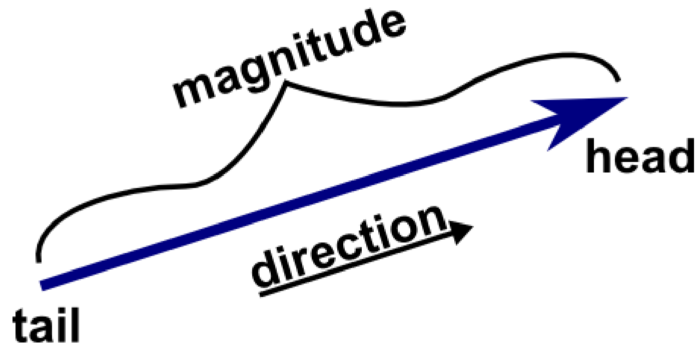
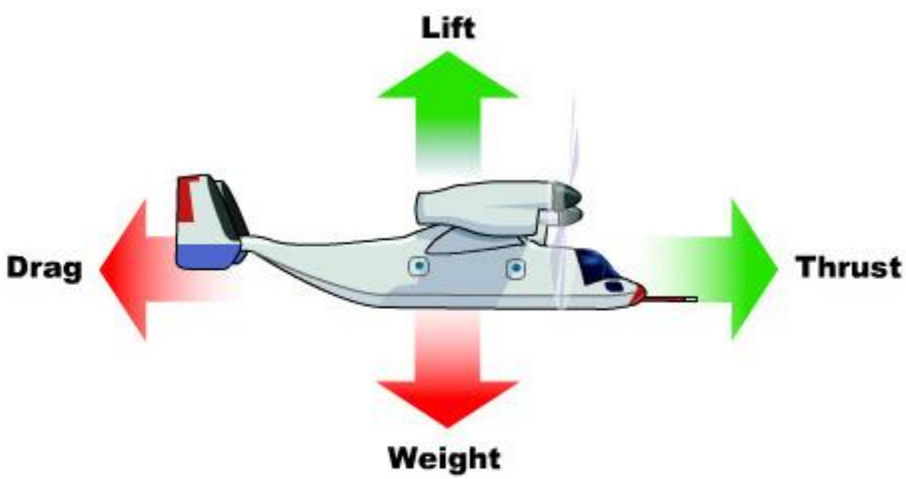
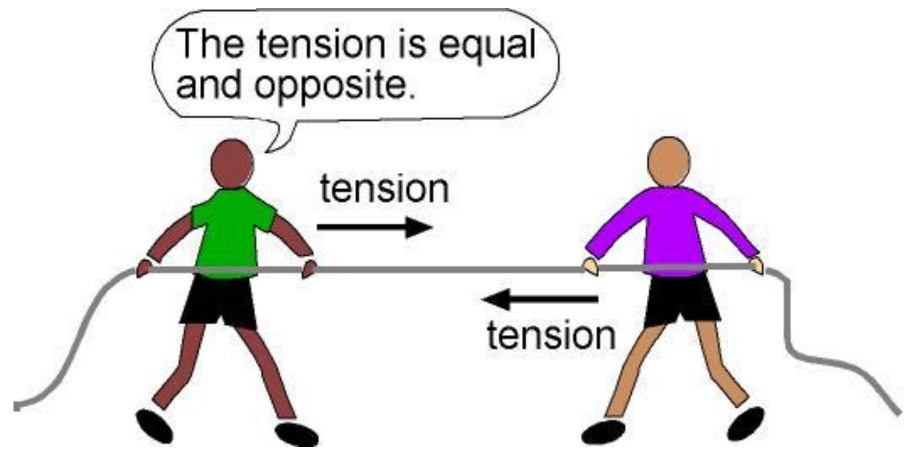


Vectors





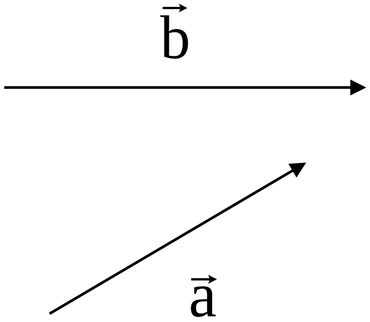
$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

