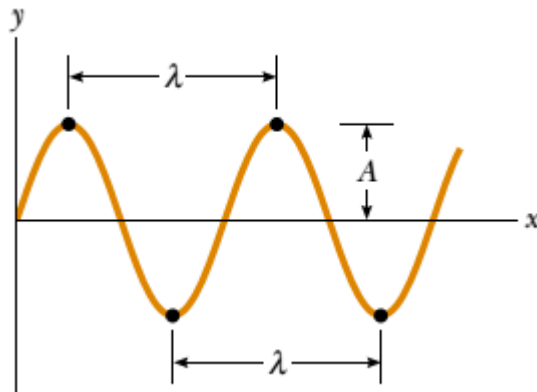
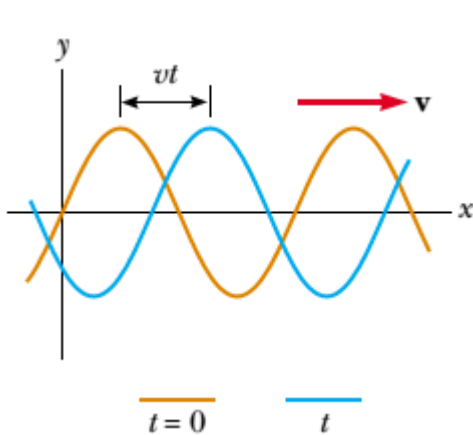
**Hullámterjedés**

# Hullámterjedés

(Hullámmozgás)



# Színusz(os) hullám



Hullámszám:  $k = \frac{2\pi}{\lambda}$

$v = \frac{\lambda}{T}$  és  $T = \frac{1}{f} \rightarrow f \cdot \lambda = v$

$v = \frac{\omega}{k}$

$$y(x, t = 0) = A \sin\left(\frac{2\pi}{\lambda} x\right)$$

$$y(x, t) = A \sin\left(\frac{2\pi}{\lambda} (x - vt)\right)$$

$$\frac{2\pi}{\lambda} vt = \frac{2\pi}{\lambda/v} t = \frac{2\pi}{T} t = \omega t$$

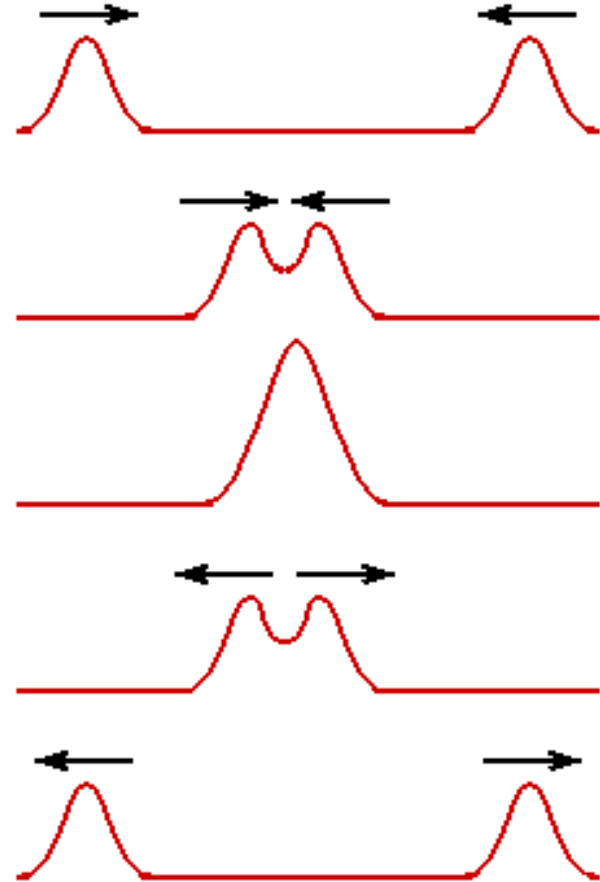
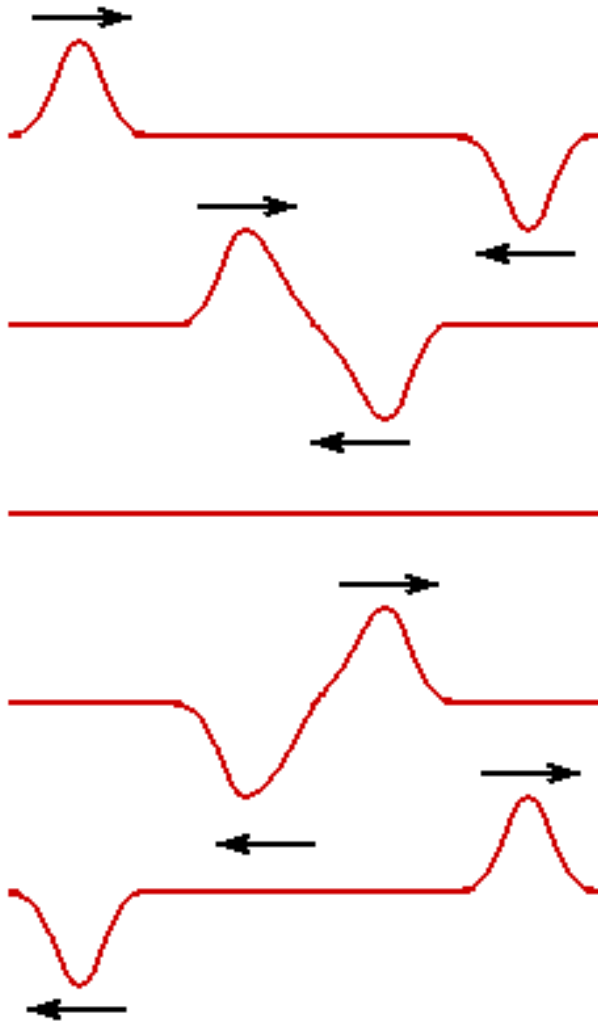
$$y(x, t) = A \sin(kx - \omega t + \varphi)$$

