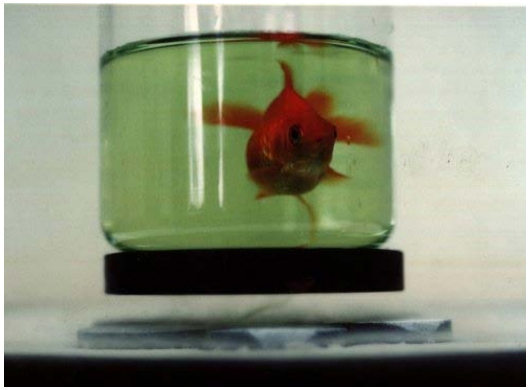
Superconductivity



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Demonstrations:
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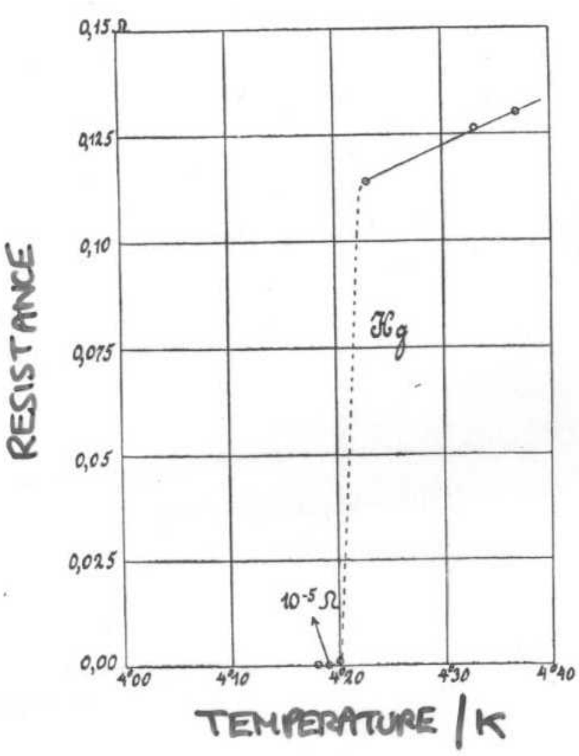
Nobel Prizes

Outline

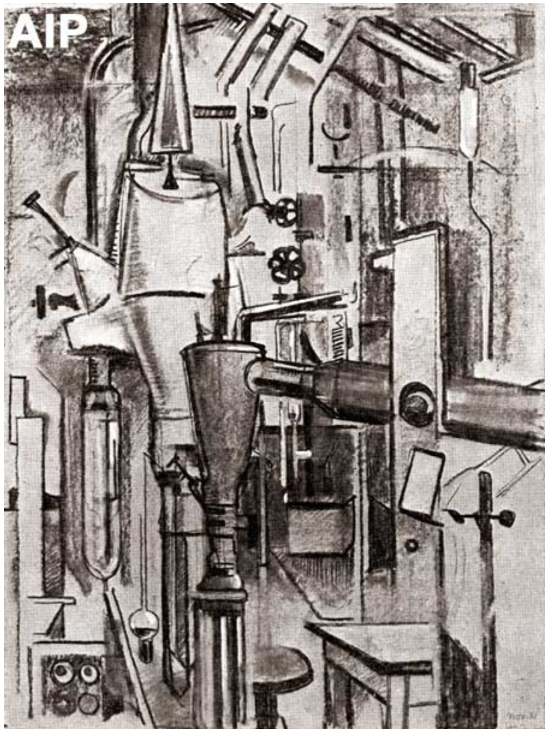
- Defining qualities of superconductors
  - Zero resistance, Meissner effect
- Type I and Type II superconductors
- Microscopic theory
- Macroscopic quantum effects
- Application of superconductors

Discovery of superconductivity

Heike Kamerlingh Onnes,
Leiden, 1911
Nobel Prize, 1913



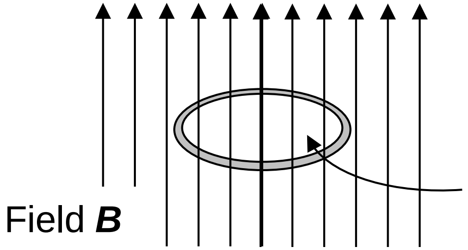
Resistance of
mercury as a
function of
temperature
(handwriting of
K.-O.)
No measurable
resistance below
T_c = 4.2 K



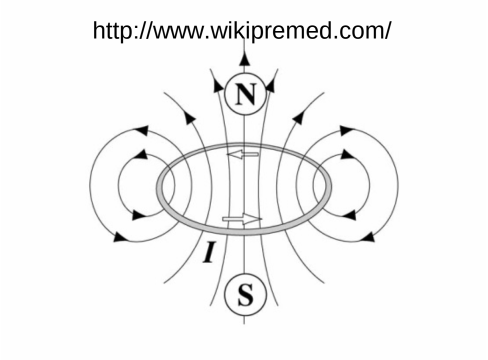
Sketch of the first helium
liquefier of the world.
K.-O., Leiden, 1904.
Boiling point at ambient
pressure: 4.2 K .

Experiment

Accurate measurement of the resistance of a superconducting wire



Cool down a superconducting loop in a magnetic field to below T_c



http://www.wikipremed.com/

Turn off the magnetic field at low temperature.

Faraday law of induction: $V = -\frac{d\Phi}{dt} = 0 \Rightarrow \Phi = const.$

Current induced in the superconductor: $I = \Phi / L$

Decay of the current with time: $I(t) = I_0 \, e^{-(R/L)t}$

Decay of the magnetic field of the current loop: $B(t) = B_0 \, e^{-(R/L)t}$

Estimation of the decay time constant $\tau = L/R$ for copper:

Radius of the loop: $r = 10$ mm

Diameter of the wire: $d = 1$ mm

Specific resistance of copper: $\rho = 1.5 \times 10^{-6} \, \mu\Omega\,\mathrm{cm}$

Self inductance: $L = \mu_0 r \left[\ln(16r/d) - 2\right] = 4 \times 10^{-8}$ H

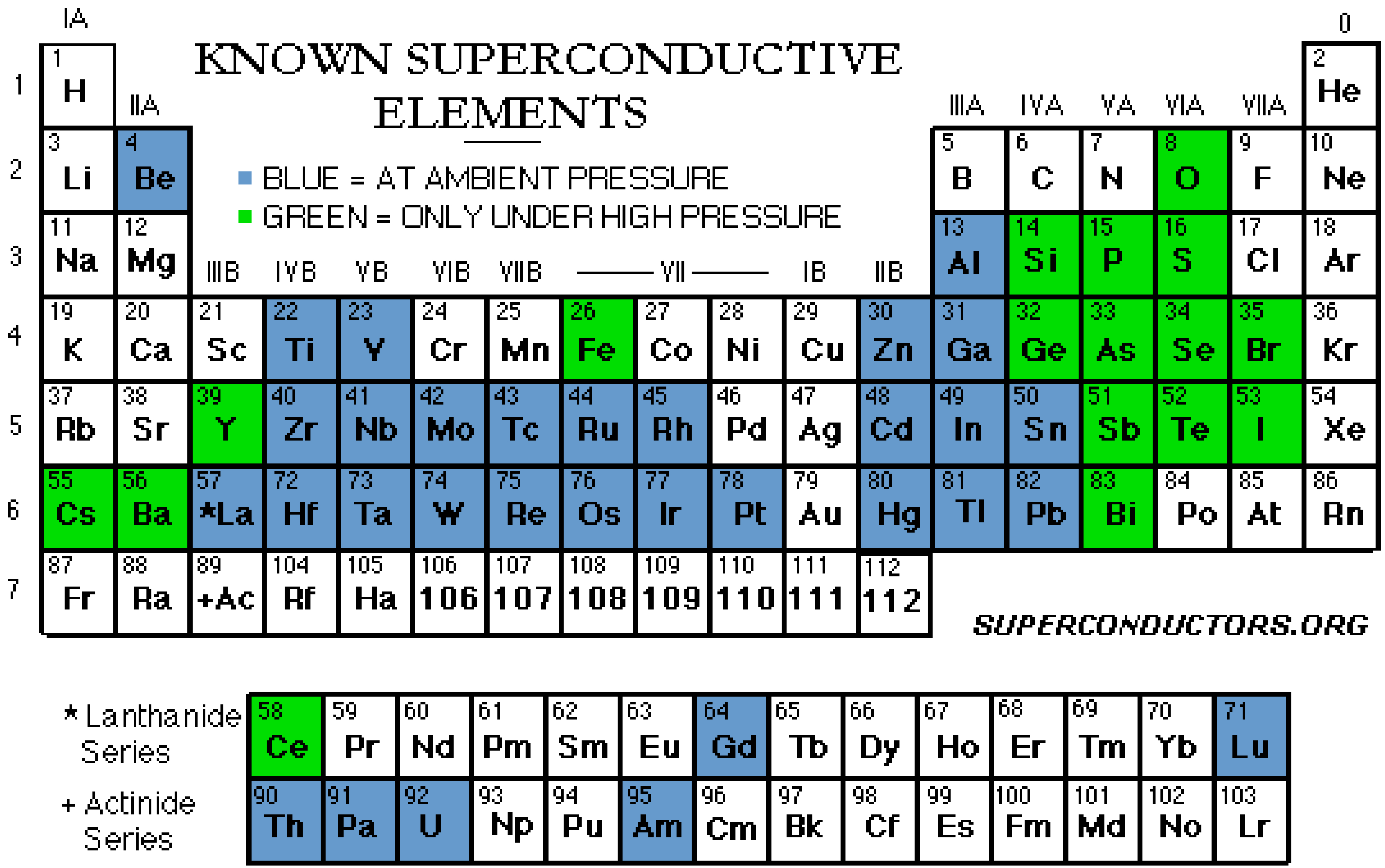
Resistance: $R = 8\rho r / d^2 = 1 \times 10^{-4} \, \Omega$