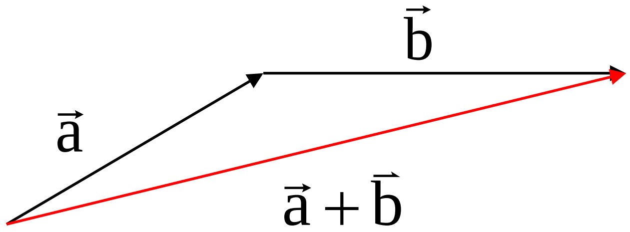
Electric force *Magnetic force*

Vectors

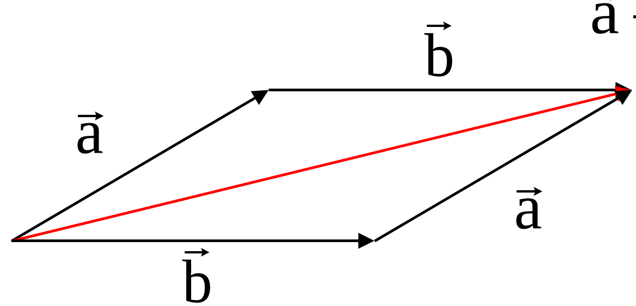
Vector geometry

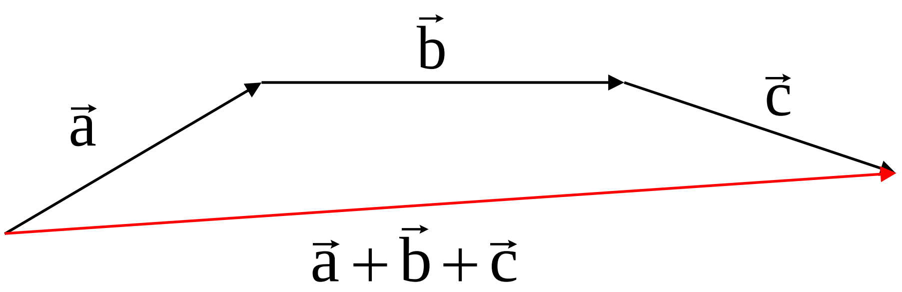
Addition of vectors:

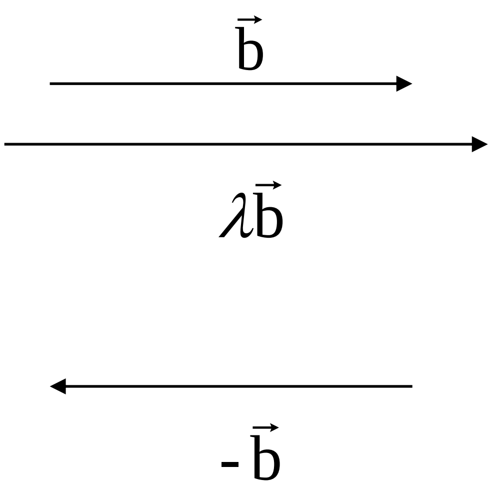




$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$



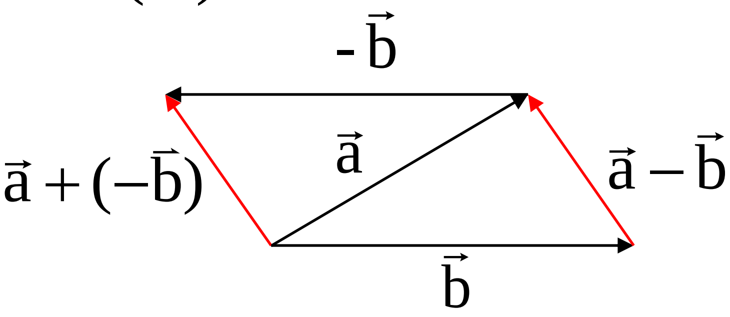


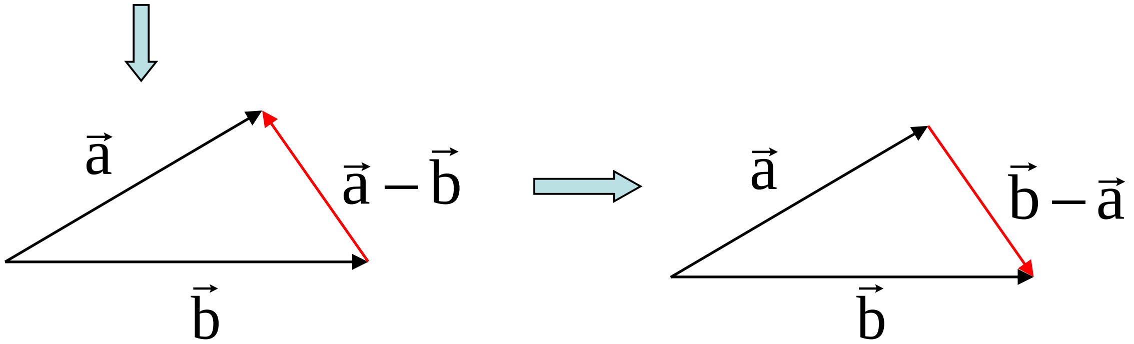


Subtaction of vectors

$\vec{a} - \vec{b} = ?$

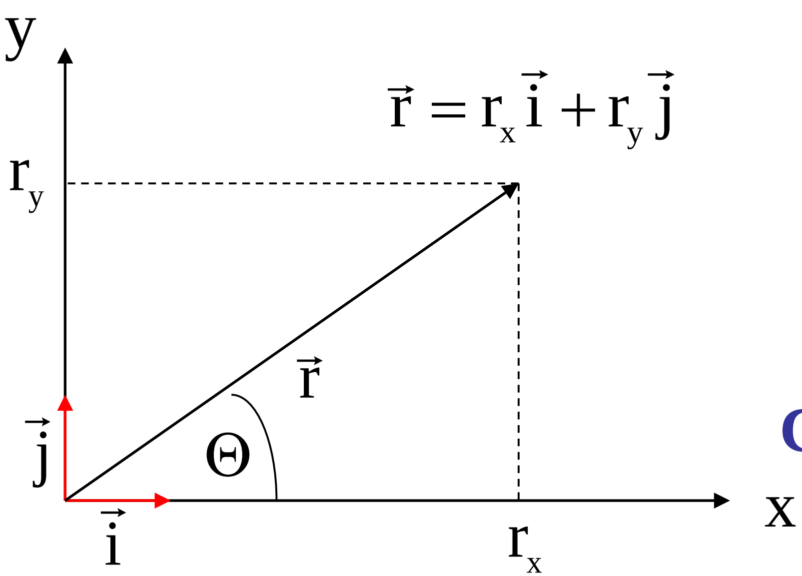
$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$





Coordinates & unit vectors

$$\vec{r} = r_x \vec{i} + r_y \vec{j}$$



$$|\vec{r}| = r = \sqrt{r_x^2 + r_y^2}$$

$$\tan\Theta = \frac{r_y}{r_x}$$

Cartesian coordinates: r_x & r_y

$$|\vec{i}| = |\vec{j}| = 1$$

$$\vec{r} = (r_x, r_y)$$

Polar coordinates : r & Θ

$$r_x = r \cdot \cos\Theta$$

$$r_y = r \cdot \sin\Theta$$

$$\vec{r} = (r, \Theta)$$

Vector algebra

$$\vec{a} = a_x \vec{i} + a_y \vec{j} \qquad \vec{b} = b_x \vec{i} + b_y \vec{j} \qquad \vec{a} + \vec{b} = ?$$

$$\vec{a} + \vec{b} = (a_x + b_x)\vec{i} + (a_y + b_y)\vec{j} = \vec{d}$$

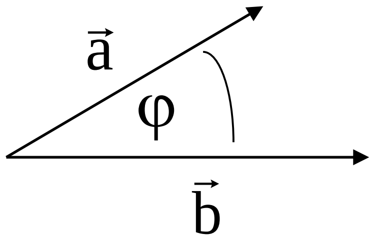
d_x d_y

$$\vec{a} - \vec{b} = (a_x - b_x)\vec{i} + (a_y - b_y)\vec{j} = \vec{c}$$

c_x c_y

$$\vec{a} + \vec{b} + \vec{c} + \ldots = (a_x + b_x + c_x + \ldots)\vec{i} + (a_y + b_y + c_y + \ldots)\vec{j}$$

Scalar product



Def.: $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\varphi$

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = 1 \quad \& \quad \vec{i}\cdot\vec{j} = 0$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} \qquad \vec{b} = b_x\vec{i} + b_y\vec{j} \qquad \vec{a}\cdot\vec{b} = ?$$

