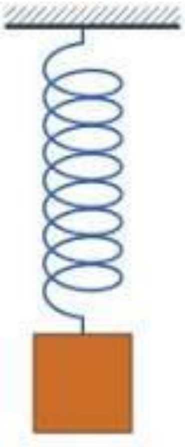
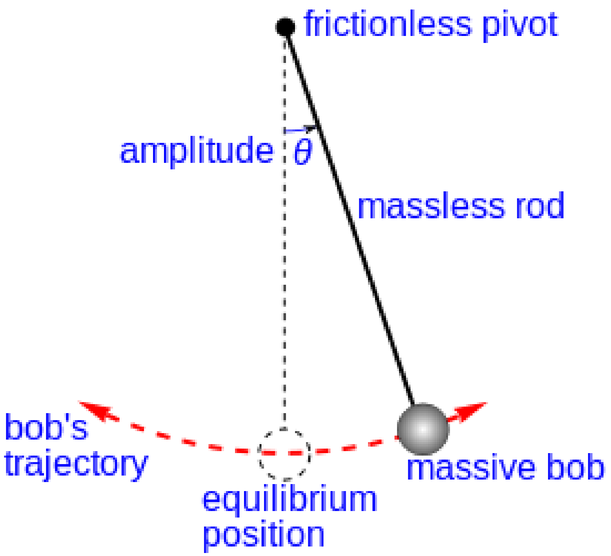
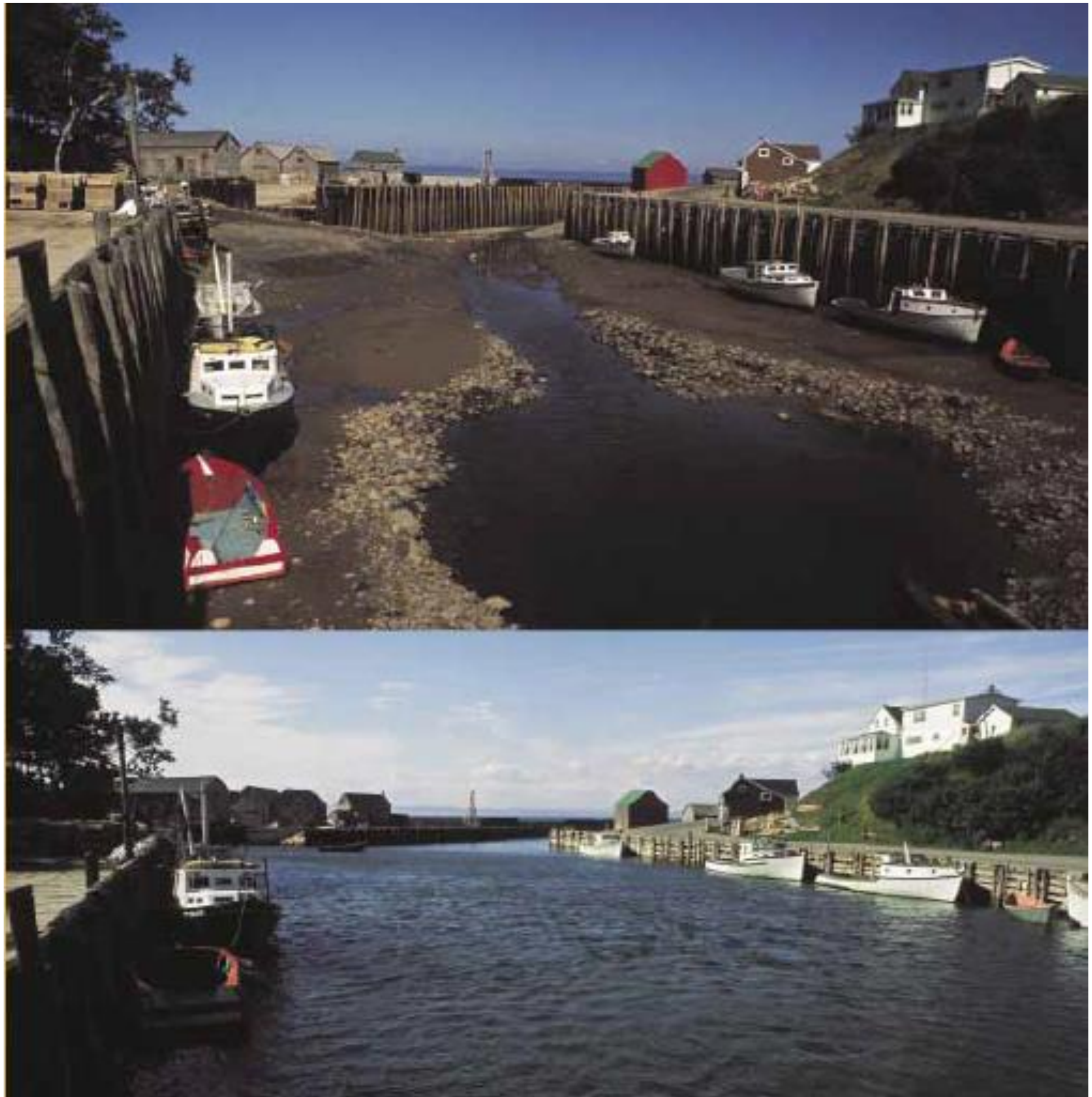
Lecture 5