Time constant: $\tau = L/R = 4 \times 10^{-4}$ s $= 400 \, \mu$s

Measured time constant in a superconductor: $\tau > 100000$ year

Theoretical estimate: $\tau >$ age of the universe

Superconducting elements



KNOWN SUPERCONDUCTIVE ELEMENTS

🟦 BLUE = AT AMBIENT PRESSURE

🟩 GREEN = ONLY UNDER HIGH PRESSURE

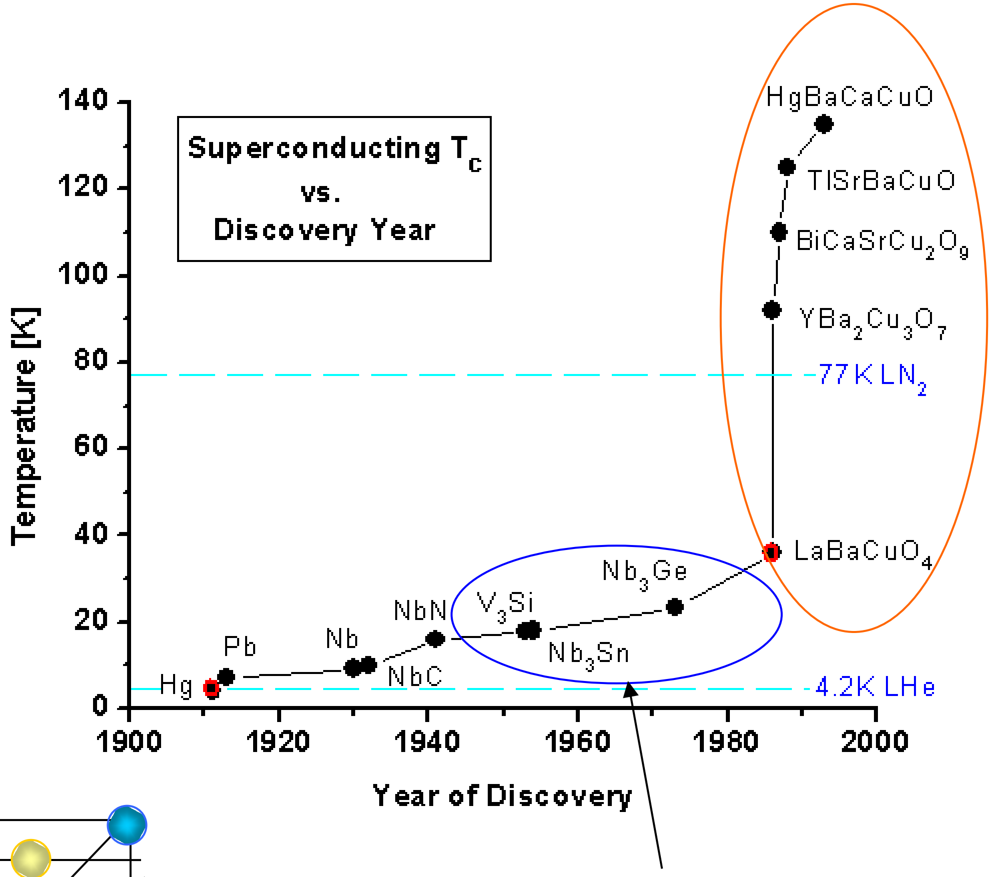
	IA	IIA	IIIB	IVB	VB	VIB	VIIB	VII	VII	VII	IB	IIB	IIIA	IVA	VA	VIA	VIIA	0
1	1 H																	2 He
2	3 Li	🟦 4 Be											5 B	6 C	7 N	🟩 8 O	9 F	10 Ne
3	11 Na	12 Mg											🟦 13 Al	🟩 14 Si	🟩 15 P	🟩 16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	🟦 22 Ti	🟦 23 V	24 Cr	25 Mn	🟩 26 Fe	27 Co	28 Ni	29 Cu	🟦 30 Zn	🟦 31 Ga	🟩 32 Ge	🟩 33 As	🟩 34 Se	🟩 35 Br	36 Kr
5	37 Rb	38 Sr	🟩 39 Y	🟦 40 Zr	🟦 41 Nb	🟦 42 Mo	🟦 43 Tc	🟦 44 Ru	🟦 45 Rh	46 Pd	47 Ag	🟦 48 Cd	🟦 49 In	🟦 50 Sn	🟩 51 Sb	🟩 52 Te	🟩 53 I	54 Xe
6	🟩 55 Cs	🟩 56 Ba	🟦 57 *La	🟦 72 Hf	🟦 73 Ta	🟦 74 W	🟦 75 Re	🟦 76 Os	🟦 77 Ir	🟦 78 Pt	79 Au	🟦 80 Hg	🟦 81 Tl	🟦 82 Pb	🟩 83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 +Ac	104 Rf	105 Ha	106 106	107 107	108 108	109 109	110 110	111 111	112 112						

SUPERCONDUCTORS.ORG

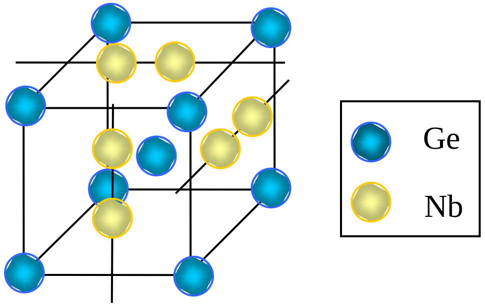
* Lanthanide Series	🟩 58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	🟦 64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	🟦 71 Lu
+ Actinide Series	🟦 90 Th	🟦 91 Pa	🟦 92 U	93 Np	94 Pu	🟦 95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

T_c history

The most important limiting factor of applications is the low critical temerature



High-Tc
superconductors



Superconductors of "A15” structure:
the most important materials
in today’s applications.