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$$

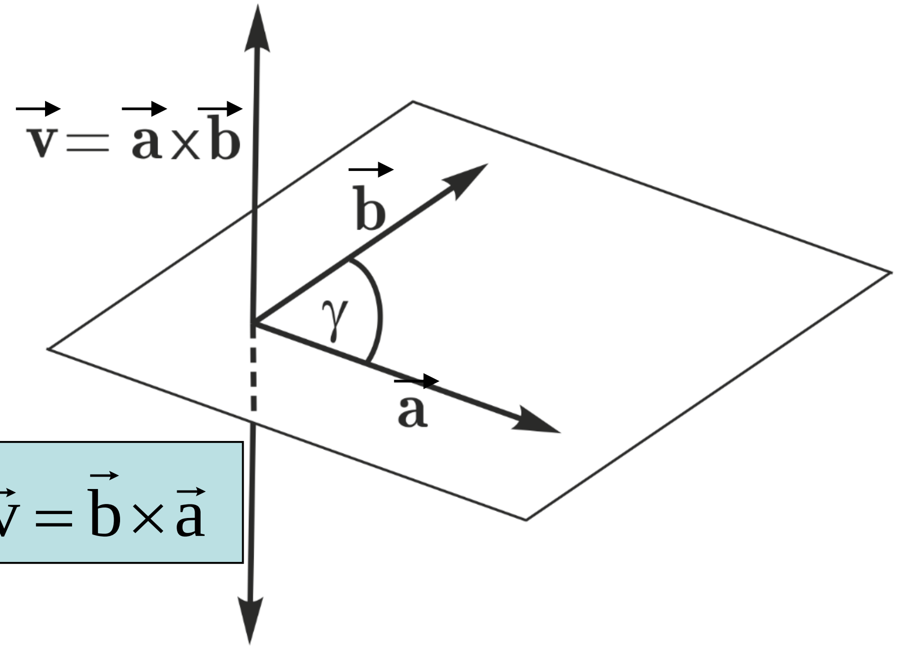
Superposition

Example: work

$$W = \vec{F}\cdot\vec{s}$$

Vector product (cross product)

$$\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\cdot\left|\vec{b}\right|\sin\gamma$$

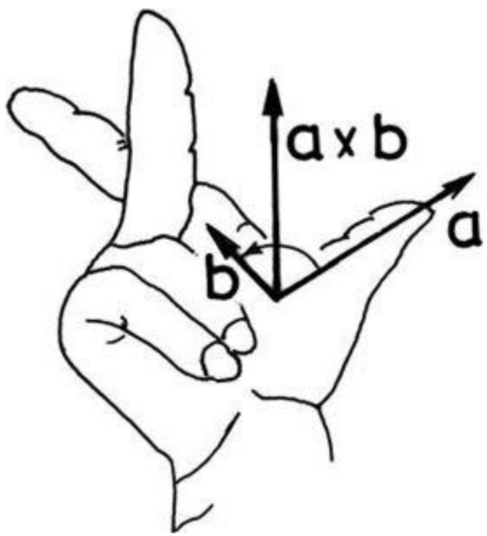


$$-\vec{v} = \vec{b}\times\vec{a}$$

$$\vec{i}\times\vec{j} = \vec{k} \quad \& \quad \vec{j}\times\vec{k} = \vec{i}$$

& $\vec{k}\times\vec{i} = \vec{j}$ **, but:** $\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0$

Right hands rule:



Example: torque

$$\vec{\tau} = \vec{r}\times\vec{F}$$

Components of vector product

Determinant

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=?$$

$$\vec{a}\times\vec{b}=\left(a_y b_z - a_z b_y\right)\cdot\vec{i}+\left(a_z b_x - a_x b_z\right)\cdot\vec{j}+\left(a_x b_y - a_y b_x\right)\cdot\vec{k}$$

⇓

Superposition

II. Trigonometry

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\mathrm{tg}(\alpha \pm \beta) = \frac{\mathrm{tg}\alpha \pm \mathrm{tg}\beta}{1 \mp \mathrm{tg}\alpha\,\mathrm{tg}\beta}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

H.W.:

$$\mathrm{tg}(2\alpha) = ?$$

$$\cos(3\alpha) = ?$$

$$\cos\left(\frac{\alpha}{2}\right) = ?$$