Oscillatory motion
(vibration)

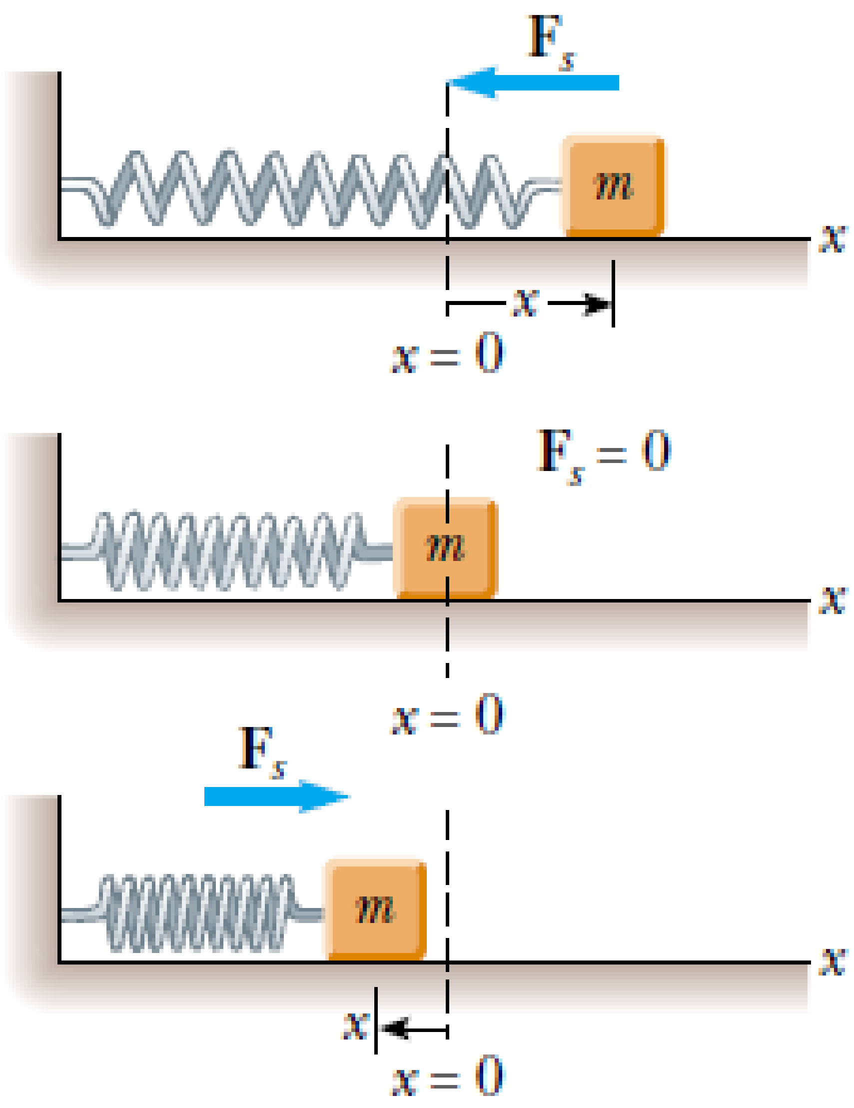
Periodic motion



▲ *In the Bay of Fundy, Nova Scotia, the tides undergo oscillations with very large amplitudes, such that boats often end up sitting on dry ground for part of the day. In this chapter, we will investigate the physics of oscillatory motion. (www.comstock.com)*