The Woodstock of Physics


Hilton New York

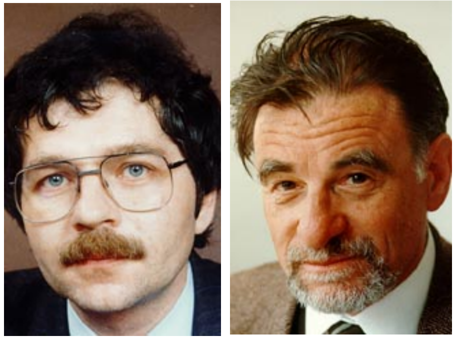
1986: Alex Bednorz and Georg Müller discovers the Cu-O based "high-T_c" superconductors
March 19, 1987, New York:
Conference of high-T_c superconductors

The New York Times, March 20, 1987

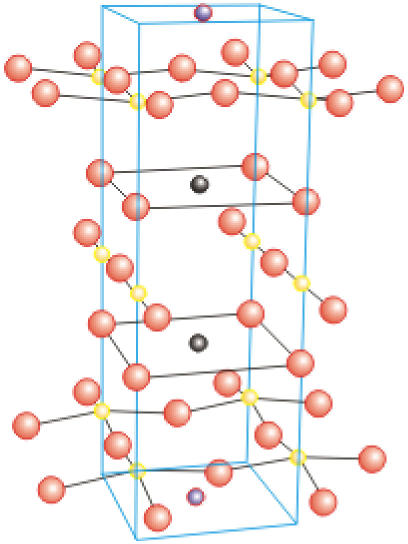
"DISCOVERIES BRING A 'WOODSTOCK' FOR PHYSICS"

"...the most extraordinary scientific meeting in memory"
"It's a phenomenon - there's never been anything like it in the history of physics,"

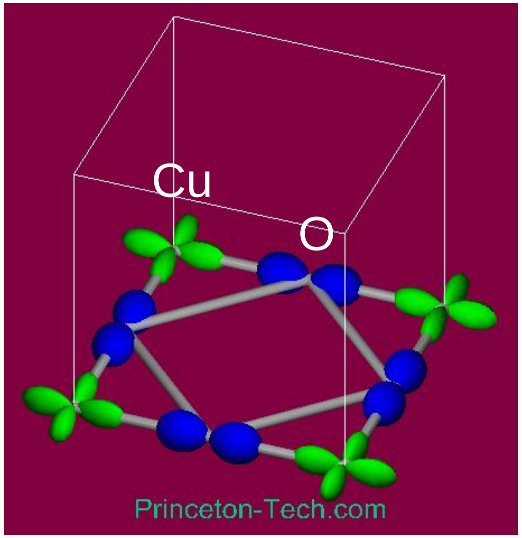
High-temperature superconductors



Nobel Prize:
Alex Bednorz,
Georg Müller,
Nobel Prize, 1986.



$YBa_2Cu_3O_{7-\delta}$



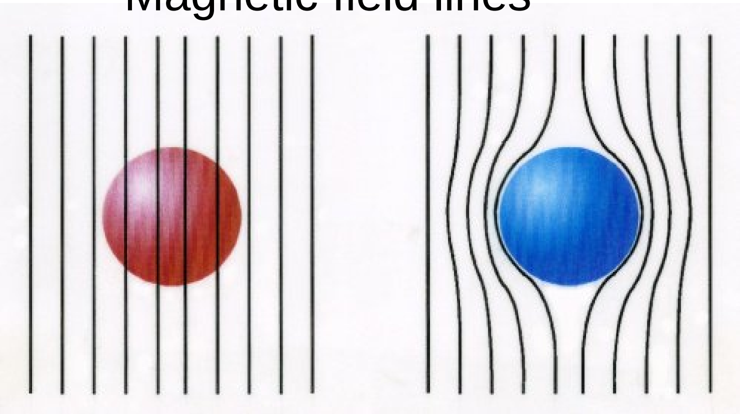
- Superconductivity occurs in CuO_2 planes. Strongly anisotropic layered structure
- Coherent Cooper pairs just like in conventional superconductors
- Origin of the attractive interaction between electrons is not known.

Meissner Effect

Walter Meissner, Robert Ochsenfeld, 1933

The superconductor expels the magnetic field.

Magnetic field lines

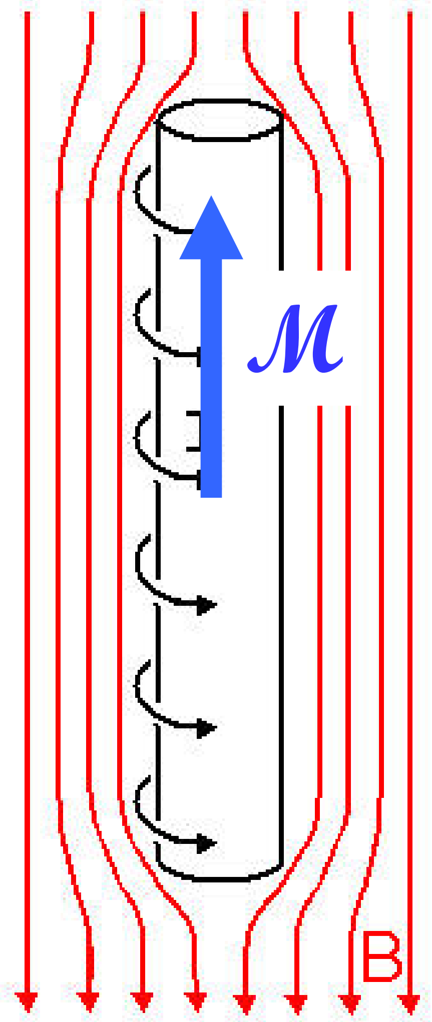


$T > T_c$ $T < T_c$

"normal” metal superconductor

The Meissner effect does not follow from perfect conductivity

Diamagnetism: long, thin cylinder



The external magnetic field is shielded by currents flowing on the surface of the superconductor

Let's introduce the magnetization as $\mathbf{M} = \mathcal{M}/V$,

where $\mathcal{M}$ is the magnetic moment of surface currents

$$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = 0$$

$\mathbf{M} = -\mathbf{H}$ c.f. $\mathbf{M} = \chi\,\mathbf{H}$, where χ is the magnetic susceptibility

$$\Rightarrow \boxed{\chi = -1 \quad \text{"perfect diamagnet"}}$$

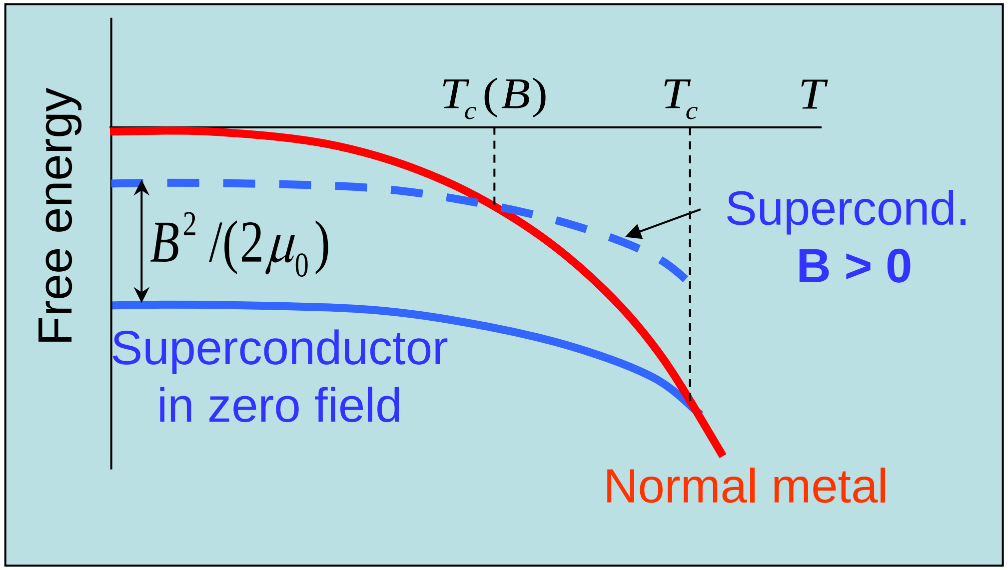
$$\mu_0\mathbf{H} = \mathbf{B} \;\Rightarrow\; \mathbf{M} = -\frac{1}{2\mu_0}\mathbf{B}$$

Variation of energy due to a small variatio of the magnetic field:

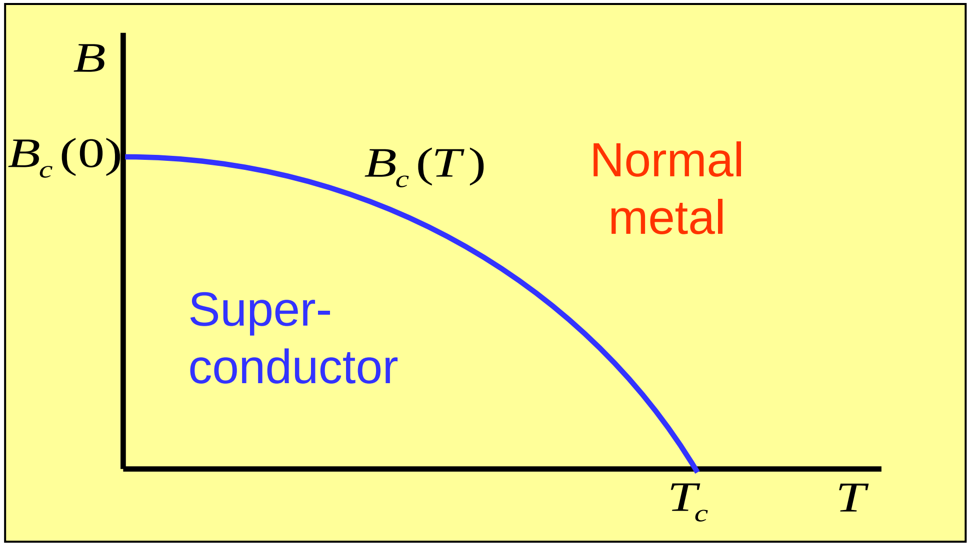
$$dE_{\text{mag}} = -\mathbf{M}\,d\mathbf{B} \;\Rightarrow\; \boxed{E_{\text{mag}} = \frac{B^2}{2\mu_0}}$$