Hullámegyenlet:

$$\frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2}$$

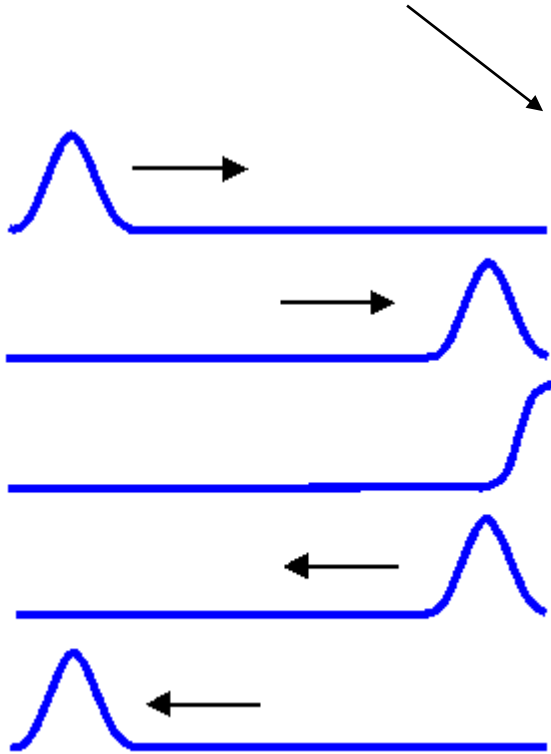
# Szuperpozíció

Linearitás!!!

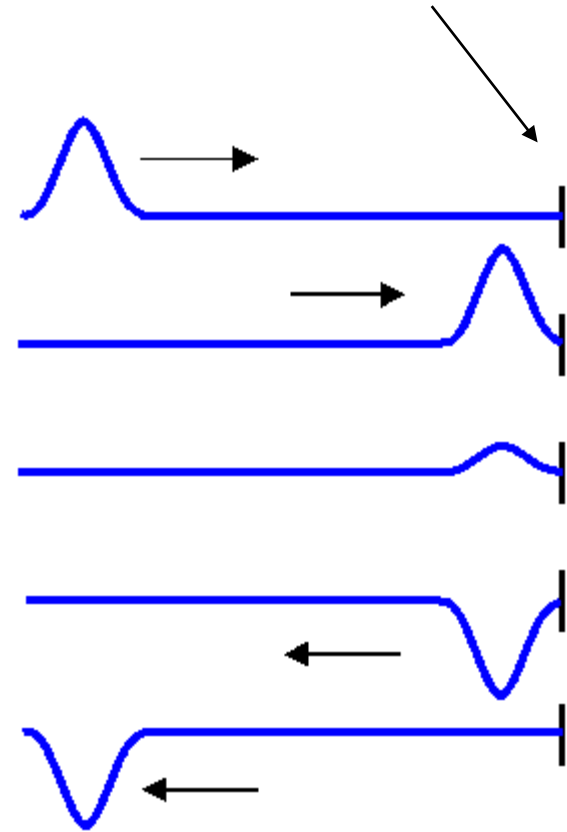


# Hullámok visszaverődése, reflexiója:

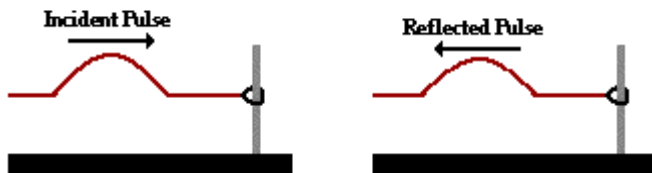
Nyitott vég



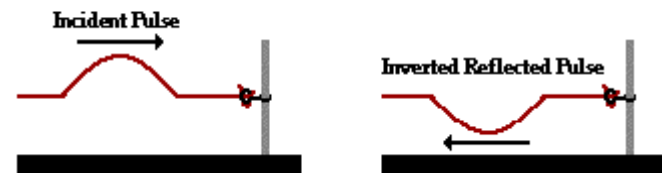
Zárt vég




Free End Reflection



Fixed End Reflection



## Állóhullám



$y_1(x, t) = A \cos(kx - \omega t)$        $y_2(x, t) = A \cos(kx + \omega t)$

$$y = y_1 + y_2$$

$$y(x, t) = 2A \cos(\omega t) \cos(kx)$$

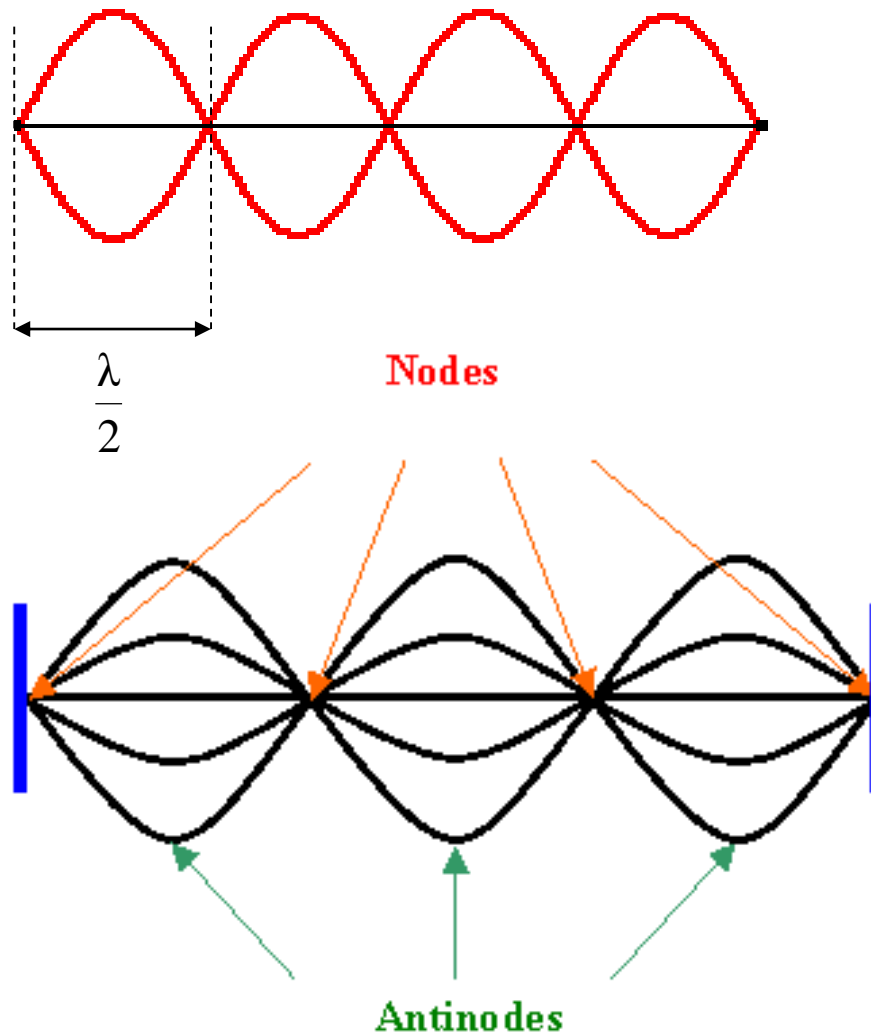
$$\omega = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda}$$