nonharmonic motion

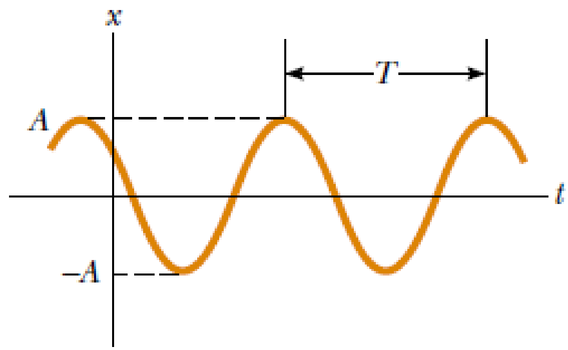
Oscillatory motion



$$F(x) = -kx$$

$$ma = -kx$$

$$a = -\frac{k}{m}x$$



Simple harmonic motion: $x(t) = A\sin(\omega t + \varphi)$

$$\omega = \sqrt{\frac{k}{m}}$$

$v(t) = A\omega\cos(\omega t + \varphi)$

$a(t) = -A\omega^2\sin(\omega t + \varphi)$

$\omega T = 2\pi$ (rad) $\Rightarrow$ angular frequency: $\omega = \dfrac{2\pi}{T} = 2\pi f$

frequency: $f = \dfrac{1}{T}$

period: T

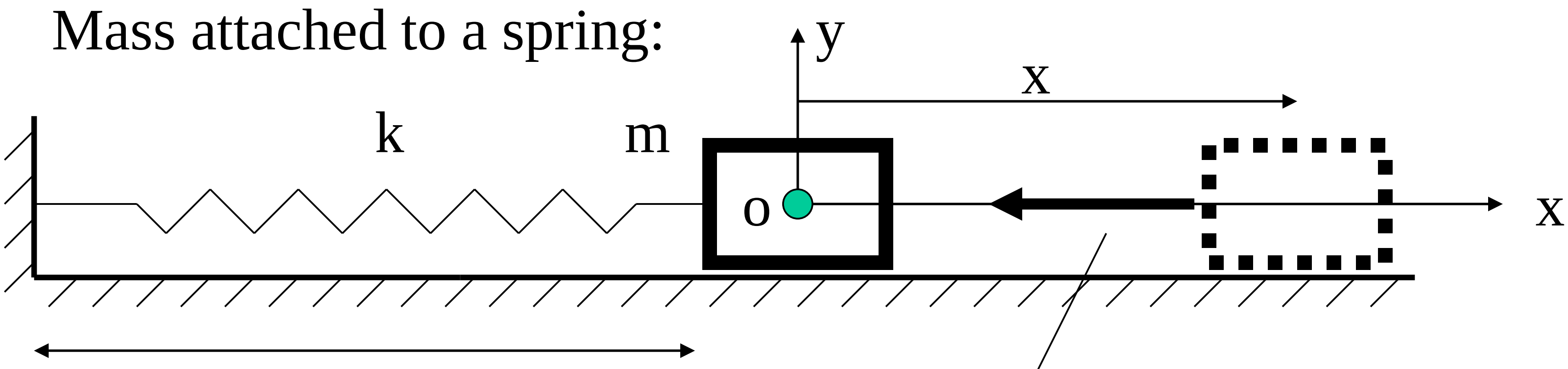
amplitude: A

(initial) phase: φ

max. speed: Aω

max. acceleration: $A\omega^2$

Mass attached to a spring:



unstretched length of the spring

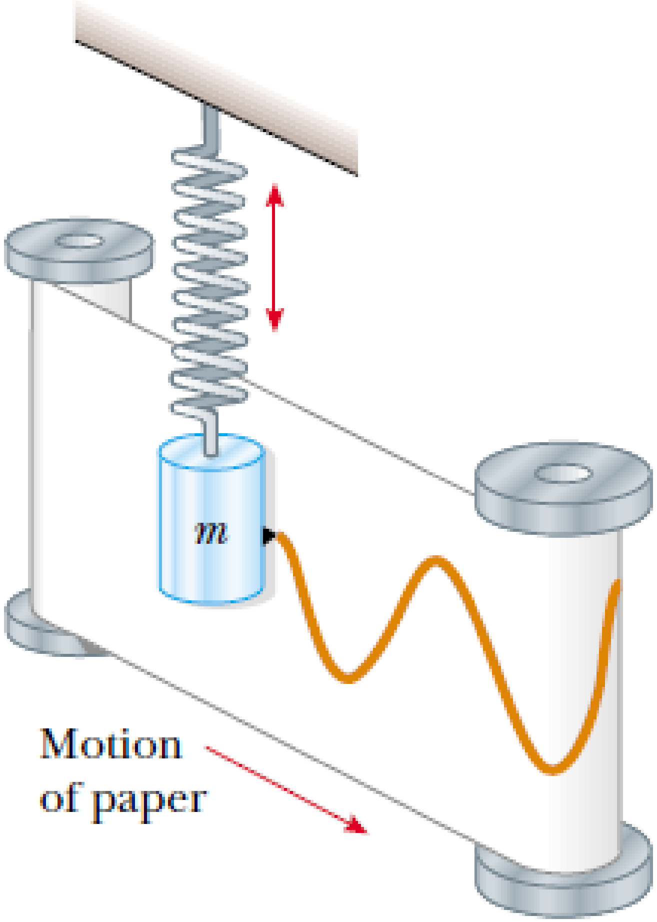
Spring force: $F(x) = -kx$

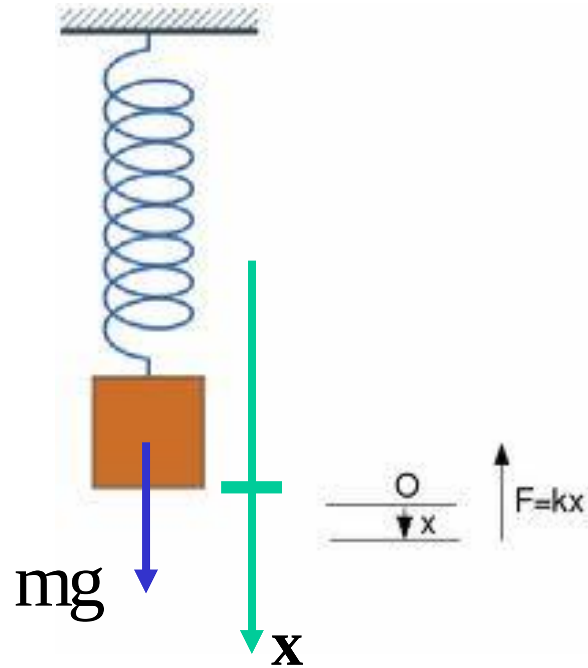
$$ma = -kx \quad \Rightarrow \quad a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$a = -A\omega^2 \sin(\omega t + \varphi) = -\frac{k}{m} A \sin(\omega t + \varphi) = -\frac{k}{m}x$$

$$\omega^2 = \frac{k}{m} \longrightarrow \omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \longrightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

period:





$$F = F_{spring} + weight = -kx + mg$$

$$ma = -kx + mg$$

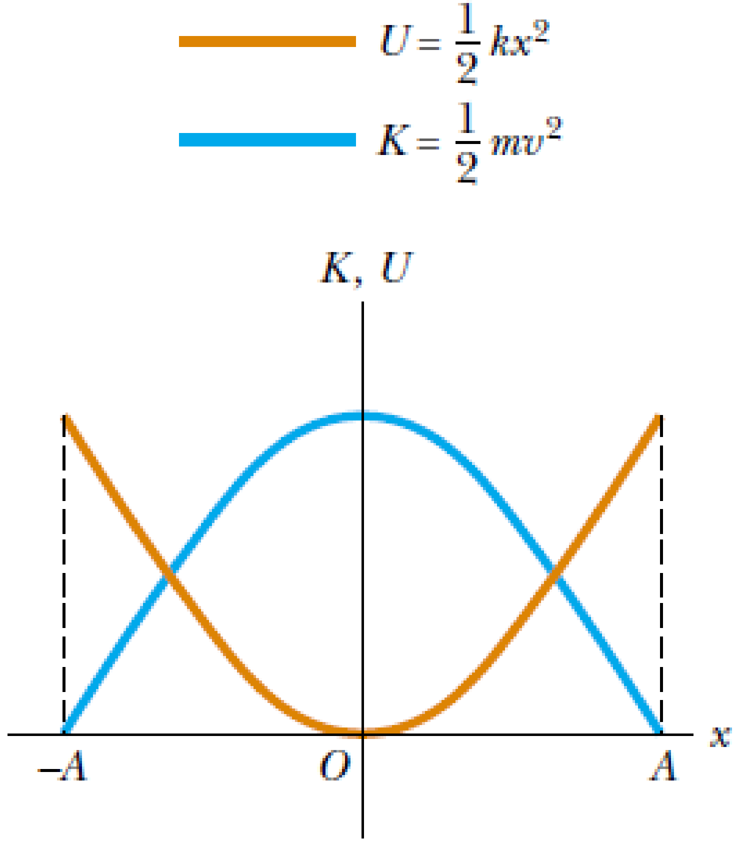
$$x = A\sin(\omega s + \varphi) + \frac{mg}{k}$$