The energy of a superconductor increases in a magnetic field

Temperature – magnetic field phase diagram



Superconductivity disappears above a temperature-dependent magnetic field B_c

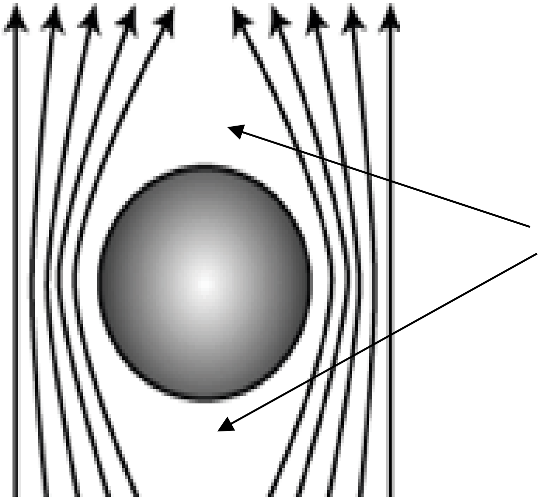


A high magnetic field "destroys" superconductivity.

Pearls of Wisdom

④ There are two types of superconductors: The magnetic field is completely expelled from *Type I* superconductors. In *Type II* superconductors the expulsion of the field is not complete: the field may penetrate the superconductor in the form of magnetic flux bundles called *vortices*. The flux associated with a vortex is the universal *flux quantum*.

Bulky (rather than thin) superconductor



Inhomogeneous magnetic field:
$B_{\text{max}} = 3/2\, B_a$ on the equator

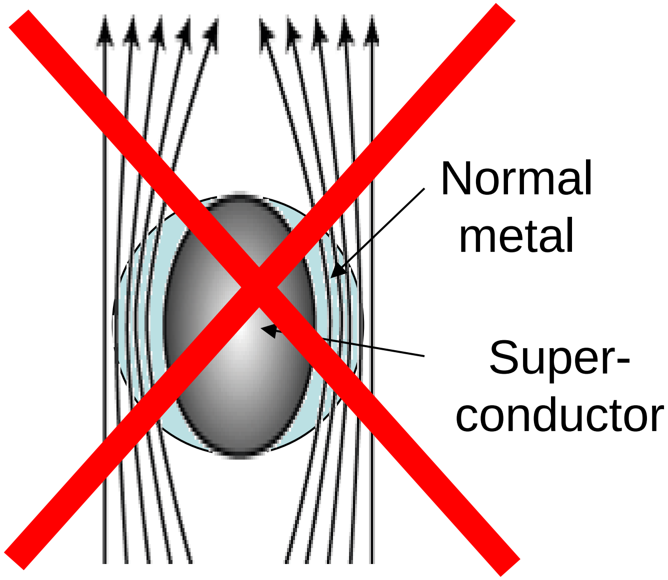
The field is expelled from a volume
larger than the volume of the superconductor

What happens if $B_{\text{max}} = 3/2\, B_a = B_c$?

Homogeneous magnetic field
$\boldsymbol{B_a}$ far from the superconductor

Will the sample transform to normal metal?
Contradiction because for $B < B_c$
the normal metal is not stable!

Hint: at the equator, where the magnetic field is highest (edge effect), a normal domain forms while the rest of the sample remains superconducting.



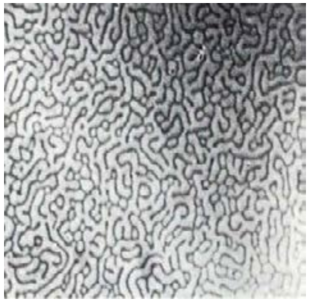
Normal metal

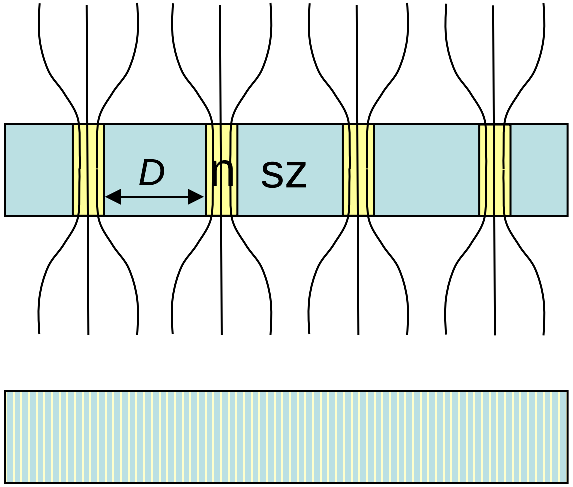
Super-conductor

Not good, because in the normal domain $B < B_c$, which is contradiction.

Solution: Fine mixture of normal (n) and superconduting (sc) domains

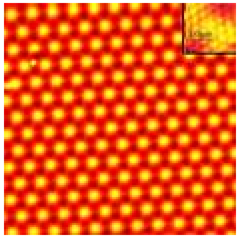
Surface energy *f* of the normal-superconductor domain wall determines the domain structure.





Type-I superconductor:
Positive surface energy $f > 0$

→ Domain wall is energetically unfavorable
→ Finite *D* distance between the domains
"**intermediate state”**.



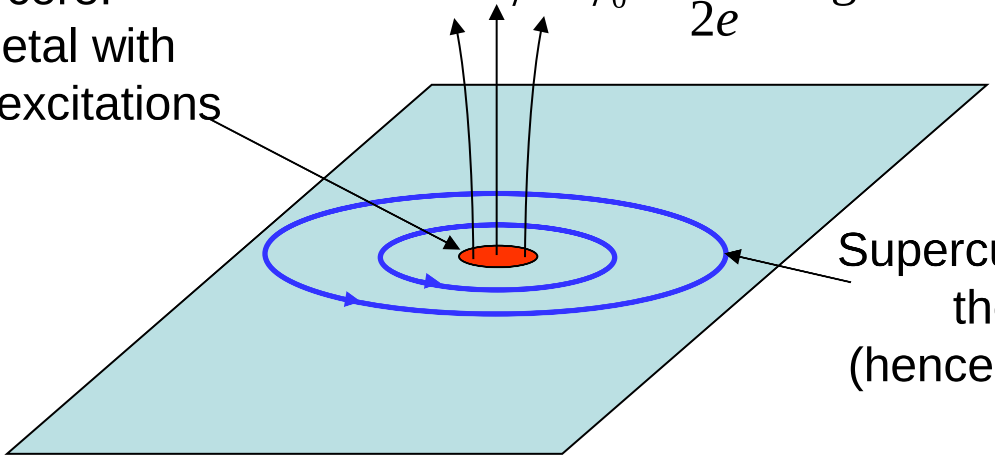
Type-II superconductor
Negative surface energy $f < 0$

→ Domain wall is energetically favorable
→ The multiplication of the domains is limited by a microscopic quantum mechanical effect :

$$\phi_0 = \frac{h}{2e} = 2.07 \cdot 10^{-16}\ \mathrm{Tm}^2$$

The magnetic flux per ϕ_0 one domain is the "flux quantum.”