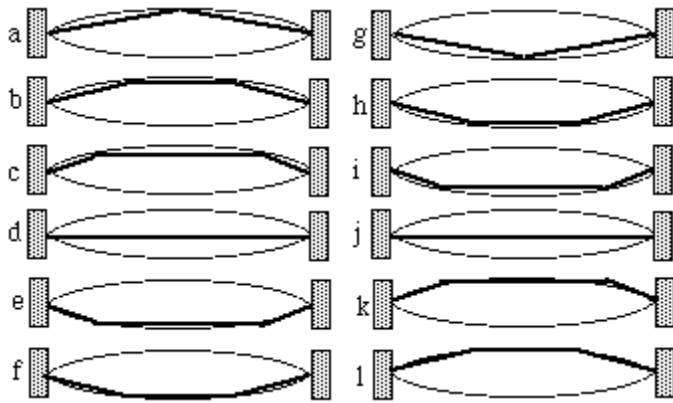


# Állóhullám:

Typical Diagram of a Standing Wave



## Alap és felharmónikusok (mindkét vég zárt)



fixed

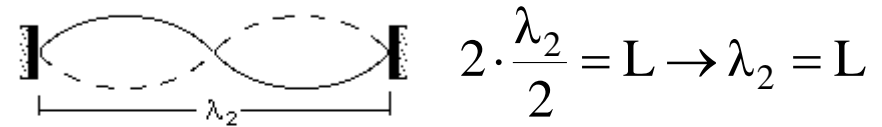
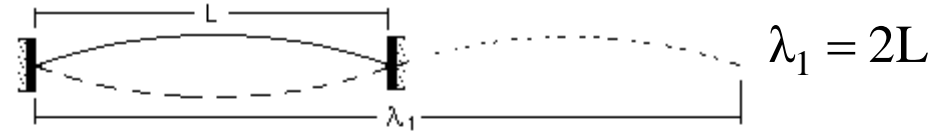
$$f = \frac{v}{\lambda}$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

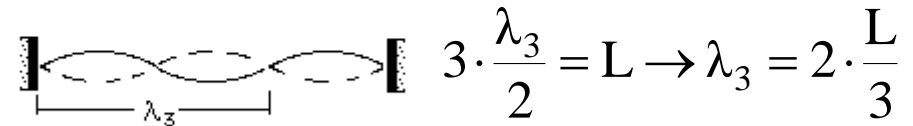
$$f_2 = \frac{v}{\lambda_2} = 2 \cdot \frac{v}{2L} = 2 \cdot f_1$$

$$f_3 = \frac{v}{\lambda_3} = 3 \cdot \frac{v}{2L} = 3 \cdot f_1$$

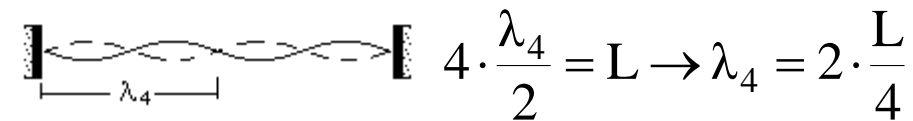
- 
- 
- 



$$2 \cdot \frac{\lambda_2}{2} = L \rightarrow \lambda_2 = L$$



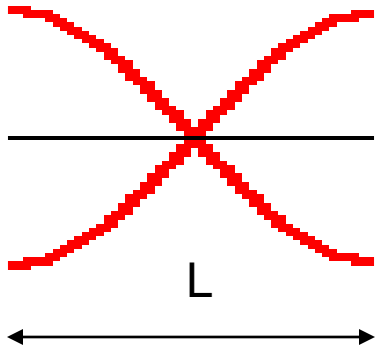
$$3 \cdot \frac{\lambda_3}{2} = L \rightarrow \lambda_3 = 2 \cdot \frac{L}{3}$$



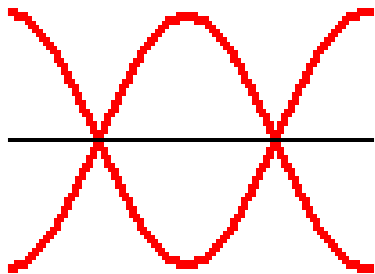
$$4 \cdot \frac{\lambda_4}{2} = L \rightarrow \lambda_4 = 2 \cdot \frac{L}{4}$$

$$f_n = n \cdot f_1$$

## Mindkét vég nyitott

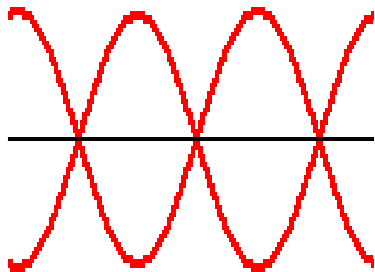


$$\lambda_1 = 2L \longrightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$