Initial condition(s): $x(t=0) = x_o$, $v(t=0) = v_o$ $\Rightarrow$ $A = \ldots\ldots$, $\varphi = \ldots\ldots$

Energy of a simple harmonic oscillator:

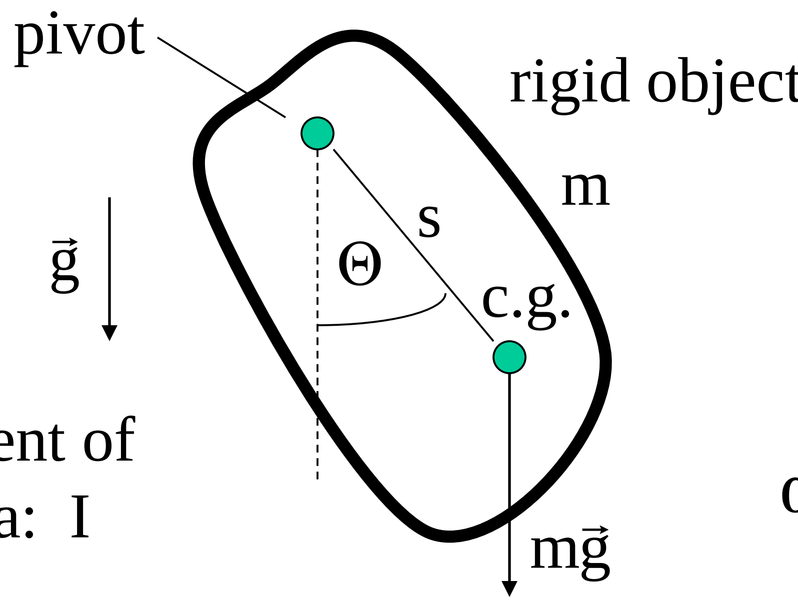
$$E_k = \frac{1}{2}mv^2$$

$$U = \frac{1}{2}kx^2$$



$$E = E_k + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$$

The physical pendulum ($\Theta_{max} << 1$ rad): / appr.: $\sin(\Theta) \approx \Theta$ /



moment of inertia: I

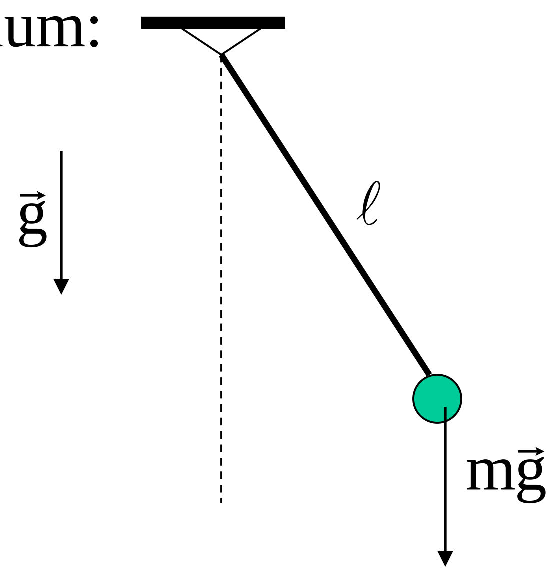
$$\tau = I\alpha \qquad \tau = -mgs \cdot \sin(\Theta)$$