Vortices as the carriers of the flux quantum in Type-II superconductors



$\phi = \phi_0 = \dfrac{h}{2e}$ magnetic flux

Vortex core:
Normal metal with
low-energy excitations

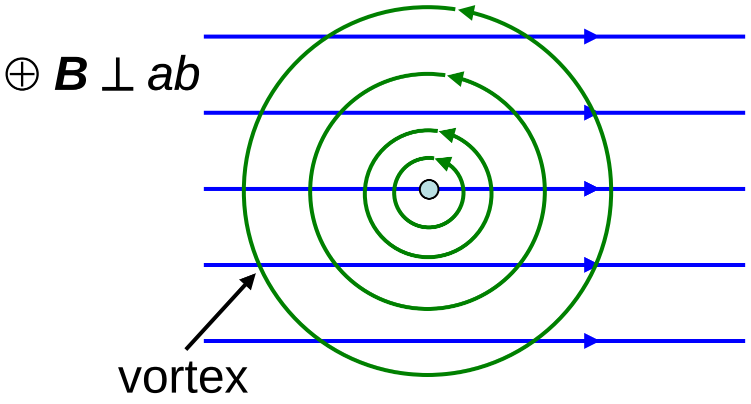
Supercurrent flows around
the vortex core
(hence the name 'vortex')

In Type-II superconductors the critical field is high because the magnetic field is only partly expelled. As a consequence Type-II superconductors are used to generate high magnetic fields.

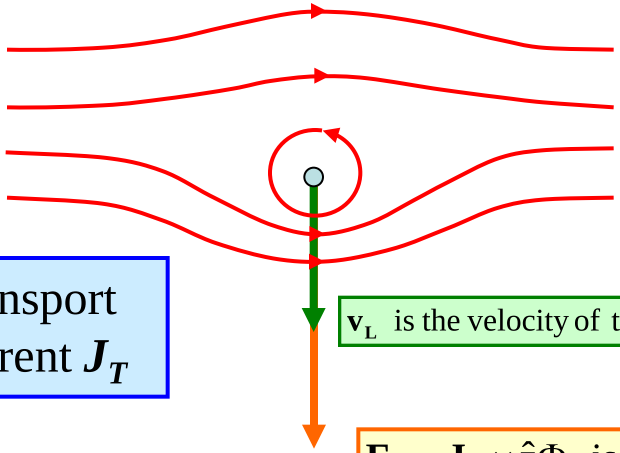
Nobel Prizes
Lev Davidovich Landau,
Vitalii Lazarevich Ginzburg, 1950

Aleksei Alekseievich Abrikosov, 1952

Dissipation in Type-II superconductors



transport current J_T



$\mathbf{v}_\mathrm{L}$ is the velocity of the vortex line

$\mathbf{F}_\mathrm{L} = \mathbf{J}_\mathrm{T} \times \hat{\mathbf{z}}\Phi_0$ is the Lorentz force
($\Phi_0 = h/2e$ is the flux quantum)

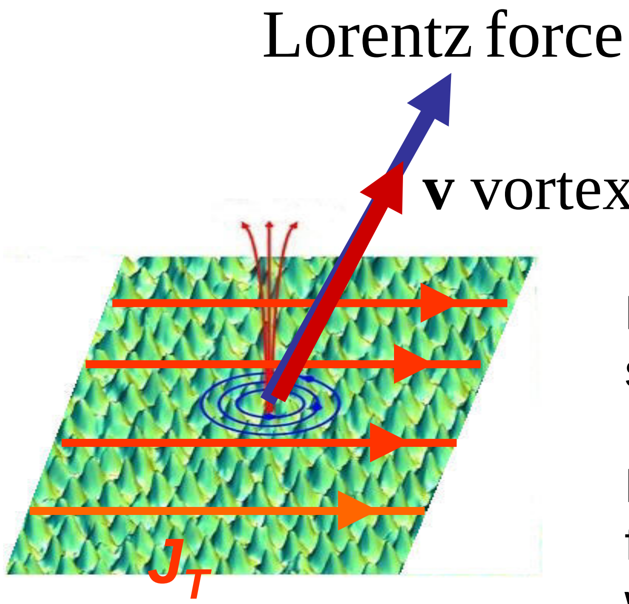
Induced electric field:

$$\mathbf{E} = \mathbf{B} \times \mathbf{v}_\mathrm{L} \parallel \mathbf{J}_\mathrm{T}$$

$\Rightarrow$ dissipation

This is the primary limiting factor
in high-field applications of superconductors

Vortex pinning



How can a Type-II superconductor superconduct?

Because of the defects of the crystal, vortices feel an energy landscape with many valleys which can pin them.

A finite force is needed to set the vortex move.

Because of vortex pinning, zero resistance is observed below a critical current.

Pearls of Wisdom

①

Defining qualities of superconductors

1. Zero resistance.
2. Meissner effect: The magnetic field is expelled from the superconductor.

②

Superconductivity sets on below a material-dependent *critical temperature*. Although there is no theoretical prediction on the highest possible critical temperature, all known superconductors become superconducting well below room temperature, which limits the practical applications of these materials.

③

As a consequence of the Meissner effect, the energy of the superconductor increases with increasing magnetic field and at a *critical magnetic field* the superconductor transforms to a normal metal.

Microscopic theory of superconductivity

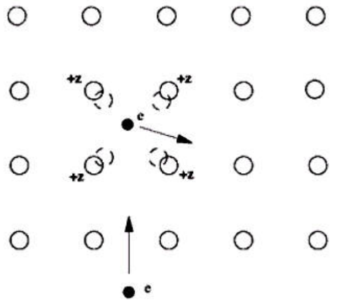
John Bardeen,
Leon Cooper,
Robert Schrieffer,1957

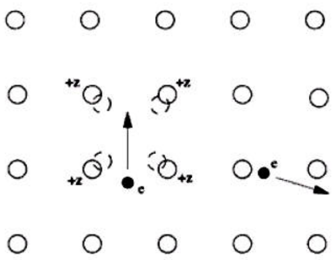
- Because of the "overscreening” of the negative charge of electrons by the positive ions, electrons experience an effective attractive interaction.
  → bound *e-e* pairs: *Cooper pairs*
- In the BCS state the wave functions of Cooper pairs are phase coherent (as though all electrons were described by the same wave function.)
  Macroscopic quantum effects:
  the laws of quantum mechanics are observed on a macroscopic length scale.

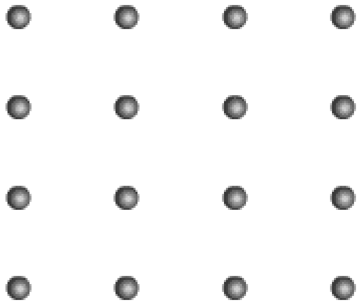
The theory

- Successfully explains perfect conductivity and the Meissner effect
- Makes several predictions on the properties of superconductors which were later verified by experiments.

Cooper pairs







ω

Cooper-modell

The heavy ions "overscreen” the electrons:
Retarded effective attraction between electrons

Origin of perfect conductivity

~~From cern.ch: "When a Cooper pair moves through the lattice, the second electron encounters less resistance, much like a passenger car following a truck on the motorway.~~

From the (fictitious) book entitled *Everything You Always Wanted to Know About Superconductors*:

~~Breaking up Cooper pairs costs finite energy therefore ther is a gap in the excitation spectrum of superconductors. Because of the gap, the electrons do not scatter on the lattice defects and carry current without resistance.~~

Not true because there are superconductors (e.g., high-T_c) in which there is no gap in the excitation spectrum.

Partial truth: the current is not carried by single electron excitations (like in a semiconductor), but by the coherent condensate of Cooper pairs.

"Phase rigidity” (P. W. Anderson): local deformation of the phase of superconducting condensate costs energy. Despite the presence of lattice defects, the phase coherence is maintained throughout the sample.

Pearls of Wisdom

⑤ There are two types of superconductors: The magnetic field is completely expelled from *Type I* superconductors. In *Type II* superconductors the expulsion of the field is not complete: the field may penetrate the superconductor in the form of magnetic flux bundles called *vortices*. The flux associated with a vortex is the universal *flux quantum*.