$$\lambda_2 = \frac{2L}{2}$$

$$f_2 = \frac{v}{\lambda_2} = 2 \cdot \frac{v}{2L} = 2 \cdot f_1$$



$$\lambda_3 = \frac{2L}{3}$$

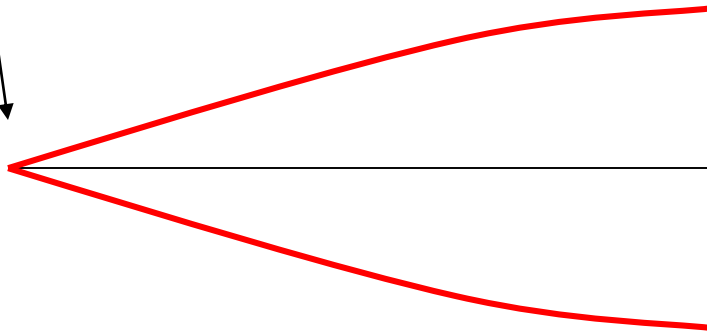
$$f_3 = \frac{v}{\lambda_3} = 3 \cdot \frac{v}{2L} = 3 \cdot f_1$$

$$f_n = n \cdot f_1$$

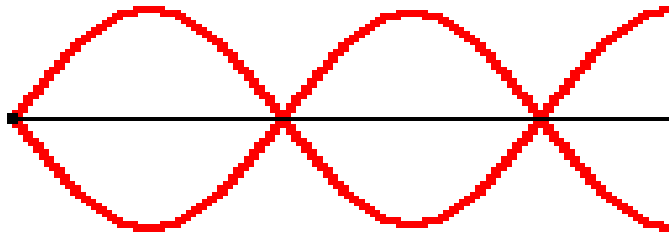
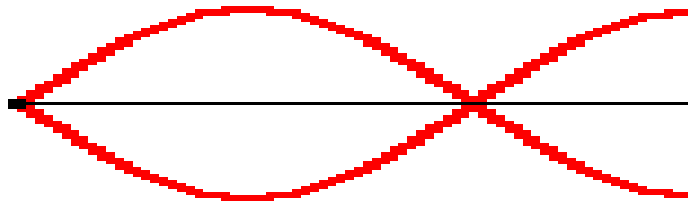
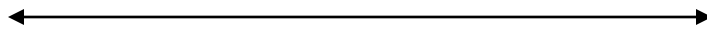
Zárt vég

Nyílt vég

**Egyik vég nyitott**



L



$$\frac{\lambda_1}{4} = L \rightarrow \lambda_1 = 4L \longrightarrow f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

$$f_2 = \frac{v}{\lambda_2} = 3 \cdot \frac{v}{4L}$$

$$3 \cdot \frac{\lambda_2}{4} = L \rightarrow \lambda_2 = \frac{4L}{3}$$

$$f_3 = \frac{v}{\lambda_3} = 5 \cdot \frac{v}{4L}$$

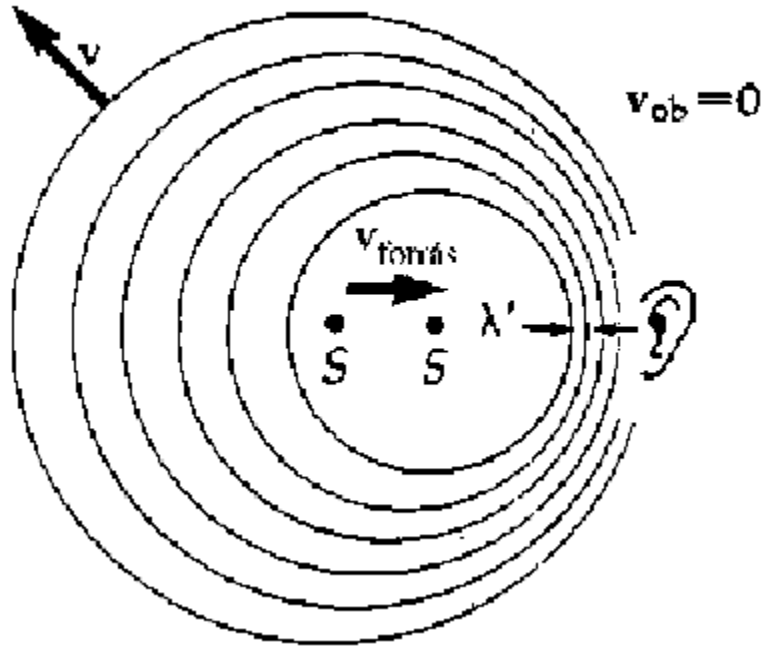
$$5 \cdot \frac{\lambda_3}{4} = L \rightarrow \lambda_3 = \frac{4L}{5}$$

○  
○  
○

$$f_n = (2n - 1) \cdot f_1$$

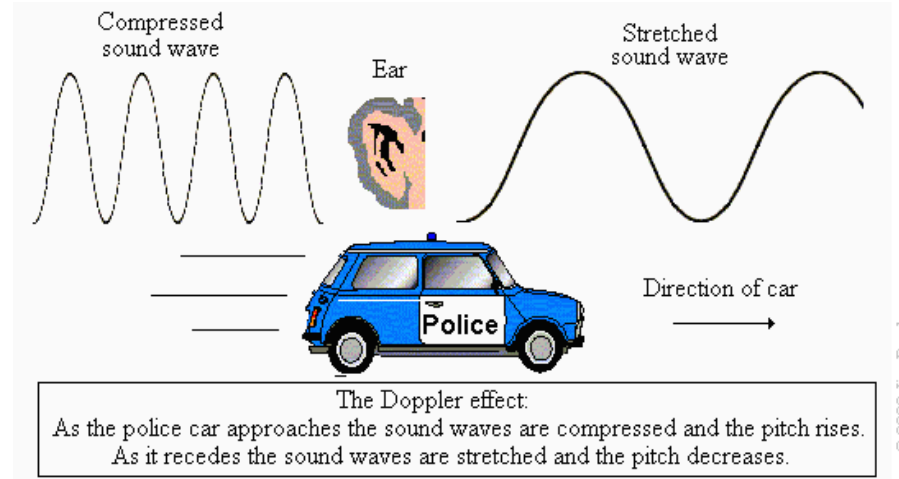
# Doppler effektus 1.