$$I\alpha = -mgs \cdot \sin(\Theta)$$

$$\alpha = -\underbrace{\frac{mgs}{I}}_{\omega^2} \cdot \Theta$$

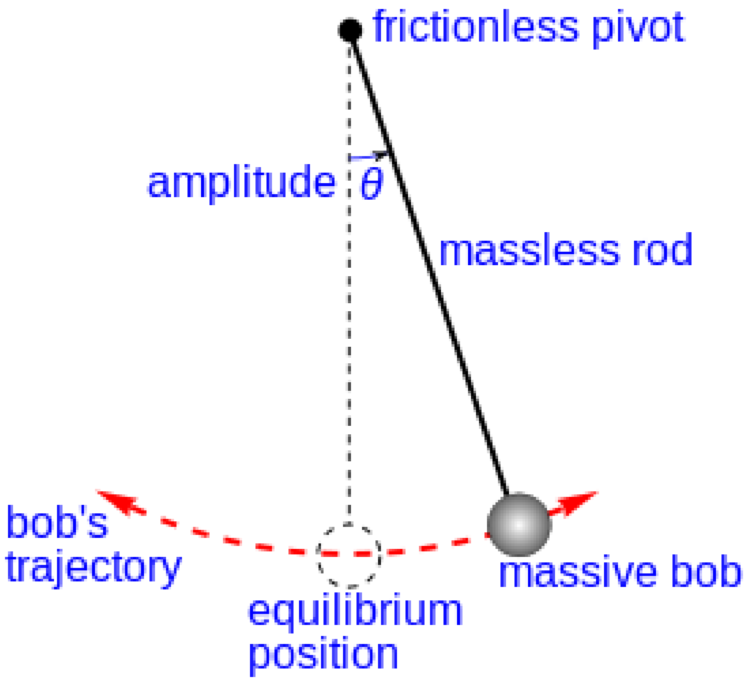
$$T = 2\pi\sqrt{\frac{I}{mgs}}$$

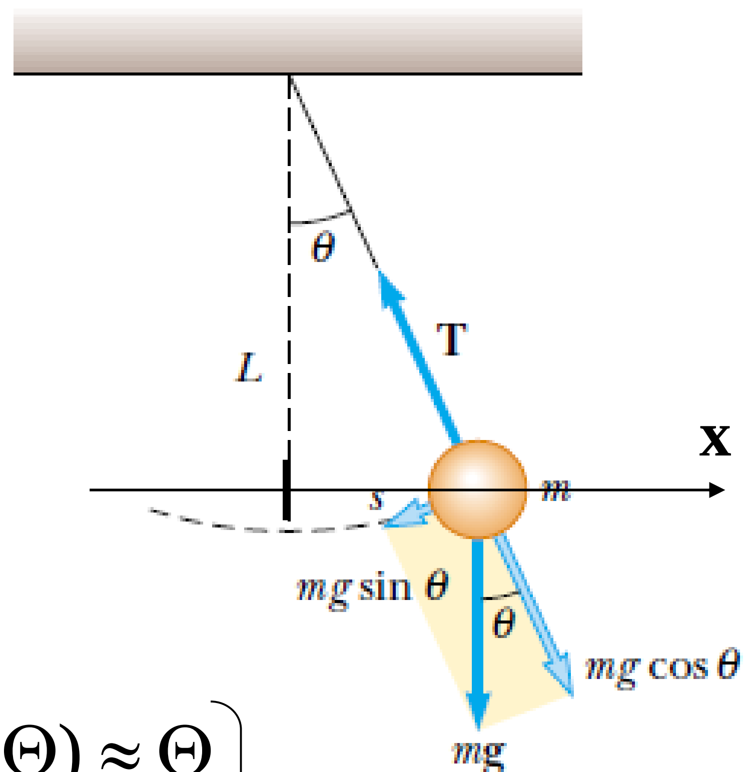
Simple pendulum:



$$I = m\ell^2$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$





$$\text{If } \Theta << 1\,\text{rad} \Rightarrow \sin(\Theta) \approx \tan(\Theta) \approx \Theta$$