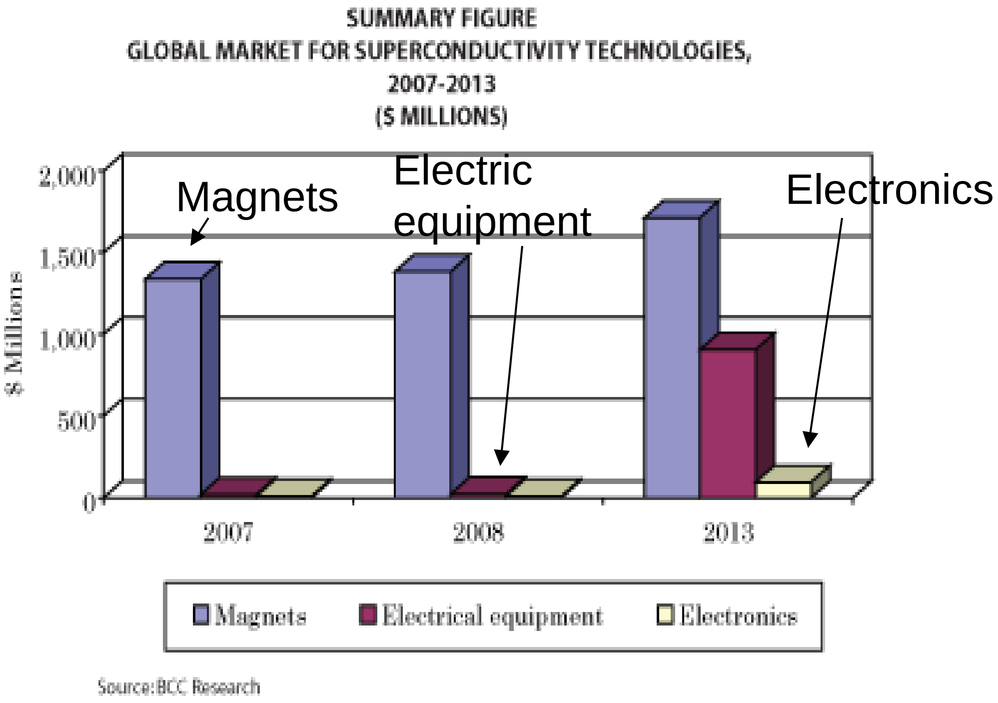
Applications of superconductors

What properties are used?
- Zero electircal resistance
- Quantum coherence

Most important areas today:
- Medical diagnostics
- Chemical/pharmaceutical industry
- Electronics

Projected applications:
- Electric equipment
- Transportation

Based mostly on high-Tc technology being developed.


SUMMARY FIGURE
GLOBAL MARKET FOR SUPERCONDUCTIVITY TECHNOLOGIES, 2007-2013
($ MILLIONS)

Source: BCC Research

Source: BCC Research
http://www.bccresearch.com/report/AVM066A.html

Most important applications

Determination of the structure of organic molecules including biological macromolecules. Sensitivity and resolution increases with increasing magnetic field.

Advantages:

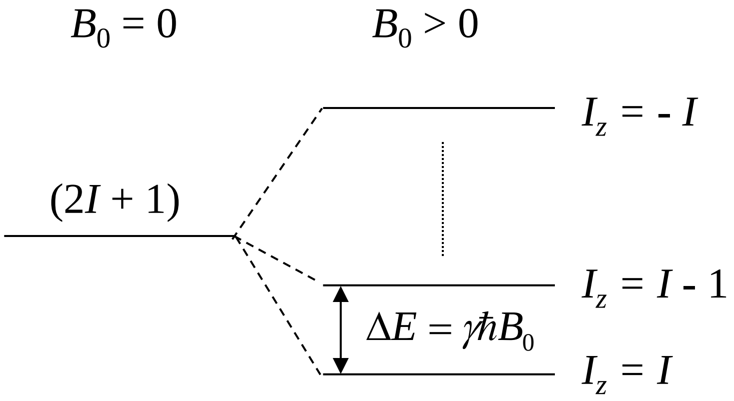
- Structure determination in solution (no need for crystallization)
- Fast and easily automated (well suited for combinatorial chemistry)

Broad range of basic research applications.

World’s first commercial 1000 MHz spectrometer:
Bruker Avance 1000
Magnetic field: 23.5 T

Nuclear Magnetic Resonance (NMR)

Nuclear spin: I ; magnetic moment: $\boldsymbol{\mu} = \gamma \hbar \mathbf{I}$



$B_0 = 0$

$B_0 > 0$

$I_z = -I$

$(2I + 1)$

$I_z = I - 1$

$\Delta E = \gamma\hbar B_0$

$I_z = I$

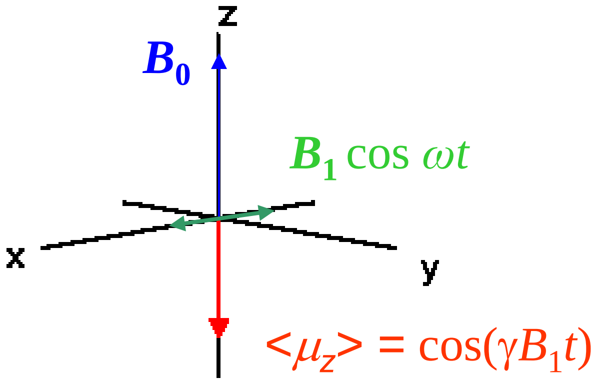
Resonance condition $\omega = \gamma B_0$

Some resonance frequencies
in B_0 = 1 T

Nucleus	$\omega/2\pi$ (MHz)
^{1}H	42.49
^{19}F	40.08
^{13}C	10.71
^{14}N	3.08
^{23}Na	11.27

Felix Bloch and Edward Mills Purcell
Nobel prize in physics 1952
(First succesful NMR experiments in 1945)

How to excite NMR transition



Selection rule for magnetic dipolar transition :

radio frequency field in the $x-y$ plane

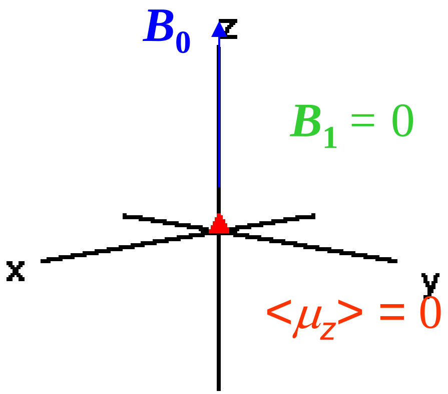
$$I_x = I_+ + iI_-$$

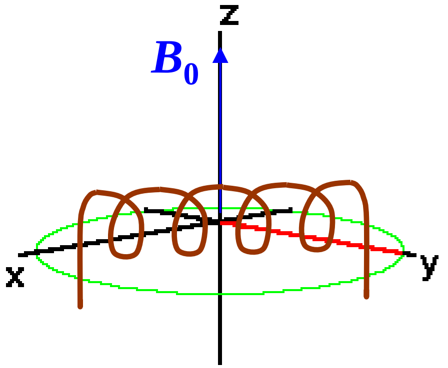
Radio frequency magnetic field :

$$\mathbf{B}_1 = \hat{\mathbf{x}} B_1 \cos \omega t$$

$$\langle \mu_z \rangle = \mu_0 \cos \omega t$$

Principle of Fourier Transform NMR spectrometer

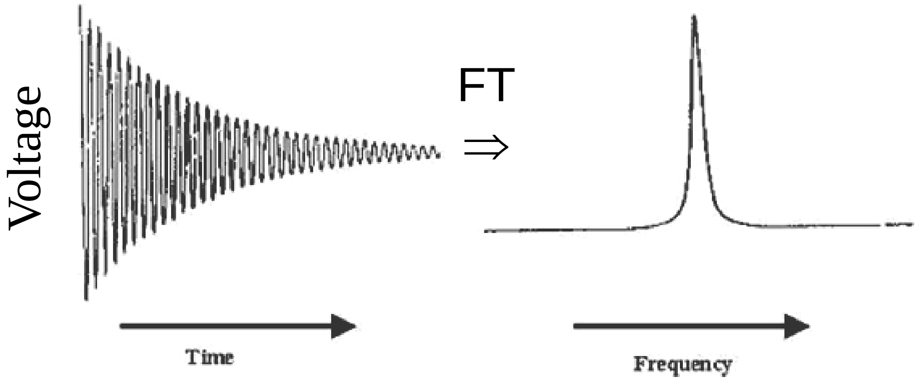




$$\langle \mu_x \rangle = \mu_0 \sin \omega t$$

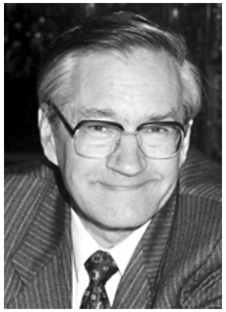
$$\langle \mu_y \rangle = \mu_0 \cos \omega t$$

Principle of FT-NMR spectrometer



Free Induction Decay

NMR spectrum



Richard R. Nernst, Nobel prize in chemistry 1991
"for his contributions to the development of the methodology of high resolution nuclear magnetic resonance (NMR) spectroscopy"

Why to measure NMR

$\omega = \gamma B_0$ Measure γ ???

$\omega = \gamma B_{\text{local}}$ Measure the local magnetic field at the nucleus!