Forrás mozog, a megfigyelő áll



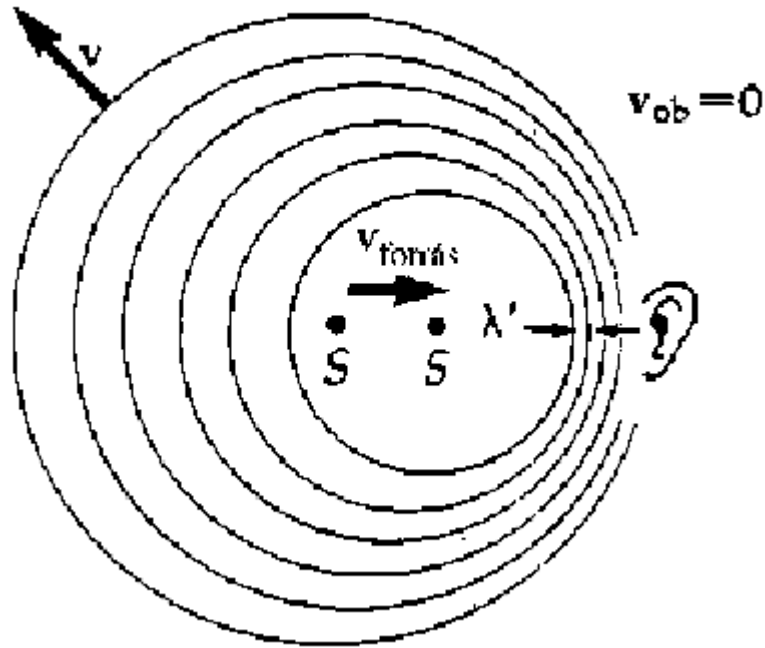
$$f' = \left( \frac{v}{\lambda} + \frac{v_{\text{megf}}}{\lambda} \right) = f \left( \frac{v + v_{\text{megf}}}{v} \right)$$

+



## Doppler effektus 2.

Forrás áll, a megfigyelő mozog



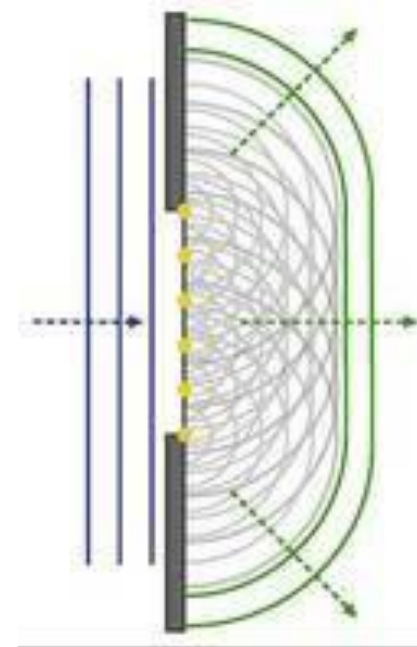
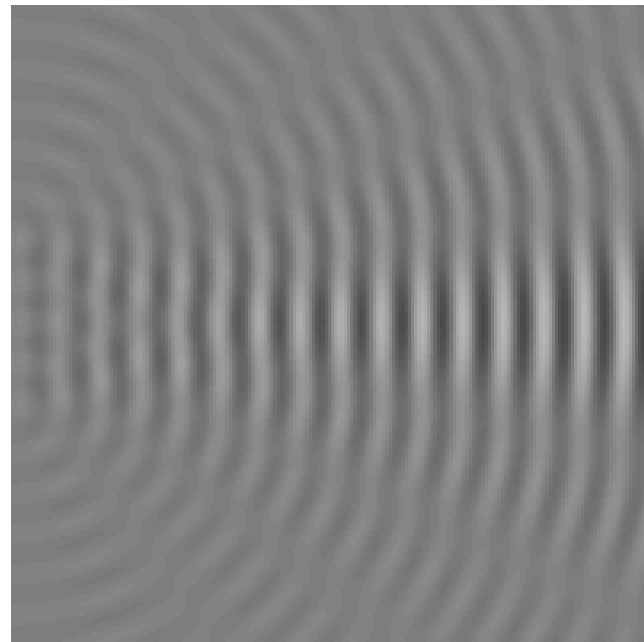
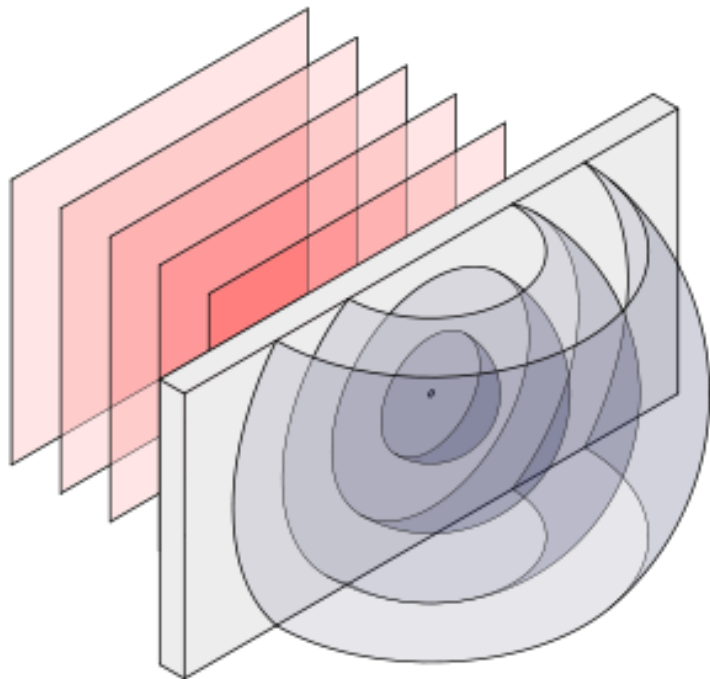
$$\lambda' = (\bar{v} - \bar{v}_{\text{forrás}}) / f_s$$