$$\text{If } \Theta << 1\,\text{rad} \Rightarrow \cos(\Theta) \approx 1$$

$$F_x = -mg\sin(\Theta)\cos(\Theta) \approx -mg\Theta$$

$$F_x \approx -mg\Theta \approx -mg\frac{x}{\ell}$$

$$F_x \approx -mg\Theta \approx -mg\frac{x}{\ell}$$

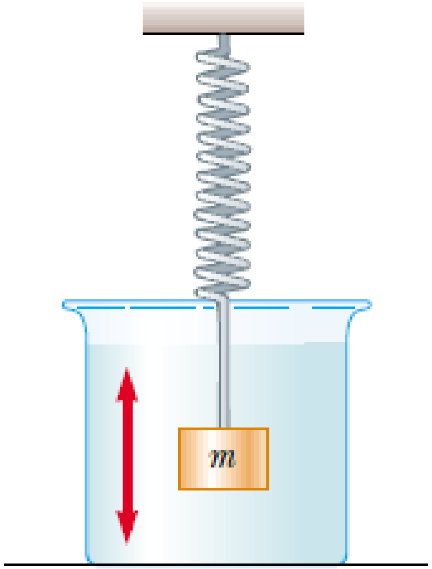
$$ma \approx -mg\frac{x}{\ell} \Rightarrow a \approx -\frac{g}{\ell}x \Rightarrow a \approx -\omega^2 x \Rightarrow \omega = \sqrt{\frac{g}{\ell}} \Rightarrow T = 2\pi\sqrt{\frac{\ell}{g}}$$

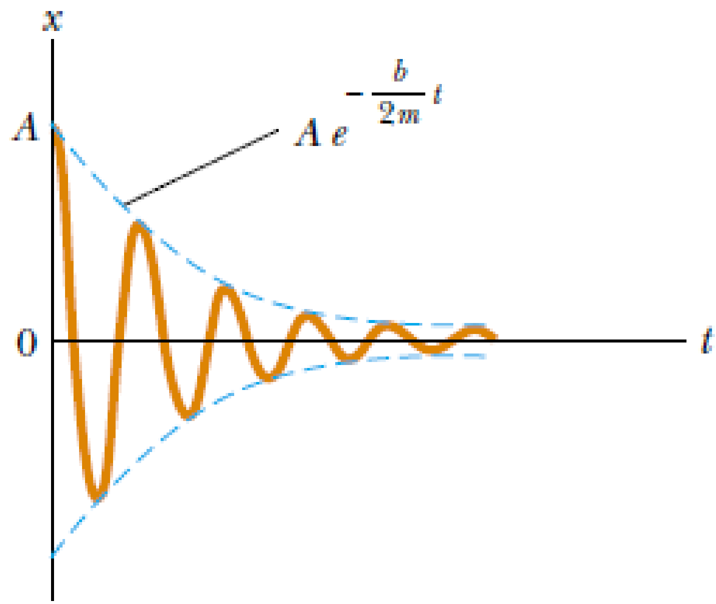
Damped oscillation

$$\sum F_x = -kx - bv_x = ma_x$$

$$x = Ae^{-\frac{b}{2m}t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$





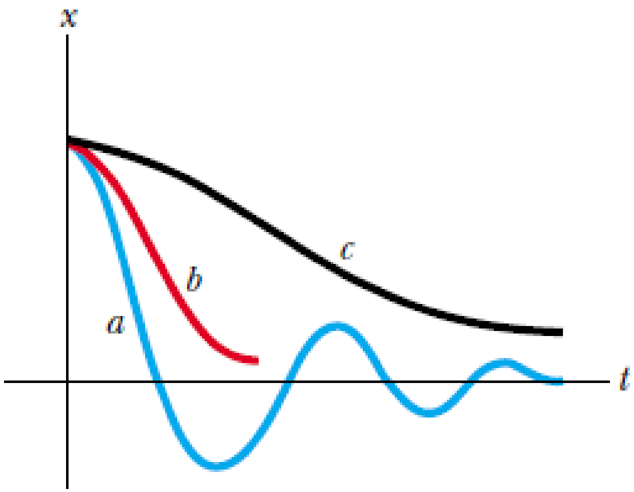


Figure 15.23 Graphs of position versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

Forced oscillation

$$\sum F = ma \longrightarrow F_0 \sin \omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$x = A\cos(\omega t + \phi) \qquad A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$