B_{local} is influenced by

- neighboring nuclei (spin-spin interaction)
- neighboring electrons (spin and orbital magnetic moment)
- orientation of the crystal with respect to the magnetic field
- motion of the nucleus

One can learn about:

- the structure of organic molecules
- electronic properties of solids
- atomic motion in molecules and crystals
- magnetic fluctuations and order
- etc.

Alternative: resistive electromagnet

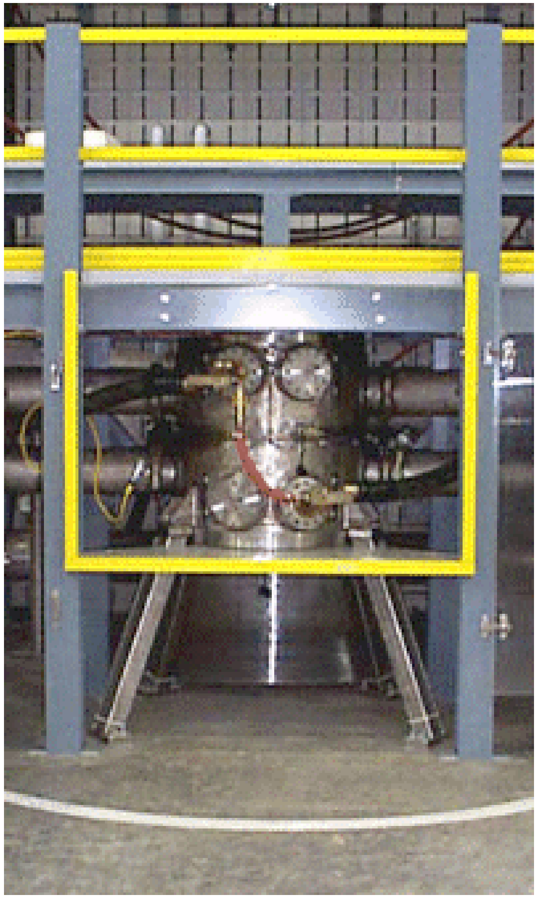
Record: 25 T (1066 MHz), National High Magnetic Field Laboratory (NHMFL), Tallahassee, Florida, USA

- Very expensive operation (35 MW + water cooling) ⇒ only for basic research
- Poor field stability because of fluctuations of the current and cooling water (~ 3 ppm with flux stabilizer insert) ⇒ not suitable for high-resolution application (10^{-8}).

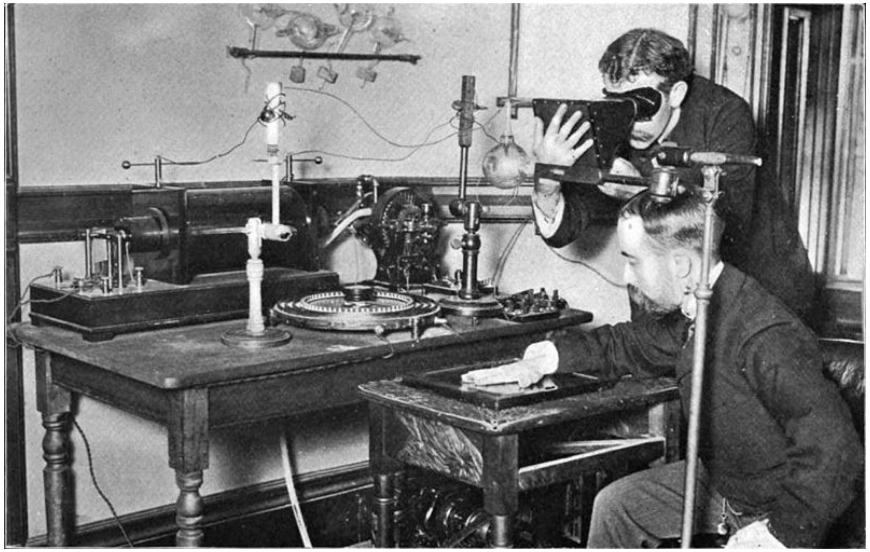
Hybride technology: resistive insert in superconducting solenoid

NHMFL 45-T hybride magnet:

34 T resistive magnet in 11 T superconducting solenoid (not for NMR)



X-ray radiography

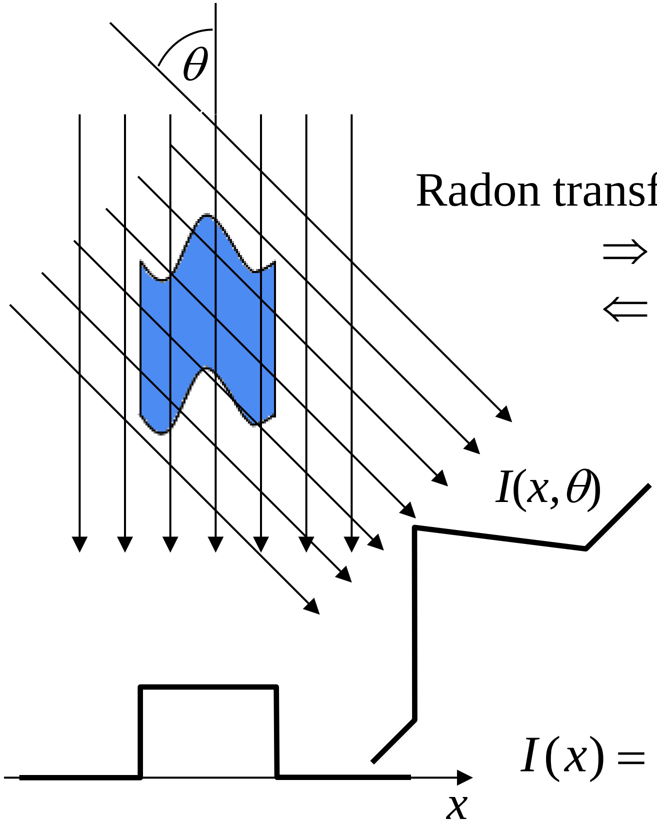


Taking an x-ray image in the late 1800’s

Wilhelm Conrad Röntgen,
The first Nobel-prize in physics in 1901

An X-ray picture (radiograph) taken by Röntgen of Albert von Kölliker’s hand on 23 January 1896

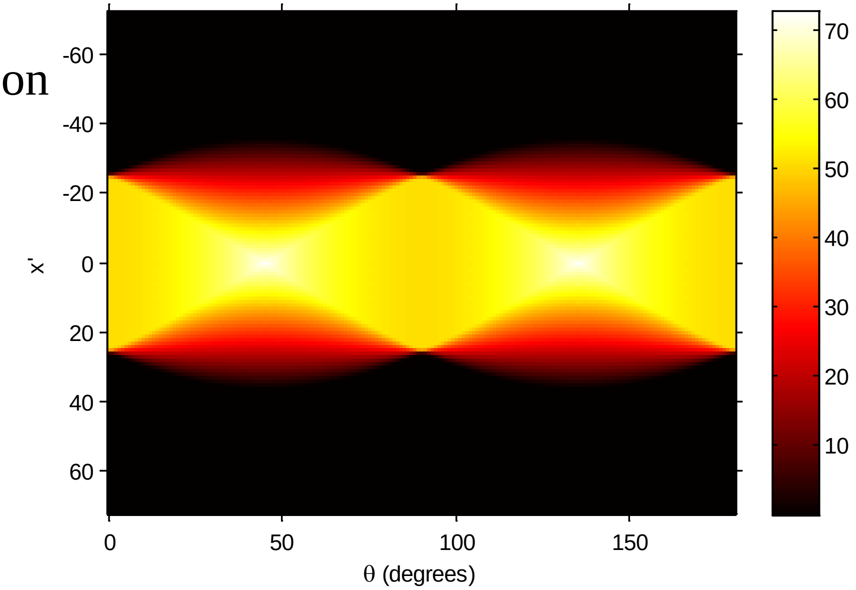
Computed Tomography



Radon transformation

⇒

⇐



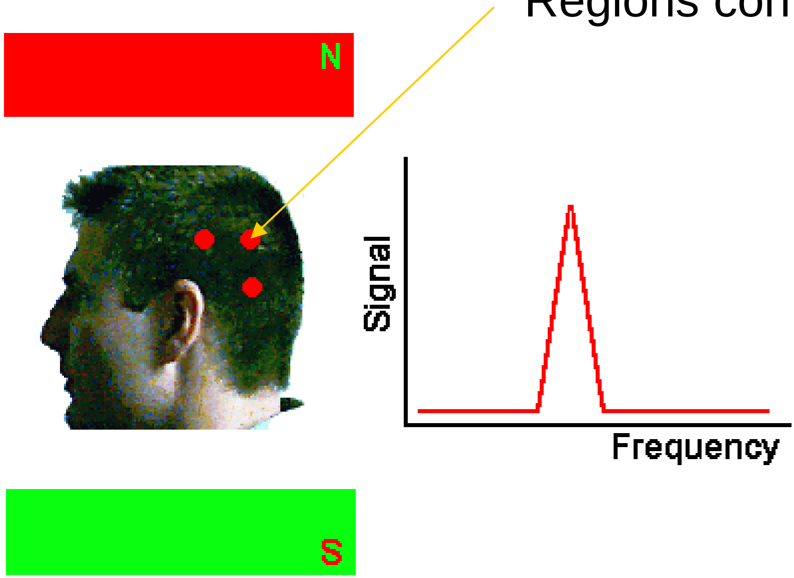
$$I(x) = \int \rho(x)^2 \, dx \quad \text{with } \rho(x) \text{ the electron density}$$