$$f' = \frac{v}{\lambda'} = f \left( \frac{v}{v - v_{\text{forrás}}} \right)$$

$$f' = f \left( \frac{v \pm v_{\text{megf}}}{v \mp v_{\text{forrás}}} \right)$$

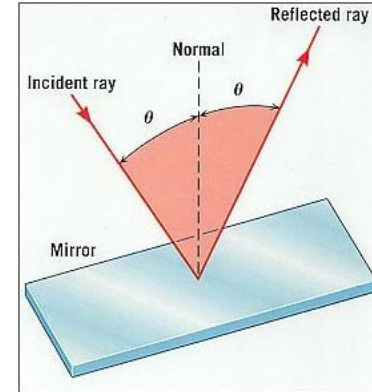
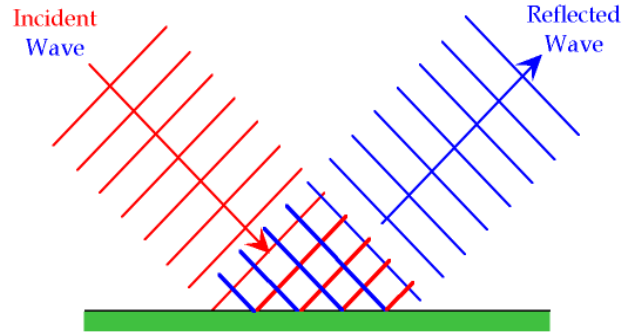
... és ha a szél fúj?

# Huygens elv 1.

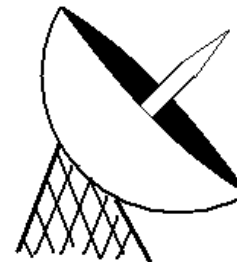
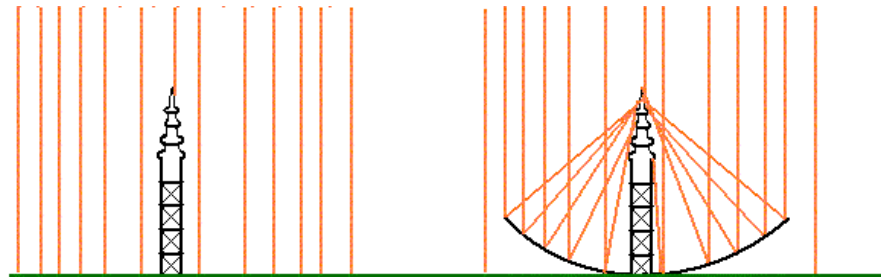
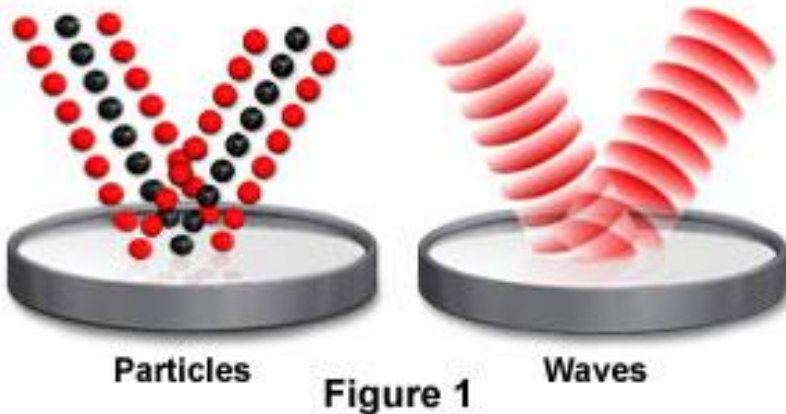


# Huygens elv 2.

reflexió



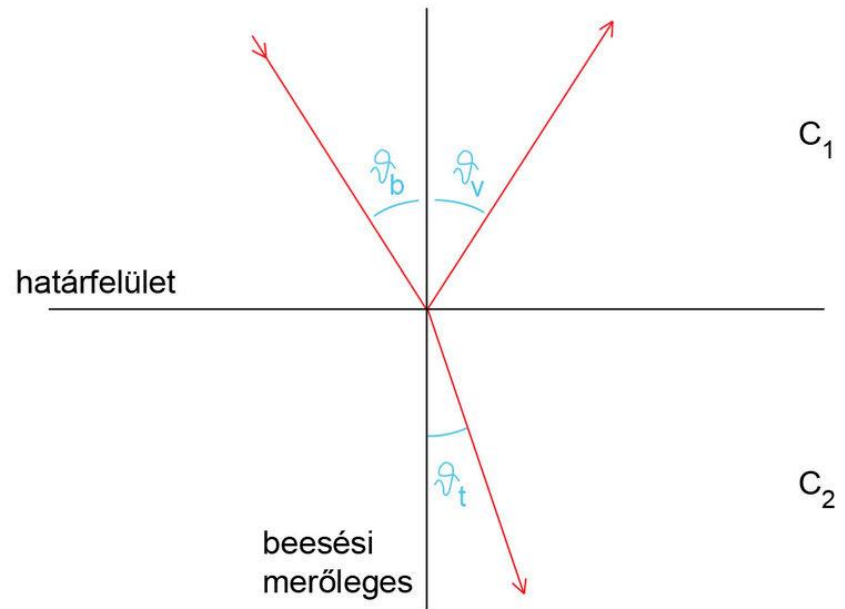
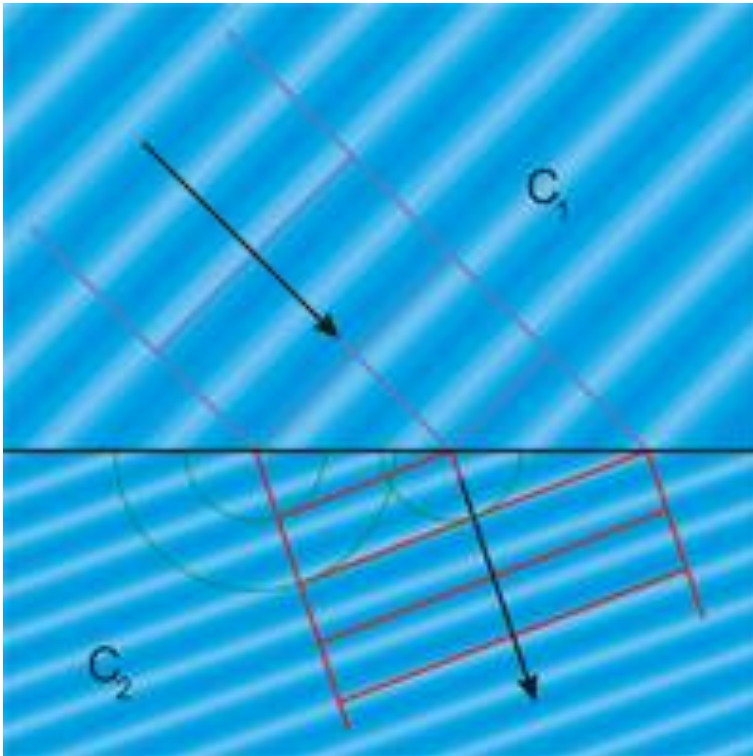
## Particles and Waves Reflected by a Mirror



This is the reason for this characteristic shape for items that receive radio waves or other transmissions from space. This particular radio receiver is pointed at a certain part of the sky so it can receive transmissions from a particular satellite.

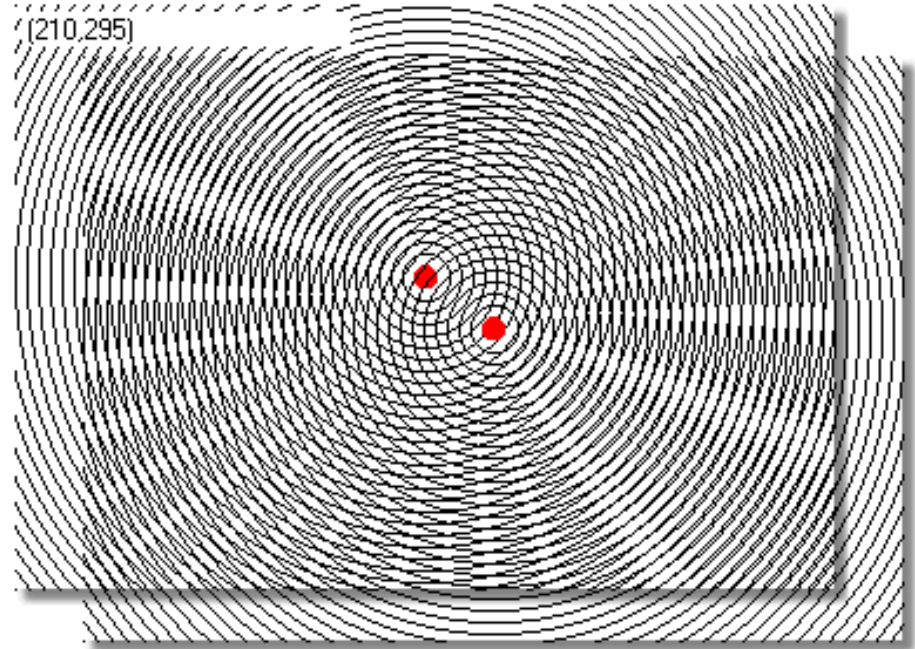
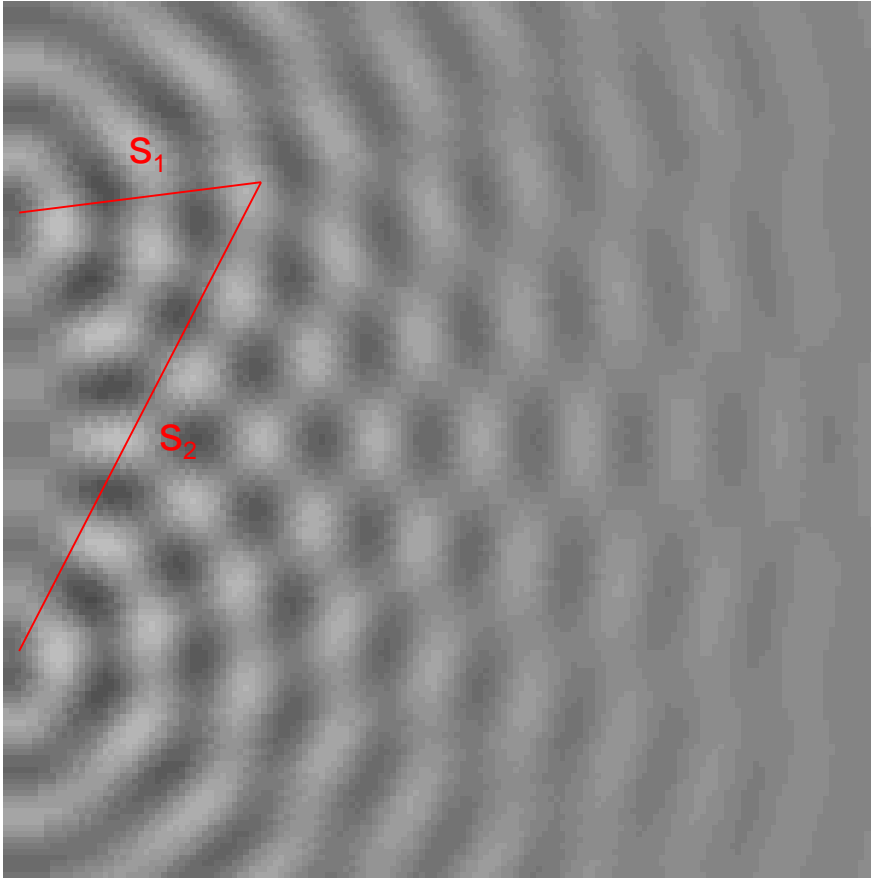


# Huygens elv 3.



Fénytörés (hullámtörés)