Allan M. Cormack, Godfrey N. Hounsfield
The Nobel Prize in Physiology or Medicine 1979

Magnetic Resonance Imaging (MRI)

(Source: *The Basics of MRI* , by Joseph P. Hornak, http://www.cis.rit.edu/htbooks/mri/)

Wave length of radio frequency radiation in NMR ~ 1 m >> required resolution (???)



Regions containing the NMR-active nuclei

Frequency encoding

Trick: use a magnetic field gradient!

$$B = B_0 + xG_x$$

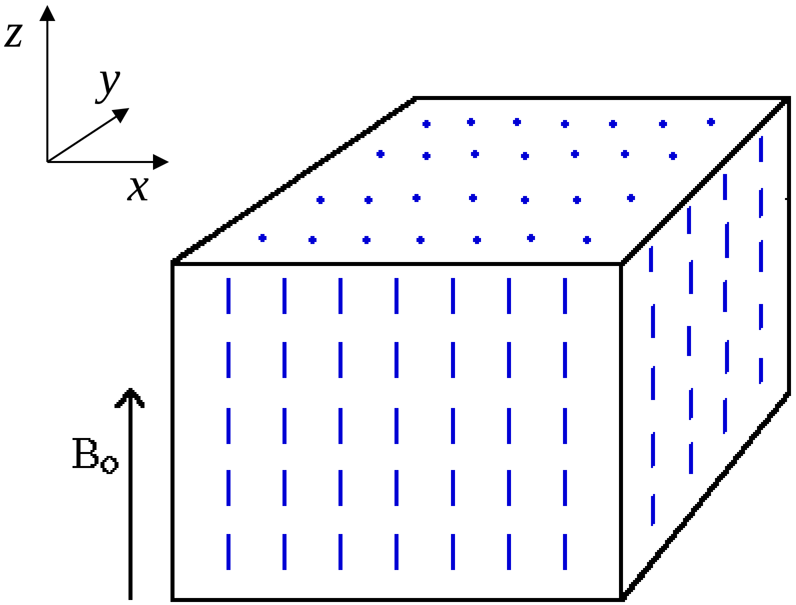
$$\omega = \gamma(B_0 + xG_x) = \omega_0 + \gamma x G_x$$

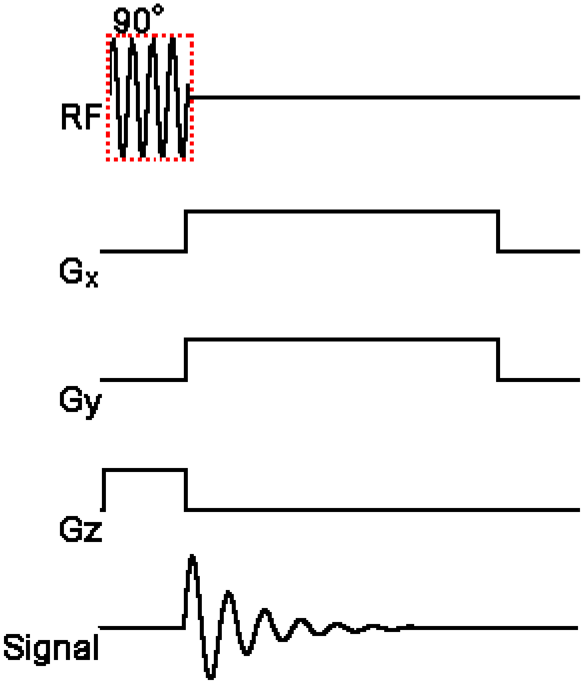
$$x = (\omega - \omega_0)/(\gamma G_x)$$

Rotate the gradient in the *x-y* plane and reconstruct the spatial distribution of the active nuclei using Radon transformation.
("Back projection technique”)

P.C. Lauterbur
Image formation by induced local interactions: examples employing nuclear magnetic resonance
Nature **242:**190-191 (1973)

Spatial resolution in the *z* direction: Slice selection





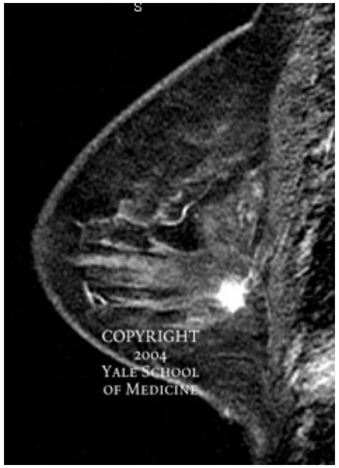
Paul C. Lauterbur, Sir Peter Mansfield
Nobel Prize in Physiology or Medicine 2003
"for their discoveries concerning
magnetic resonance imaging"

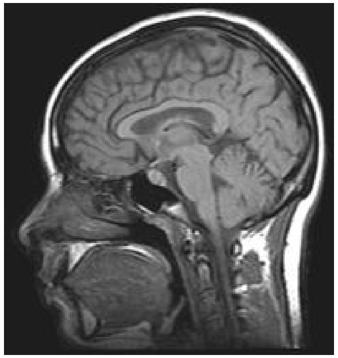
Application of MRI in medical diagnostics

High magnetic field + large diameter → resistive technology is not practical

Advantages:
- Excellent contrast in soft tissue
- Typically 50-100 MHz
  $\Rightarrow$ little if any health risk

Disadvantages:
- Expensive
- Takes long time



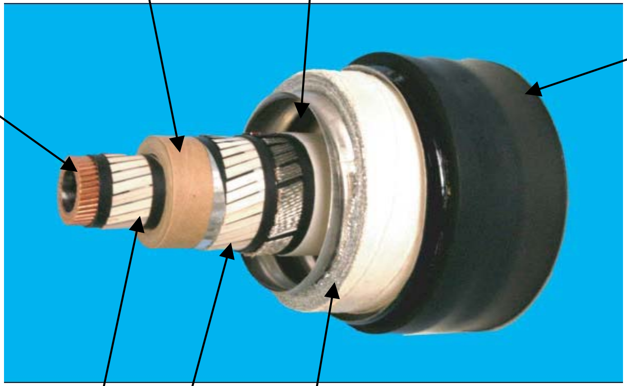


fMRI: functional magnetic resonance imaging:
Measures the metabolic activity of different brain regions

High-Tc power transmission cable

138 kV, 574 MW April 30, 2008, Long Island, USA

Prototype high-Tc cable for high-power applications



Electric
isolation

Cooling by liquid helium
(cooling stations a few kilometers away)

Copper bore

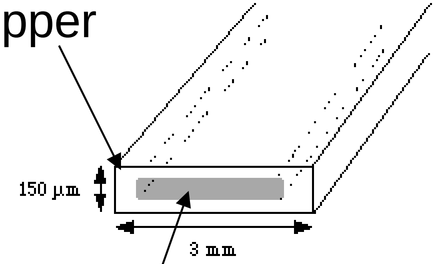
Ø 133 mm

Superconducting tape
Ceramic BSCCO
in silver matrix

Heat insulation:
vacuum +
"astronaut suit”

Superconducting
shield

Silver plated
copper



$Bi_2Sr_2CaCu_2O_8$
ceramic
superconductor
$I_c = 200$ A
$J_c = 50$ kA/cm^2

Japanese Ministry of Economics and Industry, 2004.

Magnetic levitation (maglev)

Shanghai, 2003
30 km, 430 km/h
(8 minutes)
Made by
ThyssenKrupp

Bon voyage!

THE END