# Interferencia



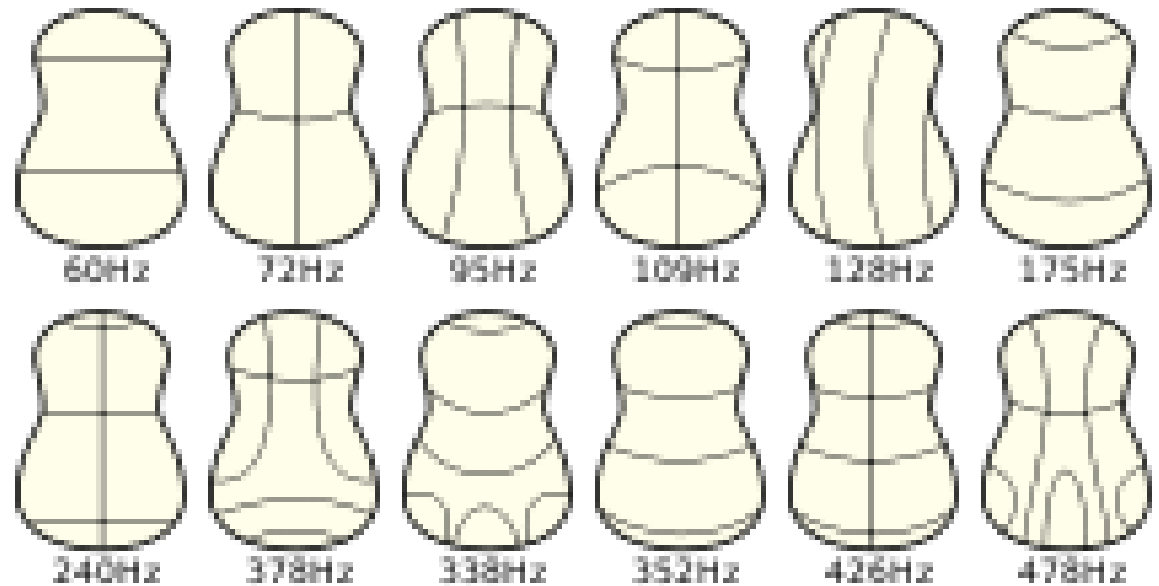
Erősítés:  $\Delta s = s_2 - s_1 = n\lambda$   
 $\Delta\varphi = n(2\pi)$

Kioltás:  $\Delta s = s_2 - s_1 = (2n + 1)\frac{\lambda}{2}$

$$\Delta\varphi = (2n + 1)\pi$$

$$n = 1, 2, 3 \dots$$

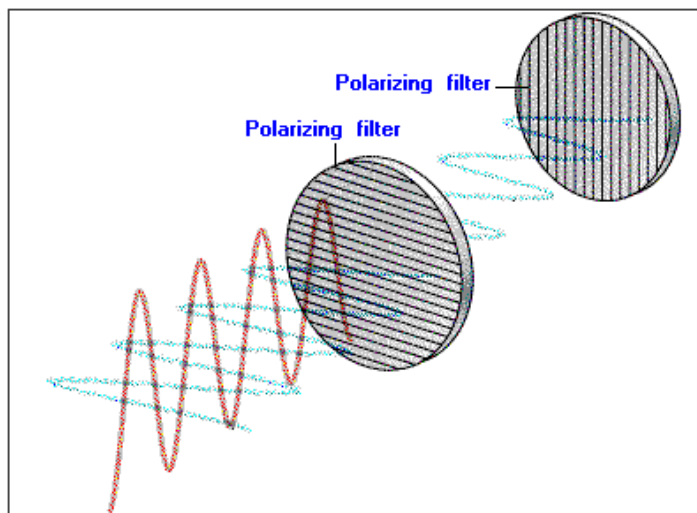
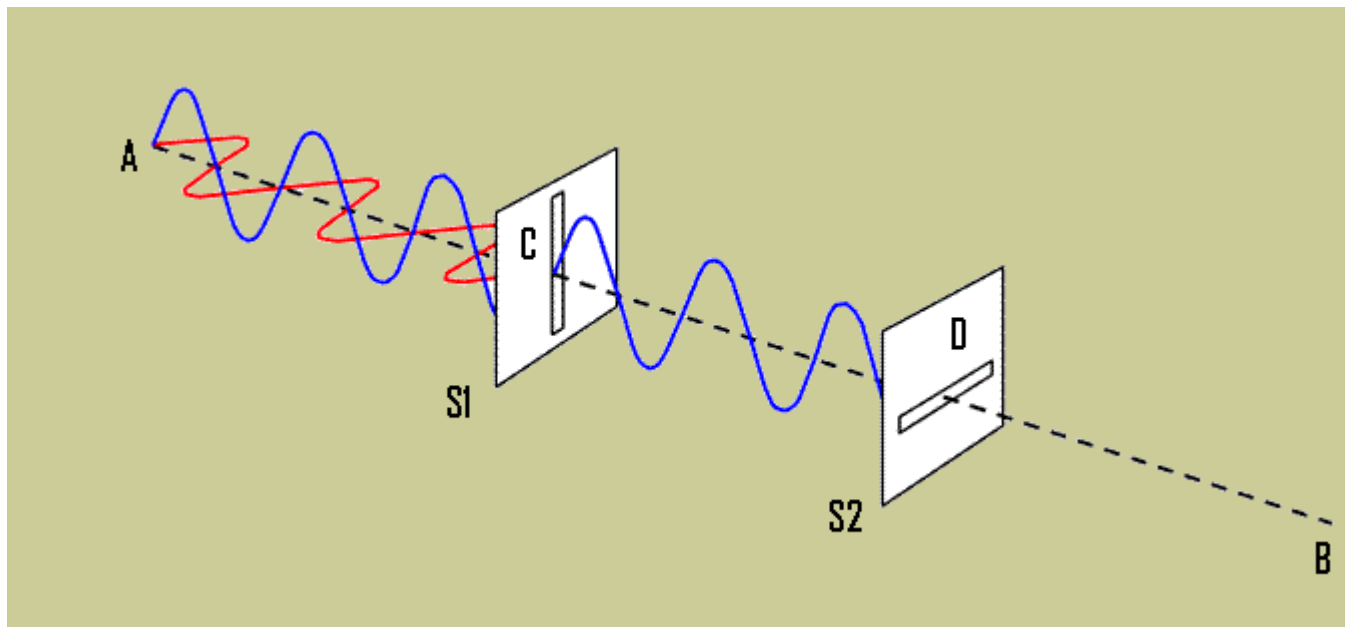
# Chladny ábrák



gitár

# Polarizáció (hullám)

kötél:



fény

